

To appear in Cedillo, T. & Isoda, M. (In press). Guidebook in Mathematics for Teacher Education, Mexico: Ministry of Education (SEP). (written in Spanish Prepared for the Work Shop written in Spanish.

How Teachers Develop Children's Mathematical Thinking in Their Classroom with Their Textbooks: What did Japanese teachers achieve through lesson study?

Masami Isoda
CRICED, University of Tsukuba, Japan

Abstract

Teaching is a cultural activity. Thus Lesson Study is a kind of enculturation based in the community. The challenge for the teacher is to learn from the classroom teaching to improve and to innovate. Through the Lesson Study activities, teachers share the materials and methods of teaching, improve the curriculum and textbooks, and theorize practical wisdom to build their own practical theories for teaching and learning. This article is prepared for the workshop which focuses on ways to develop mathematical thinking in the classroom using well-thought teaching sequence. The illustrated practices itself are the result of the 140 years of Lesson Studies and have become well known in the last decade since the curriculum reform began in 1998 in Japan. In this workshop, the participants will be given the opportunity to interpret the videos and analyze the textbooks with a view of learning how to develop mathematical thinking in the classroom. To learn about the textbooks in the workshop, this paper will begin with questions and explain the materials found in the textbook.

Let's see the video of the Teacher, Takao Seiyama, by JICA

The video of the Elementary School attached to the University of Tsukuba was developed by JICA. It has two parts. The first part explains the Japanese Teacher Training System and Lesson Studies (11 min. 52 min.). The second part explains the implementation of lesson study as the method of classroom innovation (from 11:42 to the end). The second part focuses on four aspects: Lesson Planning (from 11:53 to 16:48); the Lesson Presentation/Observation (from 16:49 to 20:35); Lesson Reflection (from 20:36 to 26:04); and Open Class (from 26:05 to 28:30) with final remarks until the end.

Here, we watch the video to learn how Japanese teachers develop children's mathematical thinking in classroom. As for the workshop, the first three aspects

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will be focused with Open Questions and Suggested Explanations.

Lesson Planning

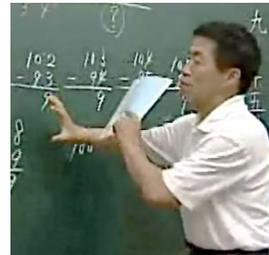
Let's see the video¹ by the teacher, Mr. Takao Seiyama, at Second Grade on the part of Lesson Planning.

Questions for the lesson planning

Q1) In the JICA video (video_e.wmv), please see the video from 11:53 (11 min. 53 sec.) to 13:34. What is written on the blackboard? Which way do you think he wrote on the board; from the left to the right or the right to the left? Please copy what is written on the board in your notebook and explain the ways of reasoning using the term 'inductive reasoning'.

Q2) Please watch the video from 13:34 to 15:03. Please explain the significance of his task using the terms 'Mathematics is the science of patterns.'

Q3) Please watch the video from 15:03 to 16:46. At 15:38, Mr. Hosomizu shows the gesture by crossing his hands. What does it mean? At 16:07-16:34, Mr.



Seiyama explains his plan of the teaching sequence and final outcome. From his explanation, please explain his tentative teaching plan and the objective of teaching at this moment.

Q4) At the beginning, the children were asked about their impressions of their teacher. Why do children love his class? Can you explain it from his ways of this preparation?

Explanation for the lesson planning

Mr. Seiyama tries to develop the class so that the children can find the pattern by themselves. The children love his class. His planned teaching sequence considers the cases 1, 2, 3,..., inductively, and if children are able to explain the case of 9, then he knows he is successful in his teaching. He wants to listen to the children, so he would wait for the children's findings. Mr. Hosomizu showed the gesture by cross his hands. If you understand the meaning of his gesture, you will understand what should happen in the class when looking for a pattern.

¹ In this explanation, the time line is used when you opened the 'video_en.wmv' directly by windows media player. The video can be seen at the normal school within Mexico with the permission of JICA. The same video can be also seen such as the following:
<http://www.youtube.com/watch?v=qSlSMYojlzw>

Lesson Presentation/ Observation

Let's see the part on Lesson Presentation/ Observation.

Questions for the observation

- 1) In the JICA video (video_e.wmv), please see from 16:49 (16 min. 49 sec.) to 17:37. Please explain the way of problem posing by the teacher especially at 17:27, and explain his intention and objective of his teaching activity from the way he posed the questions.
- 2) Please see from 17:38 to 18:08. He began the class using the case of 3, is it the same as his original plan? Please explain why he began the class using the case of 3 in relation to 18:00.
- 3) Please see from 18:08 to 19:01. At 18:34, a child said 'Teacher Seiyama, there's something wrong with them.' Why did the child say 'something is wrong' and did many children accept this?
- 4) Please see from 19:01 to 19:40. At 20:14, after the case of 5, a child said 'So if ' in the case of 1 from herself. Why did the child say say so.
- 5) At the beginning, the children were interviewed about their the impression of Mr. Seiyama. Why do the children love Mr. Seiyama? Can you explain it from the lesson itself?



Explanation for the observation

Mr. Seiyama changed his teaching sequence from the case of 3. What is amazing about the class is that the children discuss about the ways of finding patterns by re-ordering and developing other examples by themselves based on the planned sequence of teaching which began with the case of 3. Why is it that they were able to do this? It will be explained in the textbooks, later.

Now, you may understand the meaning of the gesture by Mr. Hosomizu. From his gesture, Mr. Seiyama and Mr. Hosomizu knew that ordering is the key to finding the pattern. And from the class, we also understand that the children also already knew the importance of ordering in order to find the pattern.

The children enjoyed the mathematics class and appreciate Mr. Seiyama who has enabled them to progress and develop their own world of mathematics.

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Lesson Reflection

Let's look at the part on Lesson Reflection

Questions

Q1) In the JICA video (video_e.wmv), please watch from 20:14 to 22:30. Let's discuss what is the difficulty in the task and which is better; to begin from the case of the number 3 or the number 1?

Q2) Please watch from 22:30 to 23:26. Mr. Tsubota gave a strong reaction on his objective of teaching instead of Mr. Seiyama's objective. What is Mr. Tsubota's objective of teaching?

Q3) At the beginning of the video, the children interviewed gave a good impression of Mr. Seiyama. Why do the children love Mr. Seiyama? Can you explain it from this discussion?

Explanation

There are two strong and meaningful reactions. The first reaction is from Mr. Hosomizu. He agreed with Mr. Seiyama's objective to find the pattern. Mr. Hosomizu explained that to begin from the case of 1 is easier and Mr. Seiyama explained that the case of 3 is better because of the difficulty. Another reason is that the case of 3 is the first case that allows a discussion about the ordering. To focus the discussion on ordering, the cases of 1 and 2 are inappropriate.

The second reaction is from Mr. Tsubota. He did not share Mr. Seiyama's objective. Mr. Tsubota wanted to focus on the objective 'if you want to teach how to find the pattern.' To teach 'how to find,' teachers must set the problematic situation on ordering.

Anyway, children in this classroom do not need to learn how to order because they had already learned it in the previous class. It is deeply related with the textbook. It will be explained in the next page with Japanese textbooks.

Why did the children love Mr. Seiyama's teaching? He developed the problematic that enabled the children to use what they had learned before to develop new ideas. It is cool, enthusiastic and fantastic.

The Power of Sequence of Teaching for Learning How to Learn: Structure of Japanese Mathematics Textbooks for Developing Children's Mathematical Thinking

Why do you think Mr. Seiyama can expect the children to be able to find the

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pattern even if he began from the case of 3? Here, we would like to explain the reason from what the children had learned before and the sequence of the textbooks.

Question

Q1) One year before the filming of the JICA video, when the children were in first grade, he taught the arrangement of the cards for addition using the following sequence to the same children. The cards for addition have the addition expressions on the front side and their answers on the reverse side. He made questions as follows:



'Yesterday, we've practised calculation using cards , right?'



'Today, we will line up the calculation cards'



'This is 2. Can you tell me the expression for this?'

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'What is the expression written behind the card 4?'

At the end of this teaching sequence, the children answered that behind the '4' card are '2+2,' '3+1,' and '1+3.' The children's answered in this order. '1+3' was last.

Can you say that the children considered the order of cards based on the pattern at

this moment?

Q2) Then, he asked children which one must come into the card 4 from those three, '2+2,' '3+1,' and '1+3.' Then, children began to recognize the order of cards. Children gave three reasons why '1+3' should come into the card 4. One reason is by looking at the arrangement of the cards vertically. This explains that the augends is 1,1,1 and the addend is 1, 2. Then it should be 3. Please find another two explanations how the children explained the arrangements of the cards 1+1, 1+2 and 2+1 on the board.



Hint: They used the terms, 'horizontally' and 'symmetrically.'



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3) Similar questions and answers were repeated until the “6” cards, $1+5$, $2+4$, $3+3$ and $4+2$. However one card ($5+1$) is missing. And then, he gave a different type of question which asked the children to arrange the cards which do not have augends or addend in the expression.

Even though they have blanks, the children can still arrange. Why? After the children arranged the cards even though they have blanks, Mr. Seiyama asked, “Why are you able to do this.” Explain the children’s answers.



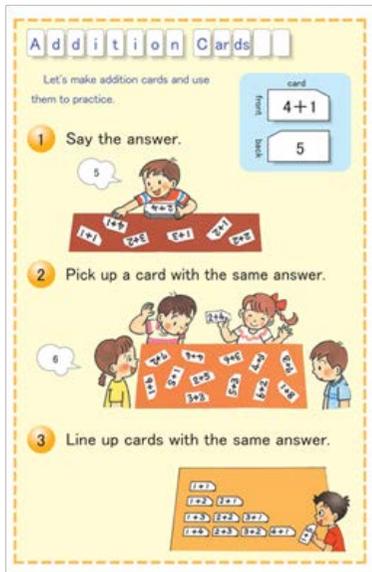
Explanation

Firstly, up until this lesson, they did not know how to arrange the cards because they just answered ' $2+2$,' ' $3+1$,' and ' $1+3$ ' for the card ' 4 .' Depending on the sequence of questioning the children will be able to recognize the ordering by constructing patterns. The three ways of explanation are related to the vertical, horizontal, and diagonal views. The diagonal view is especially related with symmetry. All the views represent beautiful structures of the world of addition. After children recognized the patterns, he gave the children the cards which were missing. Using the cards, children began to explain the structure following the patterns. Now, the children learned the way of extending the pattern of their arrangement by themselves through the ordering.

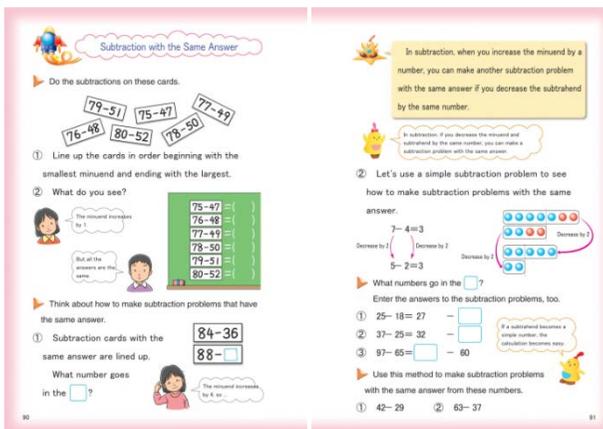
What is important is that this practice is the part of lesson study that considered how to teach the arrangement of the cards. He enabled the children to appreciate the beautiful pattern and tried to develop the mindset to find the patterns through the ordering. In the textbook, his challenges were written as follows (Grade 1,

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page 42 and Grade 2, vol.1, 88-89). If children use this textbook, they will be able to learn the arrangement of the cards to find the pattern, four times, in the first grade textbook. If the children learn from the textbooks then they can order the cards to finding the pattern. Mr. Seiyama developed the special sequence of questions for the children to recognize the structure. This is the reason why they were able to recognize the pattern.



Grade 1, p42 (There are four similar pages)



Grade 2, vol.1, pp88-89

See:

Gakkotosho (2005). *Study with your friends: Mathematics*. 11 vols. Tokyo: Gakko Tosho (English Translation of Japanese Textbooks)

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Learning How to Learn Mathematics

The sequence of the textbooks aims to develop children to learn mathematics for and by themselves. We learn from Mr. Seiyama's class that if we develop children's ability using good sequencing, we can expect the children to be able to arrange the order to find the beautiful structure. Japanese teachers develop content sequence as a product of lesson study.

Actually, the two videos are the evidence of lesson study that shows its effectiveness. The second video is the evidence to illustrate how the children learn to order the cards. They learned how to see beautiful mathematical structure when the cards are arranged appropriately. The first video is the evidence to illustrate that the children who experience this kind of activity four times can easily find the pattern. If children appreciate the beauty of mathematical structures at first grade such as in the second video, they will try to find the pattern by arranging the cards by themselves.

There are a number of explanations of inductive thinking. However, he did not choose the inductive method which begins from "1". He believed that his children can develop the pattern in the case of 3 and 5 because he had already taught the ordering activity to find the pattern. He wrote the table when he was planning the lesson. However he did not write the table during the lesson. Based on the children's explanation, he used the arrows to connect the two relational variables. His representation is necessary for the children's future learning to represent the pattern. One aim of the class is usually to prepare for future learning based on what the children have learned.

The national curriculum and textbooks has been produced as a result of lesson studies and at the same time, teachers usually recognize the objectives hidden in the textbooks and curriculum by doing lesson study. Lesson study is usually done by the teacher's group. However, the mindset of lesson study is usually in each teacher's mind even if other teachers do not observe their lesson as the teachers usually enjoy their classroom teaching and the children's ideas that are beyond their expectations but within reach of what is taught.

At the end of JICA video, you will learn about the mindset of teachers who are enjoying the challenges of their lesson study.

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Reference:

- Isoda, M. Course of Study: Mathematics (Grade 1-6). Tsukuba: CRICED.
(English Translation of Japanese curriculum, downloaded. June 1, 2012)
http://e-archives.criced.tsukuba.ac.jp/en/result_data.php?idx_key=1959
- Gakkotosho (2005). Study with your friends: Mathematics. Tokyo: Gakko Tosho
(English Translation of Japanese Textbooks)
- Isoda, M. & Murata, A. (2011). Study with your friends: Mathematics (New Edition). Tokyo: Gakko Tosho (English Translation of Japanese Textbooks)
- Isoda, M.& Katagiri, S. (2012). Mathematical Thinking: How to develop it in the Classroom. Singapore: World Scientific.
- Cedillo, T. & Isoda, M. (In press). Guidebook in Mathematics for Teacher Education, Mexico: Ministry of Education (SEP). (written in Spanish)

The videos will be distributed by CD for the participants of the workshop.