DEVELOPING GOOD MATHEMATICS TEACHING PRACTICE THROUGH LESSON STUDY: A U. S. PERSPECTIVE

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Although there is no consensus on what constitutes good mathematics teaching practice in the United States, a recent document published by the National Research Council (NRC) offers a vision that might be acceptable to the various stake holders. The NRC document considers teaching as an "interaction among teachers and students around content." Lesson study may play a significant role in developing and spreading good mathematics teaching practices that are in alignment with the vision presented in the NRC document. In this paper (and accompanying video of a lesson), we will discuss some specific features of good teaching practices and how lesson study may contribute to the development of such practices. We will conclude the paper with a brief discussion of future research that may be fruitful.

INTRODUCTION

As the participants of the APEC conference may be aware, recently there has been a lively debate about mathematics education in the United States. This debate, often called the "Math Wars," has largely focused around the new mathematics curricula developed to implement the *Standards* (National Council of Teachers of Mathematics [NCTM], 1989, 2000). These curricula de-emphasize teacher-telling as the primary mode of instruction. Rather, they organize their units around student investigations and discussions, to help students develop conceptual understandings and their own procedures, often very different from conventional procedures. Furthermore, these curricula emphasize the integration of mathematics, both within and beyond the field of mathematics. Thus teaching practices necessary to successfully implement these curricula are significantly different from that of direct instruction. Critics, such as the group called Mathematically Correct, argue that such an approach disadvantages significant segments of the student population. Thus, it should be clear that there is no consensus in the United States on what constitutes good mathematics teaching practice. However, a recent publication, *Adding It Up* (National Research Council, 2001), seems to offer a possible vision of good practice that may be agreeable to all sides of the

current debate for it is based on the work of the Mathematics Learning Study Committee consisting of people from different viewpoints.

GOOD PRACTICES

One cannot discuss good or effective instructional practices without considering the goals of instruction. In *Adding It Up*, the authors present the notion of mathematical proficiency consisting of the following five interwoven strands:

- *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *Procedural fluency* – skill in carrying out procedures flexibly, accurately, and appropriately
- *Strategic competence* – ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

(NRC, 2001, p. 116)

Thus, good teaching practices should promote the development of these strands of mathematical proficiency. Moreover, since these strands are "interwoven and interdependent" (p. 116), good teaching practices cannot focus on just one or two of these strands.

The document considers teaching as "interactions among teachers and students around content" (NRC, 2001, p. 313). In particular, they adapted the teaching triangle model developed by David Cohen & Deborah Ball shown in Figure 1.

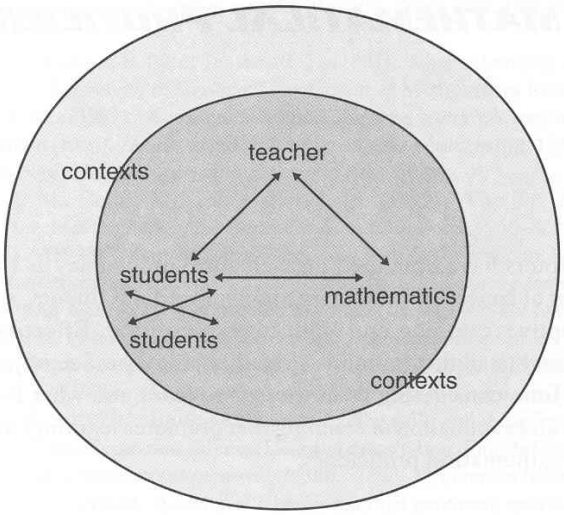

Figure 1: This model shows that mathematics teaching as the interaction among teachers, students, and mathematics, all taking place in contexts (NRC, 2001, p. 314).

Therefore, good mathematics teaching practices must go beyond simply what teachers do as they teach mathematics lessons. In particular, the document suggests,

> High-quality instruction, in whatever form it comes, focuses on important mathematical content, represented and developed with integrity. It takes sensitive account of students' current knowledge and ways of thinking as well as ways in which those develop. Such instruction is effective with a range of students and over time develops the knowledge, skills, abilities, and inclinations that we term mathematical proficiency (p. 315).

In the following section, we will illustrate some features of good mathematics teaching practices from a Grade 6 lesson taught during a lesson study open house.

AN ILLUSTRATION: RESEARCH LESSON ON AREA OF TRIANGLES

The following example is from one of the research lessons developed by a lesson study group in the U.S.. Unlike typical lesson planning, a group of teachers spent a couple of months to plan the lesson, studying the specific mathematics topic intensively and investigating the available resources, including a translated Japanese textbook. One member of the group, Mr. Jackson, was selected to teach the lesson. In order to accommodate a large number of observers, the lesson was conducted in a gymnasium.

The teacher opens the lesson by having selected students read what they wrote about what they learned in the previous day's lesson. This interaction clearly reminds the students some of the important ideas they learned in the previous day's lesson. However, in addition, this interaction illustrates how Mr. Jackson is considering his students' current thinking in setting up the main task for the lesson. Prior to the lesson, he selected which students to call on and in what order. By sequencing the students' comments (and their work from previous day's lesson), Mr. Jackson is able to provide a cohesive summary of the important mathematical ideas from the previous lesson, instead of a collection of haphazard recollections by randomly selected students, possibly leaving out some important ideas.

Another important point to notice in this opening segment is that, by integrating the students' own ideas in the lesson, Mr. Jackson communicates to his students that, in this classroom, their own ideas and methods are valued. Such an expectation has been identified as an important feature of a classroom that is set up as a community of learners (Hiebert et al., 1997).

This emphasis on students' own ideas and methods are again stressed in the next segment of the lesson, where Mr. Jackson poses the main task for the lesson – finding the area of a triangle by changing its shape. However, the first thing Mr. Jackson asks his class to do is to write down their ideas on how they might approach this task. By posing this question, Mr. Jackson communicates to his students that what is valued in this lesson is not just the answer, i.e., the area of the particular triangle, but also various ways students can determine the area. Furthermore, by having students share some of their ideas, Mr. Jackson provides an opportunity for those students who may be unsure about the task to consider ideas that they may pursue.

As the students begin their investigation, they were provided with many copies of the triangle on papers of different colours. Students are expected to cut and paste the triangles to illustrate their method clearly. The decision to use this particular set of materials in the lesson was not made lightly. During the post-lesson discussion, Mr. Jackson stated that he and his colleagues have explored a variety of materials, including the use of geoboards. However, the planning team felt the actual experience of cutting and re-arranging the given figure would provide an important foundation for the students to make sense of the area formula, which was the eventual goal of the unit. This type of careful consideration of instructional materials in light of the students' current understanding and the mathematical goals is another indication of effective mathematics teaching practices.

As the students engage in their investigations, they are free to choose whatever method that makes sense to them. As they experiment with their ideas, sometimes they attempt methods that are not productive. However, they are given the opportunity to determine whether or not their ideas are correct based on logical necessity, not because their teacher says so. Granting students such autonomy is another feature of a classroom as a community of learners (Hiebert et al., 1997).

In the next segment of the lesson, Mr. Jackson has his students share their ideas. As he did at the beginning of the lesson, Mr. Jackson carefully sequences students' ideas. By selecting and sequencing students' ideas intentionally. Mr. Jackson attempts to match students' diverse thinking processes with the development of a particular mathematical idea, illustrating interaction between the teacher, the students, and mathematics.

Furthermore, student work is clearly displayed on the blackboard, both their work with paper arrangements and mathematical expressions. In many so-called reform mathematics lessons in the United States, teachers often ask students to share their solutions publicly. However, too often, the sharing of students' solutions becomes the end of the lesson. That was not the case in Mr. Jackson's lesson. Perhaps the most important segment of the lesson is the next phase of the lesson where the teacher poses some questions to further analyse the ideas and methods shared by the students. In this particular lesson, Mr. Jackson asks students to sort the variety of methods into two types – those which transformed the given triangle to another shape without changing the area and those which created another shape by doubling the area of the given triangle. In the lessons to follow, the class will formally derive the area formula for triangles, but the experience gained in this lesson is an important foundation in understanding why the formula includes the division by 2.

Mr. Jackson ends the lesson by providing a summary of the important ideas discussed in the lesson. In his summary, he connects the day's lesson with the previous lesson by referring back to the work shared at the beginning of the lesson. This segment illustrates once again how the selection of the ideas to be shared at the beginning was intentional. Moreover, making this connection communicates to the students that mathematics learning should be based on what they have learned previously.

This particular lesson is by no means perfect. We do not claim that all students in the lesson learned everything discussed in the lesson completely. However, we offer this lesson as an illustration of good teaching practice that attempts to address the five strands of mathematical proficiency in an integrated manner. What is important to consider here, however, is that Mr. Jackson was not born a master teacher. He openly admits that his teaching was very different as recently as five years ago. He learned many of the features we discussed above through his participation in lesson study at his school. Furthermore, what Mr. Jackson learned through his participation in lesson study was not his alone. Other teachers who participated in lesson study also developed deeper understanding of good mathematics teaching practices. In the following section, we will discuss how lesson study may promote the improvement of mathematics teaching.

ROLE OF LESSON STUDY IN DEVELOPING GOOD TEACHING PRACTICE

As the participants of the APEC conference know, lesson study is the primary mode of professional development in Japan. Lesson study has played an important role in professional development in Japan since the beginning of the public education system in Japan more than a hundred years ago. One of the reasons for this popularity might be that lesson study provides Japanese teachers with opportunities to do the following: a) make sense of educational ideas within their practices; b) change their perspectives about teaching and learning; c) learn to see their practices from the child's perspective; and d) enjoy collaborative support among colleagues. For example, one Japanese teacher said:

> It is hard to incorporate new instructional ideas and materials in classrooms unless we see how they actually look. In lesson study, we see what goes on in the lesson more objectively, and that helps us understand the important ideas without being overly concerned about other issues in our own classrooms (Murata & Takahashi, 2002).

Why is lesson study so appealing to so many US researchers and educators? We think lesson study has certain characteristics that set it apart from the typical professional development program in the U.S., and that these unique characteristics are what makes lesson study so popular. These characteristics are described below.

First, Lesson Study provides teachers the opportunity to see teaching and learning in the classroom in a concrete form. This is due to the fact that lesson study guides teachers to focus their discussions on planning, implementation, observation, and reflection of classroom practices. By looking at actual practices in the classroom, teachers are able to develop a common understanding or image of what good teaching practice entails, which in turn helps students understand what they are learning.

Another unique characteristic of lesson study is that it keeps students at the heart of the professional development activity. Lesson study provides an opportunity for teachers to carefully examine the student learning and understanding process by observing and discussing actual classroom practices. Understanding student misunderstandings is

often examined in the process of observing and discussing the lesson. This also contributes to helping students construct their understanding. A third characteristic of lesson study is that it is teacher-led professional development. Through lesson study, teachers can be actively involved in the process of instructional change and curriculum development. Lynn Liptak, a retired principal at Paterson Public School No.2, Paterson, NJ, who has been implementing lesson study for over 4 years, contrasted lesson study with traditional professional development in the U.S., as summarized in Table 1.

As can be seen from Table 1, lesson study is teacher-led professional development where all the participants reciprocally learn from each others' experiences. In addition, the collaboration helps reduce isolation among teachers and helps to develop a common understanding of how to systematically and consistently improve instruction and learning by the school as a whole. Moreover, lesson study is a form of research that allows teachers to take a central role as investigators of their own classroom practices and become life-long autonomous thinkers and researchers of teaching and learning in the classroom.

Traditional	Lesson Study
Begins with answer	Begins with question
Driven by outside "experts"	Driven by participants
Communication flow:	Communication flow:
trainer → teacher	teacher ↔ teacher
Hierarchical relations between trainer & learners	Reciprocal relations among learners
Research informs practice	Practice is research

Table 1: Contrast between lesson study and traditional U.S. professional development (By Liptak, as included in Lewis, 2002, p. 12)

It is because of these features that lesson study has the potential to influence the quality of mathematics teaching practices in the United States, and elsewhere. Lesson study offers opportunities for participants to critically evaluate teaching practices. Such critical dialogues may take place during the planning period among the planning group members, or they may take place during the post-lesson discussion. In either case, these dialogues take place in the context of actual lessons, developed carefully and intentionally. Through such critical evaluations, lesson study provides a concrete image of effective instructional practice. Hiebert, Gallimore, and Stigler (2002) suggested that lesson study might be a potentially useful way of sharing good teaching practices. However, we suggest that lesson study is not only a useful tool to disseminate effective teaching practices but also a powerful mechanism to develop such practices.

As we noted at the beginning of this paper, the mathematics education community in the United States is in the midst of debate. Although people may disagree with each other, they are all concerned about students' mathematics learning. In order for us all to learn from these debates, we need to make sure that the debates are deeply rooted in the actual teaching of mathematics, and lesson study offers a systematic forum where such debates to can take place.

FUTURE RESEARCH

Just as mathematical proficiency involves interwoven and interconnected strands, good teaching practices, that is, teaching practices that promote mathematical proficiency, also involve interrelated components. *Adding It Up* lists the following five components for proficiency in the context of teaching:

- *Conceptual understanding* of the core knowledge required in the practice of teaching;
- *Fluency* in carrying out basic instructional routines;
- Strategic competence in planning effective instruction and solving problems that arise during instruction;
- Adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices so as to improve them; and a
- *Productive disposition* toward mathematics, teaching, learning, and the improvement of practice (NRC, 2001, p. 380).

Although lesson study seems to offer a promising pathway to an improvement of mathematics teaching practice, there are yet many questions that need to be addressed through further research. In particular, if we are to accept the notion of mathematics teaching *proficiency*, we must investigate how teachers develop such proficiency. One important question that needs to be addressed is the relationship between teacher knowledge and teacher practice. What knowledge do teachers draw on to teach mathematics more proficiently? How do they develop such knowledge? Other questions will have to address the effectiveness of educational policies in promoting proficient teaching practices. What support must school systems provide to practicing teachers so that they can continue to develop their proficiency? What are the appropriate responsibilities of teacher education institutions in preparing the beginning teachers?

As we engage in this research in the future, we should also keep in mind the value of cross-system research. Although teaching occurs in contexts, and our contexts vary significantly, our future research can nevertheless inform each other. One recommendation for improvement of mathematics teaching practices offered in *Adding It Up* states that professional meetings should be used for "more serious and substantive professional development" (p. 430). Likewise, when mathematics education researchers throughout the world come together, we should use those

occasions for sharing and planning further collaborative efforts to improve mathematics teaching practices everywhere.

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