

Grade 6 Mathematics Lesson Plan

Date: December 5, 2006 (Tuesday)
Place: Sapporo City Hokuto Elementary School
Class: 6B, 27 students (14 boys and 13 girls)
Instructor: Atsutomo Morii

1. Name of the lesson: "Thinking Systematically"

2. Intention of this lesson and mathematical thinking would like to foster among the students

During 4th grade, students learned how two quantities change in the unit called "investigating changes in quantities." In the 4th grade, they also learned how to express the relationship between two quantities using tables and math sentences. In addition, the students had experience reading the changes of the quantities and their characteristics.

In the 5th grade, based on their experience in 4th grade, students learned to solve problems by finding the relationship between two quantities and their regularity using tables.

The aim of this lesson is to use knowledge from prior grade levels to solve problems using tables that have more items. This lesson is included in the mathematics textbook. This lesson is not included as a part of a unit but it is set up as individual lesson. Title of the next unit is "proportional relationships." In the unit, students will construct tables, finding regularity, and expressing the relationship using math sentences. I believe this lesson is included here to help students prepare to learn about proportional relationships.

In this lesson, I anticipate that the students might solve this problem by coming up with an appropriate value and then calculating or by constructing a table. I believe that constructing a table is not a difficult task for the students because of their prior learning experiences. Moreover, I believe that many of the students will use a table to solve the problem.

The table in the textbook shows the number of pencils and ballpoint pens from 1 to 9, but in this lesson I decided to use the number from 0 to 10. This is decision relates to my hope for a certain kind of mathematical thinking that I want my students to acquire.

I would like to focus on a kind of mathematical thinking, i.e. hypothetical thinking.

Something like, "If it is then"

By changing the quantities of the items in the problem on their own, the students can come up with better solution methods. In order to do that I think it is important for the students to see an extreme case in the table such as "I bought 10 items of one kind and 0 items of the other kind."

Moreover, in order for students to find better ideas to solve the problem, it is important for the students to have an opportunity to feel that they really want to do so.

Starting in April (beginning of the school year), I taught the students to look at something from a particular point of view such as "faster, easier, and accurate" when they think about something or when they compare something.

If you think about the method that uses the table from this point of view, students might notice that "it is accurate but it takes a long time to figure out" or "it is accurate but it is complicated."

In order to solve a problem in a short time and with less complexity, it is important for the students to notice that calculation using a math sentence is necessary.

To do so, how to find regularity from the table becomes a key to finding a better solution. I would like to make sure that all students understand that "the price increases or decreases by 30 yen."

Lastly, by applying the idea of "if it is ... then ..." to a similarly structured problem that has a different situation, I believe the students will understand the merit of doing so and build the students' desire to want to use the idea.

3. Goals of the lesson

- (1) For students to notice that using a table helps them understand easily and try to use a table to solve the problem. (Interest/desire/attitude)
- (2) For the students to be able to use the table to organize and categorize and investigate in logical order (expression/manipulation)
- (3) For the students to find a regularity (pattern) from the table and be able to use it. Moreover, I hope the students will understand that you can solve the problem using calculations instead of using the table (mathematical thinking)

4. Process of the lesson

Students' activities and thinking process	Teacher's support																																				
<p data-bbox="159 280 997 425">We bought pencils and ballpoint pens and the total number of items were 10 and the price was 460 yen. The price of each pencil was 40 yen and the ballpoint pen was 70 yen. How many pencils and how many ballpoint pens did we buy?</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="215 448 486 504">If we calculate it...</div> <div data-bbox="534 448 829 504">If we make a table...</div> </div> <table border="1" data-bbox="183 526 1228 660"> <tr> <td># of pencils</td> <td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td> </tr> <tr> <td># of ballpoint pens</td> <td>10</td><td>9</td><td>8</td><td>7</td><td>6</td><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td> </tr> <tr> <td>Total price (yen)</td> <td>700</td><td>670</td><td>640</td><td>610</td><td>580</td><td>550</td><td>520</td><td>490</td><td>460</td><td>430</td><td>400</td> </tr> </table> <p data-bbox="303 683 909 739">The answer is 8 pencils and 2 ballpoint pens!</p> <p data-bbox="191 750 566 772">I wonder if there are any rules?</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="159 795 422 952">If the number of pencils increases one the total price decreases 30 yen.</div> <div data-bbox="438 795 758 952">If the number of ballpoint pen increases one the total price increases 30 yen.</div> <div data-bbox="774 795 997 952">It is increase or decrease by 30 yen.</div> </div> <p data-bbox="191 996 949 1097">Do we need to draw the table to find the answer? Let's find the solution methods that are "faster," "easier," and "accurate!"</p> <div style="display: flex; justify-content: space-between;"> <div data-bbox="151 1120 574 1276"> <p>If we buy only ballpoint pens the total price would be 700 yen. $(700 - 460) \div 30 = 8$ Thus, the answer is 8 pencils and 2 ballpoint pens.</p> </div> <div data-bbox="598 1120 997 1456"> <p>If we buy them the same number (5 pencils and 5 ball point pens) then price would be 550 yen. The total price that you actually paid was 460 yen and it is: $460 < 550$ Therefore the number of pencils is more. $(550 - 460) \div 30 = 3$ Thus the answer if 8 pencils (5 + 3) and 2 ballpoint pens (5 - 3).</p> </div> </div> <div data-bbox="151 1310 574 1456"> <p>If we buy only pencils the total price would be 400 yen. $(460 - 400) \div 30 = 2$ Thus, the answer is 2 ballpoint pens and 8 pencils.</p> </div>	# of pencils	0	1	2	3	4	5	6	7	8	9	10	# of ballpoint pens	10	9	8	7	6	5	4	3	2	1	0	Total price (yen)	700	670	640	610	580	550	520	490	460	430	400	<p data-bbox="1077 280 1412 504">Listening to the students' muttering (or voices) and pick up the idea to use a table to solve this problem. Then ask the students to fill in the table on the worksheet.</p> <p data-bbox="1077 716 1380 840">The price difference between the pen and the ballpoint pen is 30 yen. Therefore:</p> <ul style="list-style-type: none"> <li data-bbox="1077 840 1412 952">○ If the number of ballpoint pens increases by one, the total price increases 30 yen. <li data-bbox="1077 974 1412 1064">○ If the number of pencils increases by one, the total price decreases 30 yen. <p data-bbox="1077 1086 1412 1332">Ask the students to look at various solution methods from the point of view of "faster," "easier," and "accurate" and ask them to think about calculation methods that do not require the table.</p>
# of pencils	0	1	2	3	4	5	6	7	8	9	10																										
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<p data-bbox="199 1500 973 1579">If we use the idea of "If it is ... then ..." then we can find the answer without using the table.</p> <p data-bbox="191 1601 813 1635">Let's solve another problem using the idea we used!</p> <p data-bbox="175 1646 989 1803">We bought colored pencils and markers and the total number of items were 12 and the price was 820 yen. The price of each colored pencil was 60 yen and the marker was 80 yen. How many colored pencils and how many markers did we buy?</p> <p data-bbox="223 1814 949 1870">This time the total price increase or decrease by 20 yen.</p> <p data-bbox="151 1870 726 1915">$(960 - 820) \div 20 = 7$ $(820 - 720) \div 20 = 5$</p> <p data-bbox="247 1915 845 1982">We bought 7 colored pencils and 5 markers.</p>	<p data-bbox="1077 1478 1380 1601">Help students to see the value of idea for thinking the following: "If we buy only ballpoint pens..."</p> <p data-bbox="1077 1635 1412 1937">Asking the students to solve another problem that helps the teacher and the students to evaluate student learning. Also, providing another opportunity for the students to experience the merit of solving the problem with out creating a table.</p>																																				