Mathematical Thinking

Big Ideas in Elementary School Mathematics

Yeap Ban Har
National Institute of Education
Nanyang Technological University
Singapore

Purpose of School Mathematics

The development of a way of thinking that forms the foundation for competencies of a high-quality workforce.

Mathematical Thinking

This way of thinking is
mathematical thinking.

Purpose of This Lecture

To offer another way of
thinking about
mathematical thinking.

Synopsis of this Lecture

In the first part, the different aspects of mathematical thinking will be described.

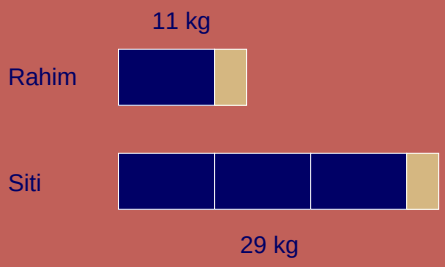
Synopsis of this Lecture

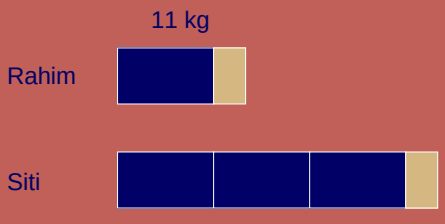
In the second part, the supporting generic competencies will be described.

Task 1

Siti packs her clothes into a suitcase and it weighs 29 kg.
Rahim packs his clothes into an identical suitcase
and it weighs 11 kg.
Siti's clothes are three times as heavy as Rahim's clothes.

What is the mass of Rahim's clothes?
What is the mass of the suitcase?






11 kg
Rahim
Siti
29 kg


11 kg
Rahim
Siti
29 kg

Mathematical Thinking

Big Idea 1

visualization

The Role of Visualization

Solve $x + y = 11$ **and** $3x + y = 29$

$x + y = 11$

$3x + y = 29$

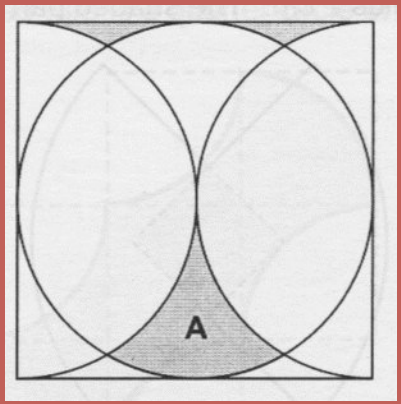
$2x + x + y = 29$

$2x + 11 = 29$

$2x = 18$

$x = 9$ **and** $y = 2$

Task 2



The figure shows a square of side 14 cm, two semi-circles and a circle.

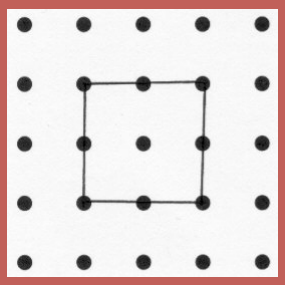
Find the area of the shaded portions.

Find the perimeter of the shaded part labelled A.

Visualization

The ability of the mind to work with abstract ideas – to analyze and synthesize ideas.

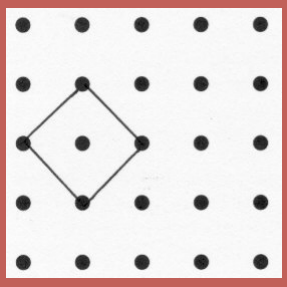
Task 3



There is 1 dot inside the square.

There are 8 dots on the perimeter of the square.

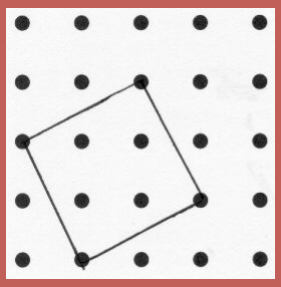
Task 3



There is 1 dot inside the square.

There are 4 dots on the perimeter of the square.

Task 3



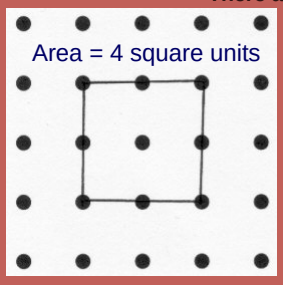
There are 4 dots inside the square.

There are 4 dots on the perimeter of the square.

Task 3

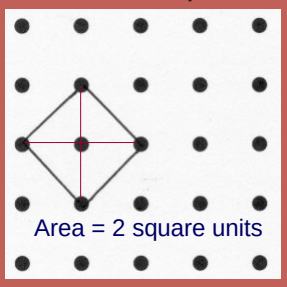
There is 1 dot inside the square.

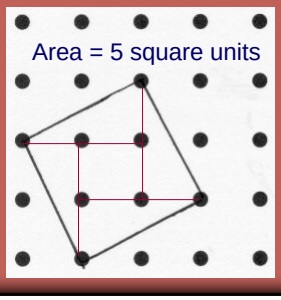
There are 8 dots on the perimeter of the square.



There is 1 dot inside the square.

There are 4 dots on the perimeter of the square.





There are 4 dots inside the square.

There are 4 dots on the perimeter of the square.

Area (unit^2)	i	p
4	1	8
2	1	4
5	4	4

Area (unit^2)	i	p
4	1	8
2	1	4
5	4	4

Area (unit²)	i	$p \div 4$
4	1	2
2	1	1
5	4	1

Area (unit²)	i	$p \div 2$
4	1	4
2	1	2
5	4	2

$$A = \frac{1}{2}p + i - 1$$

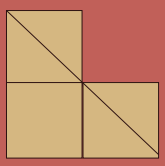
Mathematical Thinking

Big Idea 2

patterning

Task 4

$$1+2+3+4+\ldots+n=\frac{n(n+1)}{2}$$

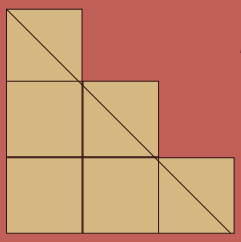


n = 2

Sum = ½(2)(2) + 2(½)

Task 4

$$1+2+3+4+\ldots+n=\frac{n(n+1)}{2}$$



$n = 3$

Sum = ½(3)(3) + 3(½)

Task 4

$$1+2+3+4+\ldots+n=\frac{1}{2}(n)(n)+n(\frac{1}{2})$$

$$1+2+3+4+\ldots+n=\frac{n^2}{2}+\frac{n}{2}$$

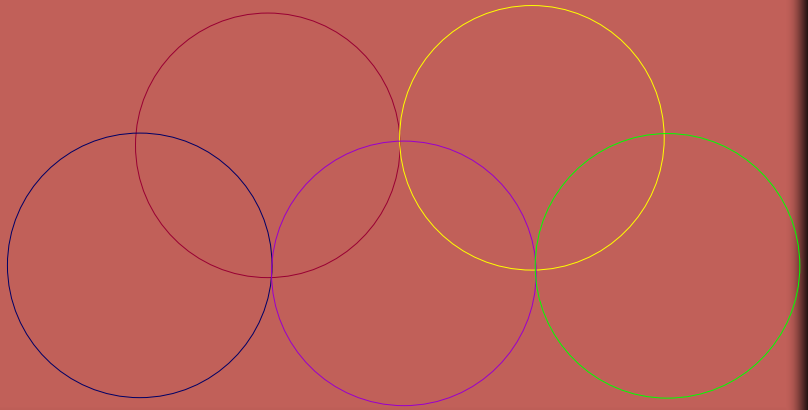
Patterning

The ability of the mind to see trends and identify relationships between and among variables.

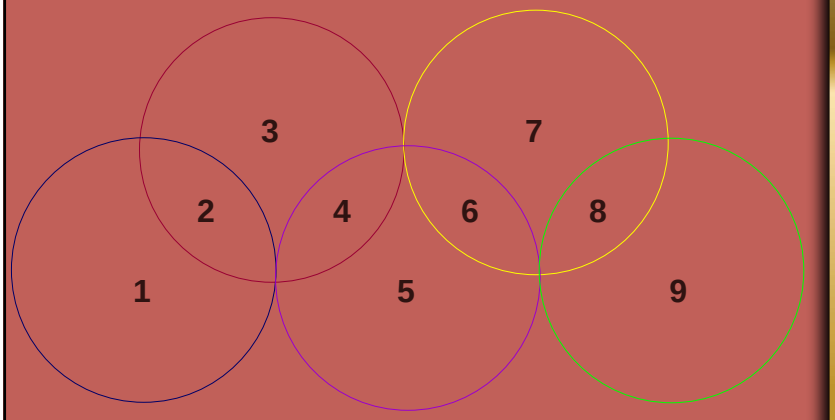
Task 5

There are 9 regions.

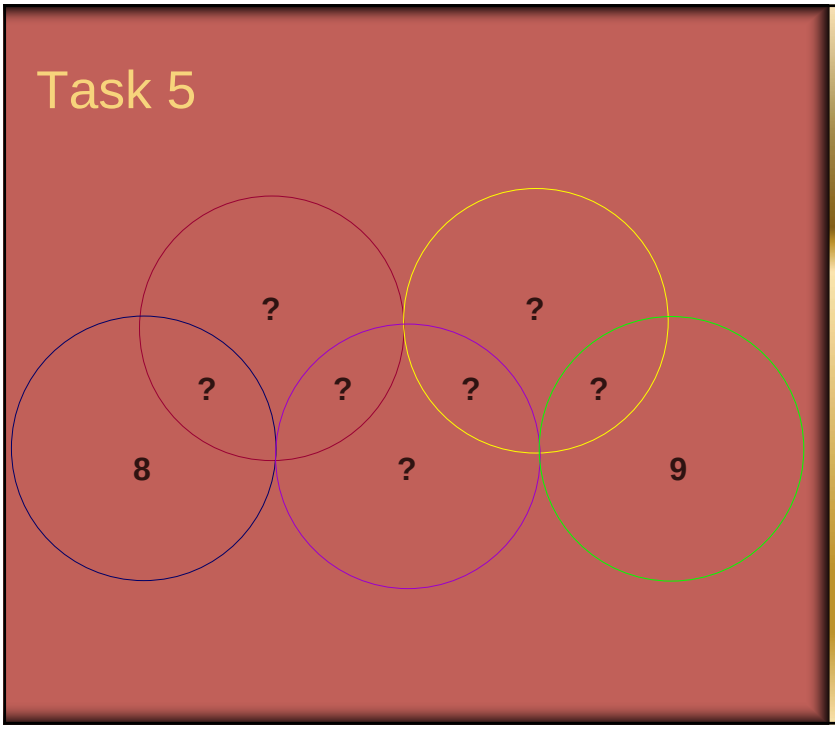
Put whole numbers 1 to 9 in the 9 regions such that the sum of numbers in every circle is the same.



Task 5



Task 5



Mathematical Thinking

Big Idea 3

numbersense

Task 6

Each of the three cards shown is printed with a different whole number. The smallest number is 23. When these numbers are added two at a time, the sums are 61, 71 and 86. What is the largest number on the cards?

Number Sense

The ability of the mind to make generalizations based on the specific cases done.

Summary

In the first part, the three aspects of mathematical thinking are described.

visualization

patterning

number sense

In addition

Generic competencies
complement these aspects of mathematical thinking

Generic Competencies

- Heuristics
  - Using a diagram
  - Using a table
  - Solve simpler problems
- Habits of Mind
  - Creative thinking
  - Critical thinking
  - Metacognition

Thank You

banhar.yeap@nie.edu.sg