DEVELOPING MATHEMATICAL THINKING IN SINGAPORE ELEMENTARY SCHOOLS

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INTRODUCTION

The role and value of school mathematics is often questioned and debated. In this paper, the role of mathematics in the Singapore education system is discussed. Education in Singapore has an economic function. Education is perceived as preparing pupils to develop competencies that the future workforce needs to have. In particular, education is the platform to prepare pupils to become knowledge workers who are capable of innovative thinking and communicating such thinking.

Thus, mathematical thinking is a focus of the Singapore mathematics curriculum. Since 1992, the main aim of school mathematics has been to develop mathematical problem solving ability among pupils (Ministry of Education, 1990). The curriculum was revised in 2001 and will be revised again in 2007 but the main aim remains the same (Ministry of Education, 2000). Figure 1 shows the framework for the Singapore mathematics curriculum (Ministry of Education, 2006)

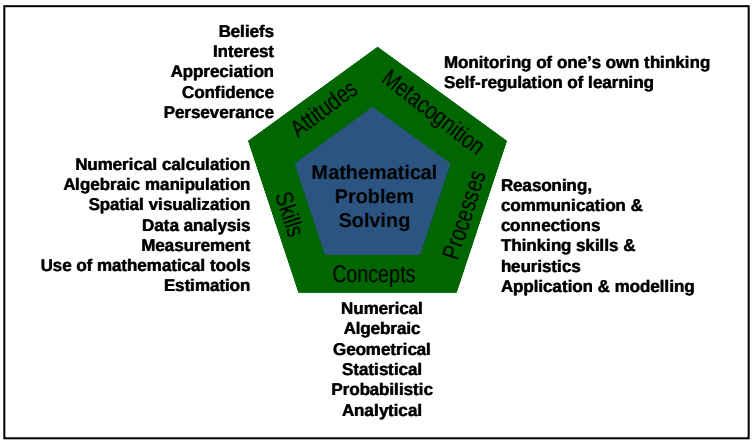


Figure 1 The Singapore mathematics curriculum

This paper comprises three parts. In the first part, I discuss how mathematical thinking is defined in the curriculum document in Singapore. In the second part, I describe and discuss one perspective of understanding the mathematical thinking. In the third part, I describe some ways mathematics lessons can be used to develop mathematical thinking. In particular, I focus on lessons that make use of instructional materials commonly used by Singapore schools.

MATHEMATICAL THINKING IN THE SINGAPORE CURRICULUM

In 1997, the then Prime Minister of Singapore announced that Singapore schools should help their pupils develop the ability to think. The Thinking Schools, Learning Nation initiative was started in 1997 for this purpose (Goh, 1997). Generic thinking skills such as classifying and comparing were taught to pupils. These thinking skills were also infused into key subjects including mathematics. Thinking skills are considered to be part of processes required in problem-solving efforts. In 2003, another initiative Innovation and Enterprise was introduced to encourage schools to develop good habits of mind or thinking habits among their pupils (Tharman, 2004). Along with information technology and national education, thinking is considered one of the key components of the education system.

In particular, pupils are expected to be able to engage in problem solving, routine as well as novel problem solving, in mathematics. This includes mathematical investigations. Mathematical thinking is the process that pupils engage in when they solve mathematics problems. According to the curriculum framework, mathematical problem solving requires five inter-related components – skills, concepts, processes, attitude and metacognition. Pupils are expected to possess mathematical skills and concepts. Skills include computation including mental computation and visualization. Key concepts in elementary school include numerical, geometrical and algebraic concepts. Pupils are also expected to possess the ability to engage in processes such as reasoning, communicating, making connections, modeling, and using thinking skills and heuristics. This aspect is the focus of Thinking Schools, Learning Nation. Pupils are expected to possess good problem-solving attitudes and habits as well as the ability to engage in metacognition. These aspects are the focus of Innovation and Enterprise. In a way, the Singapore mathematics curriculum defines mathematical thinking as the orchestration of mathematical skills, concepts and processes to handle a situation which could be novel.

This is reflected in the national examination. In the Primary School Leaving Examination (PSLE) taken by pupils at the end of six years of primary education, about half of the maximum marks available for the mathematics test are from a section comprising thirteen problems. In this section, pupils must be able to show the method they used to solve the problems.

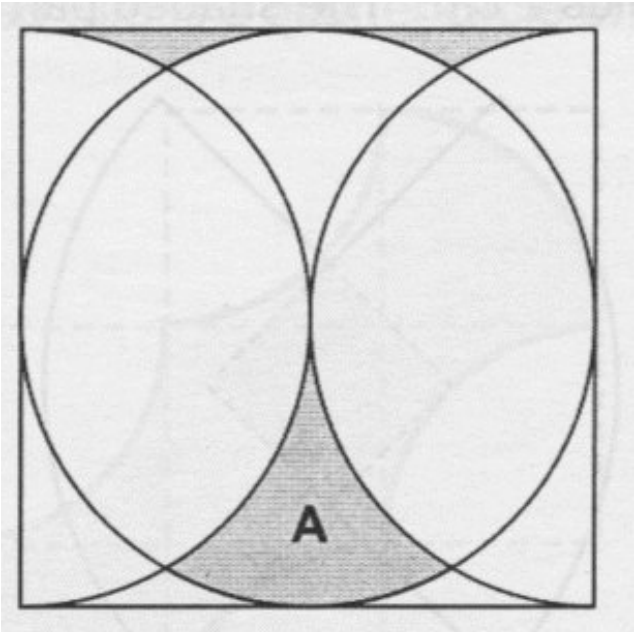
ASPECTS OF MATHEMATICAL THINKING

The perspective of mathematical I present in this paper views mathematical thinking as a juxtaposition of mathematical competencies and generic competencies when pupils handle a mathematical situation such as mathematical problem solving.

The mathematical competencies include visualization, patterning and number sense (Yeap, 2005). These mathematical competencies are referred to as ‘big ideas’. These are the essence distilled from specific mathematical work that pupils engage in.

Generic competencies are those that support the use of big ideas in mathematical problem solving. Generic competencies include skills such as the use of heuristics as well as habits of mind such as metacognition.

In order to find the area of the shaded portions in the here-Region Problem (Figure 2), one does not need to find the area of each individual portion. Pupils who are able to engage in visualization are able to use their skills in finding the area of squares and circles to solve this problem. In the same way, such pupils are able to see that the perimeter of the required region is part of the circle. The mathematical competency, or big idea, required is visualization.



The figure shows a square of side 14 cm, two semi-circles and a circle.
Find the area of the shaded portions.
Find the perimeter of the shaded part labeled A.

Figure 2 Three-Region Problem (SEAB, 2006)

In the Pick's Theorem[1] task, pupils are required to see the relationship between the area and the number of dots inside and on the perimeter of the polygon. The simple use of basic four operations is all that is needed as far as skills are concern. However, the ability to do the four operations is necessary but not sufficient. Another competency is needed. The mathematical competency, or big idea, needed is patterning. This is the ability to see trends and identify relationships between and among variables.

The use of a heuristics such as the use of a table facilitates patterning ability. The use of table is an example of a supporting generic competency. Also, one has to be flexible in order to be able to see the relationship. When one sequence of operation

[1] The Pick's Theorem describes a relationship between the area of a polygon formed on a geo-board, A, and the number of dots inside the polygon, I, and on the perimeter of the polygon, p, as $A = \frac{1}{2} p + i - 1$.

does not yield the relationship, one has to try to look at the same data set from a different perspective. Good habits of mind such as flexibility, which is a component of creative thinking, facilitate the ability to see the relationship. Flexibility is another example of a supporting generic competency. Some of the supporting generic competencies are heuristic skills while others are habits of mind.

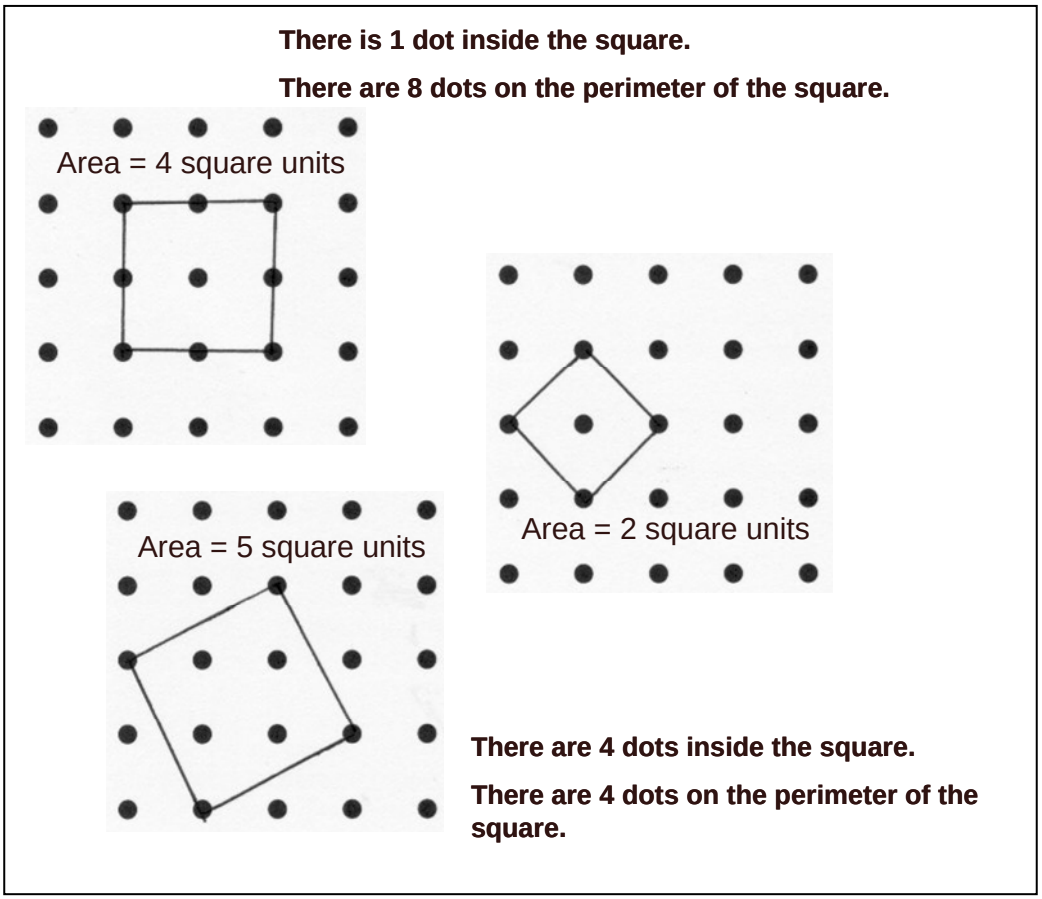
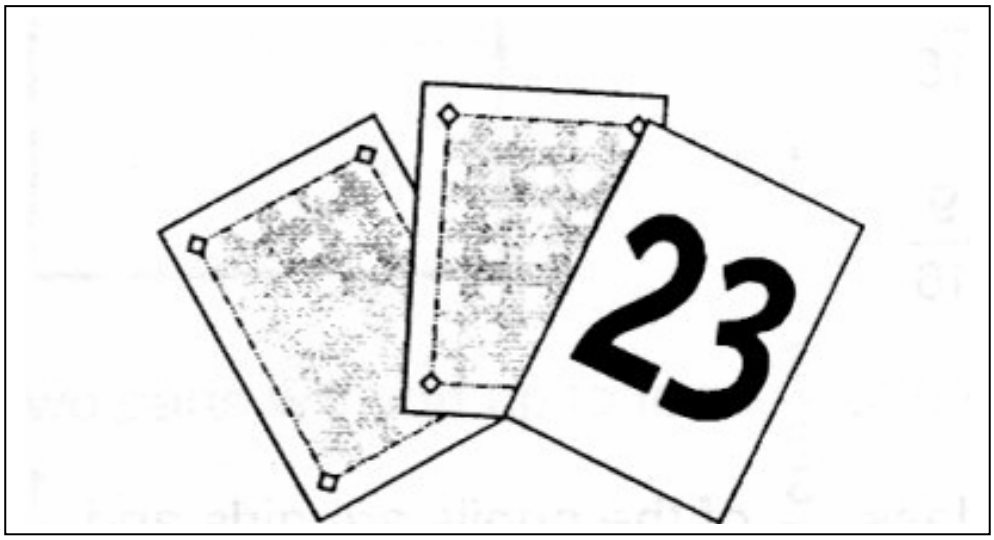


Figure 3 Pick's Theorem Task

In finding the largest number on the three cards in the Three-Card Problem (Figure 4), some pupils knew that 23 is not used to get a sum of 86. They are using the idea that the largest sum is obtained from the largest two numbers. Some pupils are able to form such ideas after doing addition for some time. They are able to distill the essence of the specific mathematical asks that they engage in. The essence is a big idea, or mathematical competency, referred to as number sense.

In this section, I present another perspective of mathematical thinking as comprising of mathematical competencies and generic competencies working together. I describe the three mathematical competencies to some extent. I also give examples of generic competency which could be a heuristic skill or a habit of mind.



Each of the three cards shown is printed with a different whole number.
The smallest number is 23.
When these numbers are added two at a time, the sums are 61, 71 and 86.
What is the largest number on the cards?

Figure 4 Three-Card Problem (SEAB, 2006)

LESSONS TO DEVELOP MATHEMATICAL THINKING

In this part, I use common instructional practices in Singapore elementary schools to show how mathematical thinking may be developed.

One common feature of the Singapore textbooks is the use of visuals. In particular, the 'model method' is used extensively. The 'model method' is essential the use of diagram to solve algebraic problems. Word problems such as the one shown in Figure 5 are often solved using this method.

Siti packs her clothes into a suitcase and it weighs 29 kg.
Rahim packs his clothes into an identical suitcase
and it weighs 11 kg.
Siti's clothes are three times as heavy as Rahim's clothes.

What is the mass of Rahim's clothes?
What is the mass of the suitcase?

Figure 5 Suitcase Problem (SEAB, 2006)

Figure 6 shows the diagram used to solve the Suitcase Problem.

The extensive use of visual method such as the 'model' method can help pupils develop visualization ability. Although the mere use of visuals somewhat facilitates the development of visualization, there are other teaching techniques that enhance the development of visualization.

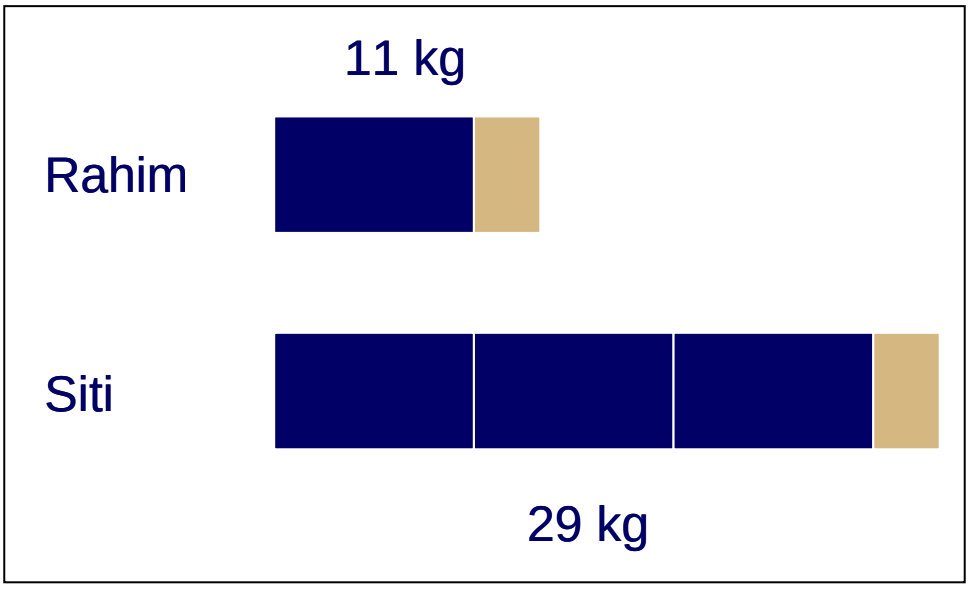


Figure 6 Model Method

Among the common teaching techniques can be used include:

(a) Providing – The teacher provides opportunities such as providing tasks and problems that include the use of mathematical thinking.

(b) Telling – The teacher tells or explains how a problem is solved. In a way, the teacher is telling the product of the teacher's own mathematical thinking.

Mathematical thinking can also be developed through other teaching techniques including:

(c) Modeling – The teacher think aloud during the problem-solving process. In this way, pupils get to see how the teacher figures things out including running into obstacles and overcoming the obstacles.

(d) Coaching – The teacher gives instructions so that pupils engage in a certain type of thinking. In a way, the teacher is scaffolding pupils' thinking. The role of the teacher here is to provide an instruction that can spark off a thinking step.

CONCLUSION: A RESEARCH AGENDA

Lesson study is to be used as a tool for teachers to develop or improve their ability to help pupils develop mathematical thinking. As part of their professional development, a group of teachers in several elementary schools in Singapore will engage in a lesson study to understand the idea of mathematical thinking and how pupils can develop mathematical thinking. The study will take place between February and April 2007.