DEVELOPING MATHEMATICAL COMMUNICATION IN PHILIPPINE CLASSROOMS

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This paper describes the importance that the basic education curriculum places on developing in Filipino students effective communication skills. It also presents the different components of mathematical communication and the teaching strategies to develop it including those that address specific teaching and learning practices to be changed for improvement to support this development in Philippine classrooms.

CURRICULUM PROVISIONS TO DEVELOP MATHEMATICAL THINKING

What do the General and the Mathematics Curricula Provide?

Along with critical thinking, creative thinking, problem solving, decision-making, and entrepreneurial/productive skills, effective communication is one of the core life skills that every Filipino student who is competent to learn how to learn should possess. This is stated in the philosophy of the basic education curriculum that is currently being implemented. Effective communication is an important skill needed for life-long learning (Department of Education 2002, p.8).

The goal of the elementary mathematics curriculum is for pupils "to demonstrate understanding and skills in computing with considerable speed and accuracy, estimating, communicating, thinking analytically and critically, and in solving problems in daily life using appropriate technology" (Bureau of Elementary Education 2002, p. 8). Meanwhile the secondary mathematics curriculum states that at the end of fourth year, the students are expected to be able to compute and measure accurately; arrive at reasonable estimates; gather, analyse, and interpret data; visualize and explain abstract mathematical ideas; present alternative solutions to problems using technology and apply them in real life situations (Bureau of Secondary Education 2002, p.1). Representation which involves visualization and explanation of abstract mathematical ideas are both components of mathematical communication. These provisions show that communicating is an important skill.

How do the General and Mathematics Curricula Promote Communication?

Both the general and the mathematics curricula promote communication through interaction. They stress mutual interaction between students and teachers, between students themselves in collaborative learning, and between teachers of different disciplines in collaborative teaching. They view the teacher as a manager of the learning process that enables the students to become active constructors of knowledge and not just passive recipients of information. “The ideal teacher helps students to learn not primarily answers but how to reflect on, characterize and discuss problems, and how on their own initiative, form or find valid answers (Department of Education 2002, p.9)”. These characterizations imply that both the teacher and the students are actively engaged in communication.

Among others, the general curriculum recommends teaching that focuses in inquiry that uses questions to organize learning. Such involves students in conducting investigations where they formulate problems, design how they would gather and interpret information, generate answers, communicate to others what they have learned, and formulate extension problems (Department of Education 2002, p.32). So again, there is emphasis on communication.

COMPONENTS OF MATHEMATICAL COMMUNICATION

Isoda (2007) proposed several components of mathematical communication. The foregoing discussion is based on his comprehensive list.

Using the Appropriate Language to Promote Conceptual Understanding and Discourse

Filipino is the national language. But Mathematics is required to be taught and hence, students’ learning of it, in English. However as borne by the Learner’s Perspective Study, students often had to contend with English in order to understand the mathematics concepts that were expressed in this foreign language (Ulep 2004). When they were asked to answer the teacher’s questions which only required short or factual answers, they spoke in English. But when they were asked to explain their answers, although they were aware that they were expected to speak English, they used Filipino but retained the mathematical terms in English. Interestingly, before they explained, there were students who even asked permission from the teacher for them to use Filipino. These findings show that students were able to express their thinking in the language that they truly understood and were comfortable with. Such is understandable because when they were not engaged in public talk in class, students talked in Filipino or in their native dialect, just like they did at home and anywhere else except in their English and Science classes in school.

During the author’s discussions with teachers when she observed classes, they admitted that when their principal or supervisors observed their classes, they spoke English and asked their students to do the same. But when these observers were not around, they explained mathematics in Filipino and allowed their students to speak in Filipino. They did this especially in classes consisting of low ability students to ensure that they understood the lesson.

Teachers claimed that due to students’ poor comprehension, a topic which they found difficult to teach and students found difficult to learn was solving word problems (High School Mathematics Group 1995). To help students analyse word problems, the elementary mathematics curriculum recommends that students answer the following guide questions: (1) What is asked (A)? (2) What is given (G)? (3) What is the word clue/operation to use (O)? (4) What is the number sentence (N)? (6) What is the answer (A)? AGONA implicitly shows how a word problem should be analysed (Bureau of Elementary Education 2002). Moreover, teachers ask students to look for key words that would suggest the operation to use in solving the problem. However, besides taking a lot of time, going through AGONA does not ensure the needed understanding and by simply relying on key words, students tend not to try to understand the problem anymore.

The above accounts imply that for conceptual discourse to take place, it is necessary to encourage students to use the language that they best understand and with which they can ably express their mathematical ideas (Setati 2003). It is also necessary for teachers to help students conceptually analyse word problems instead of asking them to routinely do procedures like AGONA and just depend on key words (Ulep 2007).

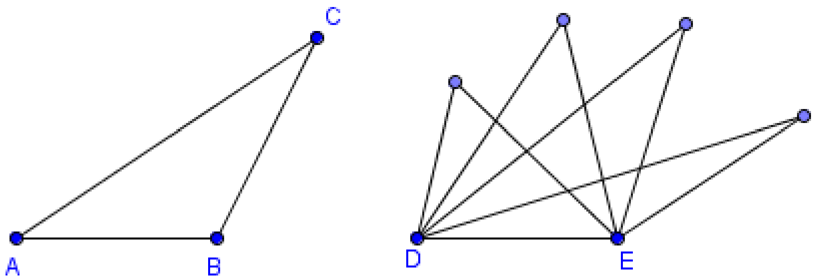
Emphasizing Logical Reasoning

Geometry is usually viewed as the school mathematics subject where logical argumentation is expected to be emphasized. But oftentimes this does not happen. In secondary school mathematics for example, it is defined that two triangles are congruent if their vertices can be made to correspond such that three pairs of corresponding sides and three pairs of corresponding angles or a total of six pairs of corresponding parts are congruent. Then the SSS, SAS, and ASA conditions are presented as triangle congruence postulates. Later the AAS triangle congruence theorem is proved. Unlike the curriculum in other countries, in the Philippines there is no transformation geometry so it cannot be used to develop the conditions for triangle congruence.

Students are not encouraged to find out why from the required six pairs of corresponding parts being congruent only three pairs are needed to show that two triangles are congruent. Moreover, they are not challenged to determine why only these four sets of three-conditions each and no other such sets of three conditions can enable one to show that two triangles are congruent. So teachers teach triangle congruence as it is usually presented in most textbooks.

A group of geometry teachers involved in a lesson study developed a lesson that was intended to make students systematically discover why the least number of conditions for two triangles to be congruent is three. The students first considered the least to be one pair of corresponding side then one pair of corresponding angle. First they drew a triangle. By construction, they copied one of its sides and tried to construct another triangle congruent to the original triangle. But they could not. They did similarly using one angle and got the same result.

Then they considered the least number of corresponding congruent parts to be two, specifically two pairs of corresponding congruent sides. One group of students had this work shown below.



The group members realized that having two pairs of corresponding sides that are congruent to each other in two triangles does not make the two triangles congruent. Specifically they considered $\overline{AB}$ and $\overline{DE}$ as one pair. But $\overline{BC}$ could be paired with many sides each of which has E as one endpoint and the third vertex of each triangle with $\overline{DE}$ as one side, as the other endpoint.

The activity enabled the students to discover counterexamples. One counterexample was enough to show that the conditions being considered would not result to having two congruent triangles. By systematically considering the other cases, the students found out that it was only when they constructed using the conditions SAS, SSS, and ASA that they obtained a triangle that was congruent to the given triangle each time. The activity had at least provided the students a basis for accepting the postulates. They realized that although postulates do not require proof, they also have a basis.

Differentiating between Conceptual Explanations and Procedural Descriptions

When students are asked to write or pose their work on the board and explain it to the class, what they do most of the time is to read what they have written. They do not really explain the thinking that they used which enabled them to develop a solution or obtain the required answer. This was the case that happened during the lesson implementation of a lesson study group in an elementary school. The students correctly represented the given word problem by the number sentence shown on the right and correctly determined the missing digits shown on the left, below:

$$\begin{array}{r} 76\_4 \\ -\ 388\_ \\ \hline 3\_86 \end{array} \qquad\qquad \begin{array}{r} 76\underline{7}4 \\ -\ 388\underline{8} \\ \hline 3\underline{7}86 \end{array}$$

When they were asked to explain how they determined the missing digits, many students gave these procedural descriptions: Four minus 8 cannot be. So borrow 1 from 7. Four becomes 14 and 7 becomes 6. Fourteen minus 8 is 6. Six minus 8 cannot be. So borrow 1 from 6. Six becomes 16 and 6 becomes 5. Sixteen minus 8 equals 8. And so on. There were a few students who gave these conceptual explanations: Four minus a certain number equals 6. But 6 is bigger than 4. So we need to borrow one ten from the digit in the blank in the tens digit of the minuend. So instead of 4 we now have 14. Now what number should be subtracted from 14 so that the answer is 6? So the missing digit here must be 8. And so on.

To enhance mathematical communication and thinking, it is important that teachers require students to provide reasons for what they did and not just to relate the procedures that they used to solve problems (National Council of Teachers of Mathematics 2000).

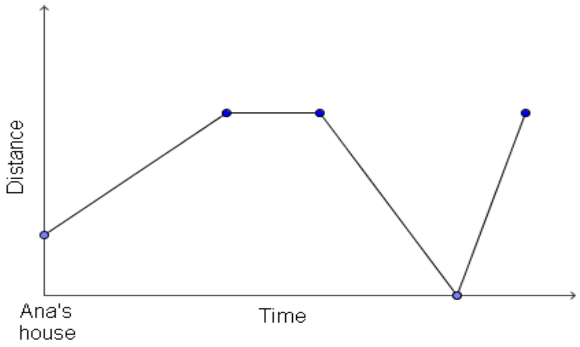
Making Meaningful Representations

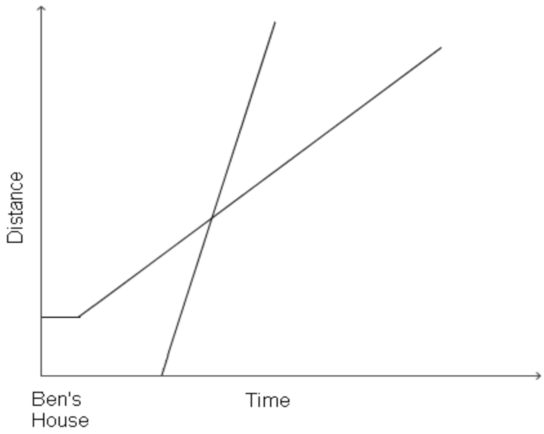
There are different ways of representing the same mathematical idea. For example, a relationship between two changing quantities may be verbally described or shown using diagrams, tables, graphs, and equations. Students should see the connections among equivalent representations of the same ideas (National Council of Teachers of Mathematics 2000).

One form of representation is simulation. For instance, in a certain teacher training program where the topic was on experimental probability, the teachers were asked how they could use the scientific calculator to obtain the possible results if an actual fair coin was tossed many times, since scientific calculators were available anyway. Scientific calculators have become increasingly more available in schools so it was expected that the teachers could do the activity in their own classes. First the teachers were made to analyse the characteristics of the coin tossing experiment. They mentioned that there are two possible outcomes and these are randomly generated. They were then asked which key in the calculator would enable them to simulate the experiment. Almost all of them were unfamiliar with the random number generator. And so they were asked to observe what would happen each time they pressed this key. They noticed that the numbers were different each time which meant that the results were random and that they were either odd or even which meant that there were only two possible outcomes.

The teachers were asked why one might prefer to use the scientific calculators if these were available instead of actually tossing a fair coin. One reason that they gave was that it could save time. Another was that it could provide uniformity in performing the experiment. If one actually tossed a coin, s/he might not uniformly do it each time so the results might be affected. In contrast, in the simulation, the conditions could always be the same. The teachers decided that when they do the simulation, each time the result was an even number, they would take it to mean that a head came up. If it was an odd number, then it would mean that a tail came up. Conducting the simulation elicited good discussions as the teachers compared it with what it represented.

Another way of promoting mathematical communication is through interpreting graphs and making inferences based on these interpretations. In a certain teacher training program, the teachers were asked to write stories that might be associated with graphs below. Likewise, given a story, they were asked to draw the associated graph.

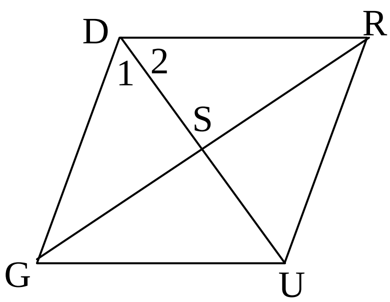




From the stories or graphs given, certain misconceptions surfaced and hence, were addressed.

Fostering Sympathy

Considering other people's ideas and ways of thinking with respect is important in creating a classroom environment that promotes communication (Silver & Smith 1996). This situation could be exemplified by how a class dealt with the erroneous answers committed by students. For example, in a grade 8 class in the Learner's Perspective Study, the students were asked to find the measures of the angles of a parallelogram. Different groups were given different items to work on. After the groups had finished working, they were asked to present their work to the whole class. Thus, each group's work became the object of both reflection and evaluation of the other groups. Since the other groups had done different items, they had to listen carefully and attentively to the group's presentation in order to fully understand it. One group was assigned to work on the following.



DRUG is a rhombus.

If m $\angle 1 = 5x - 10$

and m $\angle 2 = 3x + 20$,

find the measure of $\angle$D, $\angle$R, $\angle$U, and $\angle$G.

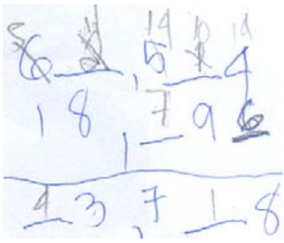
The representative of the group said that together $\angle 1$ and $\angle 2$ measured 180° since they were supplementary. Pursuing this thinking, he obtained incorrect values for the angle measures. The student did not notice that even based on the figure alone, his thinking was not correct. A classmate called the teacher's attention and said that $\angle 1$ and $\angle 2$ were not supplementary but they were congruent. The teacher analysed the item and remarked that since the diagonals of a rhombus bisect opposite angles, then $\angle 1$ and $\angle 2$ were congruent. Only then did the class including the one who presented their supposedly group output but which he alone did, realized that his answer was incorrect. The student felt embarrassed but the teacher assured him that it was alright and asked him and his group to work on the item again and correctly this time.

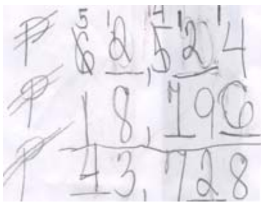
The situation exhibited the spirit of sympathy where people tried to sincerely understand what others had done and how they thought about it. Based on such understanding, they offered useful evaluation for them to learn if they did not get the expected answers and get help when needed (Gallos & Ulep 2007).

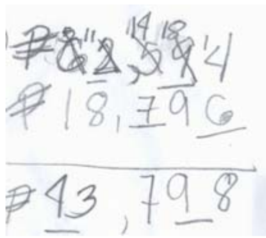
TEACHING THROUGH PROBLEM SOLVING: A STRATEGY TO DEVELOP MATHEMATICAL COMMUNICATION IN THE CLASSROOM

What have been Accomplished so Far?

To a large extent, mathematics teaching in the Philippines is still characterized as teaching for problem solving. The definitions, concepts, or procedures are all presented first by the teacher and then several illustrative examples are given. After this, several so called "problems" which should be more appropriately referred to as exercises, are provided to which the students are expected to use the concepts or apply the procedures that they have been taught. Increasingly though very slowly, teaching through problem solving is being promoted through teacher training programs and curriculum materials development at UPNISMED and more recently through lesson study in four schools. Here, a problem which is an unfamiliar situation that needs a solution for which students may not have readily available prescribed procedures to use, is presented and the students are expected to generate their own original methods of obtaining an answer/s. Solving open-ended problems, that is problems which have many different solutions or even correct answers, provide rich opportunities for mathematical communication. Following are examples of the different answers that the students got using different reasoning in solving word problems on subtraction with regrouping involving missing digits. The students were asked to explain their work (Ulep 2007).







In the schools where lesson study has been ongoing, with much effort, teachers are already beginning to use problem solving to develop concepts or make students investigate mathematical relationships. In these classes where there are on the average 50 to 60 students which is typical in the Philippines, to encourage communication related to open-ended problem solving, students are placed in smaller groups for them to collaboratively come up with the desired answer. So students are also now learning to discuss in groups. Though these practices may not yet be claimed to be already a part of their classroom culture, there are already instances where students help one another to: clarify their interpretation of the problem, restate it in their own words to make sense of it, sometimes draw to help them visualize what it means, use symbols to represent relationships, device their own ways of recording what they are doing, make conjectures based on the results or data that they gather, test them, and when appropriate make generalizations, and verify if their answer is correct.

During the presentation of their group output, representatives of the different groups explain their work in ways that they can clearly be understood by the other groups. They reason to support their answer and to convince others that what they had done is correct.

Teaching mathematics through problem solving has been changing the ways teachers are teaching as well as the ways that the students are learning. For instance, in one elementary class, the teachers had decided to do away with the drill and review and to start right away with asking students to solve the problem realizing that they would need more time to think about the solution which was more important than practicing them with drills. Another example was in a high school class, where during the post observation discussion, an observer asked the teacher why she omitted the review and gave the students the problem right away. The teacher responded that this was the way that they were trained in the training where lesson study was introduced. They themselves did not know what to do and how to deal with the problem. They later realized that this was part of solving the problem.

What Else Need to be Done?

The teacher has a big role to play in developing mathematical communication in class. There are already some practices that they are changing to achieve this. A good example is that through the students' working in groups, they are no longer doing most of the talking in class. But there are still many practices that they need to change. For example, the tasks that they give should be challenging enough to demand collaborative work. Another is that the quality of the questions that they ask needs to be raised and be made more open. Still another is that, they should be able to model how from a single problem, other extension problems can be formulated. Furthermore, at appropriate parts of the lesson, they need to synthesize important ideas that emerged.

Teachers also need to improve how they handle students' incorrect responses. When the response is written or posted on the board, they should not just erase or mark them wrong or when it is given orally, they should not ignore it and call on other students until s/he gets the correct answer. Instead, they can involve the whole class in analysing why it is incorrect. Through probing, the students' understanding can be clarified and deepened.

Together with students, teachers also need to establish socio-mathematical norms. When students present their work to the whole class, it becomes public and hence the object of other people's evaluation. So their presentation also encourages communication. From their different solutions or answers which represent different ways of thinking, there are those that are correct or incorrect. And among the correct ones, there are those that are more efficient or represent more sophisticated ways of thinking. While all these different correct solutions or answers are acceptable, students can compare them and discuss which are better based on criteria such as efficiency, sophistication and others. Such expectations should become part of classroom practices (Yackel & Cobb 1996).

The blackboard is a good tool for communicating the progress of mathematical thinking that happened during the lesson. Teachers need to use the blackboard more systematically and teach their students to do likewise to make the most of what can be communicated through it.

Lastly, teachers need to change their assessment practices. Currently, the type that is most commonly used in the Philippines is multiple-choice. However, it is not able to assess communication skills which the curriculum considers important to develop. Moreover, since students can get the correct answer for the wrong reason, teachers can be misled that the students have learned when in fact they have not (Cai, Lane, & Jakabcsin 2006). This was one reason why Filipino students did badly in TIMSS (Ibe 2001). They were not used to answering items that required explanation. As the emphasis in teaching is becoming more oriented towards problem solving, reasoning, and communicating, assessment has to include more open-ended items that would require students to explain their thinking or reasoning. This change or improvement and all those cited early on can be gradually achieved through lesson study.

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