The Singapore Mathematics Curriculum and Mathematical Communication

Yeap Ban Har
National Institute of Education
Nanyang Technological University
Singapore

Introduction

In this paper, we consider various aspects of the curriculum to determine the extent to which the Singapore mathematics curriculum includes mathematical communication. The term curriculum can be interpreted and investigated at different levels. First, we can consider the intended curriculum by examining the curriculum document. Second, we can consider the hidden curriculum by examining the nature of tasks in the national examination. Third, we can consider the interpreted curriculum by investigating the textbooks. Finally, we can consider the implemented curriculum by studying actual classroom lessons. In this paper, we do the first three. The proposed study, to be reported in the next meeting, focuses on the last.

Based on an analysis of the curriculum, and a review of the literature, a framework that describes various aspects of mathematical communication is described along with suggestions on how mathematical communication can be infused into the mathematics classroom. The paper concludes with a proposal to use lesson study as a platform for teachers to gain a deeper understanding of mathematical communication, its relevance in mathematical thinking and problem solving, and how to organize classroom activities to promote it.

The Singapore Mathematics Curriculum

The Singapore mathematics curriculum (Ministry of Education of Singapore, 1990, 2000, 2005) has mathematical problem solving as its primary goal. According to the curriculum, the development of mathematical problem solving is dependent on the development of five inter-related components, namely (1) concepts, (2) skills, (3) processes, (4) attitudes, and (5) metacognition.

The processes listed in the revised curriculum framework (Ministry of Education of Singapore, 2005) include reasoning, communication and connections. For primary schools, this revised framework was implemented in 2007 for grades one to four. Subsequently, the revised framework will be implemented in 2008 for grade five and in 2009 for grade six. Among the stated aims of the curriculum is to "develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently" (Ministry of Education of Singapore, 2005, p.5). As the curriculum is designed to "ensure that we have an increasingly competitive workforce to meet the challenges of the 21st century (Ministry of Education of Singapore, 2005, p. 5)", communication has to be an important part of the curriculum.

In the curriculum document, communication is used to "refer to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically" (Ministry of Education of Singapore, 2005, p.8). Communication is described as a way to help students develop their own understanding of mathematics and to sharpen their mathematical thinking.

The curriculum also made a call for changes in the way teachers teach and students learn mathematics due to advances in technology. Students should have opportunities to discover, reason and communicate mathematics by engaging in "stimulating discussions and activities (Ministry of Education of Singapore, 2005, p. 2)" to explore possibilities and make connections.

These qualitative changes require a change in the teaching and learning approaches. Lesson study holds promise as a professional development tool to effect such a change.

It is evident that the intended mathematics curriculum in Singapore places a great deal of importance on mathematical communication.

The Role of the National Examination

At the end of six years of primary schooling, students in Singapore sit for Primary School Leaving Examination (PSLE). Only 20% of total score comes from selected-response (multiple-choice) items. Among the constructed-response items, the majority require students to communicate their methods.

The following are selected-response items that assess ability to solve problems.

> Alice bought some guppies. She gave away 11 of them. Benny gave her the same number of guppies as the number of guppies she had left. She put all the guppies equally into 8 bowls. Each bowl contained 13 guppies. How many guppies did Alice buy?
>
> (1) 52
> (2) 63
> (3) 104
> (4) 115
>
> (SEAB, 2007, p. 9)

> The chairs in a hall were arranged in rows. Each row had the same number of chairs. Weiming sat on one of the chairs. There were 5 chairs to his right and 5 chairs to his left. There were 7 rows of chairs in front of him and 7 rows of chairs behind him. How many chairs were there in the hall?
>
> (1) 140
> (2) 150
> (3) 154
> (4) 165
>
> (SEAB, 2007, p. 9)

While knowledge-level items require minimal level of reading and comprehension ability, problem-solving items in the selected-response format expect students to be able read reasonably complex paragraphs.

The following are constructed-response items that assess ability to solve problems.

> Siti started saving some money on Monday. On each day from Tuesday to Friday, she saved 20 cents more than the amount she saved the day before. She saved a total of $6 from Monday to Friday. How much money did she save on Monday?
>
> (SEAB, 2007, p. 40)

> At 09 00, a lorry started from Town X and travelled towards Town Y at a speed of 55 km/h for the whole journey. At 11 00, a car started from Town Y and travelled towards Town X. The speed of the car remained the same until it passed the lorry at 13 00. At this point, the lorry had travelled $\frac{5}{9}$ of the journey. After passing the lorry, the car decreased its speed by 8 km/h and travelled at the new speed for the remaining journey. At what time did the car reach Town X?
>
> (SEAB, 2007, p. 41)

Similarly, these constructed-response problem-solving items require students to be able to read reasonably complex paragraphs. In addition, students are expected to be able to communicate their thinking and methods by showing how they reason and arrive at the answer. Typically, up to 80% of the total score for an item is awarded for the ability to communicate one's method.

In Singapore, students learn mathematics (and all other subjects except mother languages) in the English language from the first grade. This is despite the fact that only 21% of the sampled students reported that they always use English at home in PIRLS[1] 2006 (Ministry of Education of Singapore, 2007). The prevalent use of English in schools probably contributed the increased proportion of students who use English at home. In PIRLS 2001, 37% of the sampled students reported that they used English at home. In PIRLS 2006, 42% did.

The nature of the national examination has made it necessary for students to be able to function at both ends of the communication process, i.e. to be able to understand and to be able to be understood.

The Role of the School Textbooks

Textbooks used in Singapore schools have to be approved by the Ministry of Education before schools can elect to use them. A complete analysis of textbooks used in Singapore is not within the scope of this paper. However, several textbook studies had shown that one common method used and taught in the textbooks for problem solving is the so-called 'model method'. Figure 1 shows an example from a textbook for the fourth grade.

> Ravi and Samuel collected 1775 stamps altogether. Samuel bought another 85 stamps. Ravi then had thrice as many stamps as Samuel. How many stamps did Samuel have at first?

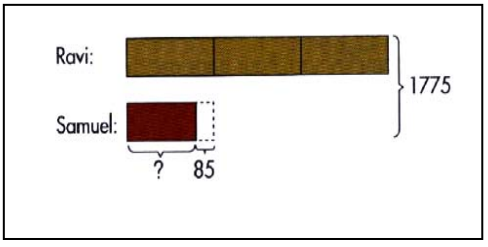

Figure 1 Word problem and the 'model method'

The use of 'model method' provides students with a means to handle information, deal with complexity and, at the same time, communicate their thinking through the use of visuals which they can manipulate.

[1] Progress in Reading Literacy Study (PIRLS) was conducted in 2001 and 2006.

The extensive use of this problem-solving heuristic in the textbooks is probably the main reason why most teachers include it in their lessons and why many students are comfortable using it as a problem-solving and communication tool.

From an investigation of the curriculum document, national test items and textbooks, it becomes clear that consistency among the three elements of the curriculum is probably the reason why you would expect mathematics teachers in Singapore to place emphasis on being able to handle the language aspects of word problems and communicating their solutions in writing and using diagrams. In this case, the curriculum document does demand mathematical communication. The national test expects students to be able to engage in mathematical communication. The textbooks provide tools for students to do so, even in fairly complex tasks.

Aspects of Mathematical Communication

In this section, we further clarify the different aspects of mathematical communication. There is the ability to understand and ability to be understood. The ability to read and comprehend word problems is an example of the former. The ability to present one's thought in solving problems is an example of the latter.

In either aspects, there are the skill aspects, the metacognitive aspects and the attitude aspects. In understanding, examples of skill aspects of communication include ability to read. In making oneself understood, examples of skill aspects include ability to draw 'models' to represent one's thoughts during problem solving. In understanding, examples of metacognitive aspects include the ability to monitor one's reading so as to achieve comprehension. In making oneself understood, examples of metacognitive aspects include the monitoring of one's solution as one is solving it. In understanding, examples of attitude aspects include not possessing negative feelings towards lengthy word problems. In making oneself understood, examples of attitude aspects include the willingness to explain oneself in writing although one may be able to see the entire solution mentally.

The above focuses on aspects of mathematical communication in the process of solving problems. There are also aspects of communication in mathematical activities other than problem solving.

Research Plan

The general research proposal is to use lesson study to help teachers learn more about mathematical communication, its development and its role in mathematical problem solving (and mathematics learning).

During the Tokyo-Kanazawa session, we will have opportunities to benefit from several keynote lectures. We will also have opportunities to see how different economies and researchers interpret what mathematical communication is, what types of instructional strategies can promote communication and the roles of communication in mathematics learning including problem solving.