Key Questions for Focusing on Mathematical Communication

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Why do we focus on mathematical communication?

Currently, the publications of massive character - as newspapers, magazines or advertising leaflets - include in the presentation of the information: geometric graphs, statistics graphics, numerical tables and other schemes of mathematical type that allow people to accede to the information in a compact, synthetic and precise way. Due to this increase in the use of symbols and mathematical concepts in the processes of social communication, the basic education in mathematics has been forced to incorporate the codification and decodificación of these elements, the interpretation of the same ones and the aptitude to argue using foundations of mathematical base. This is one of the reasons as which, in the curriculum for the Basic Regular Education in Peru, we have identified three fundamental capacities, one of which is the Mathematical Communication, another one is the capacity of Reasoning and demonstration(proof) and the Resolution of Problems. We know that, in the frame of the process of mathematical thinking, these are not capacities developed independently, but rather complementary.

We depart from the premise that the knowledge must allow to take decisions about a problem that it is necessary to resolve, also it must allow to communicate the chosen procedures, defend and validate what has been done, to confront it and to compare it to what others did. It is a question then, of generating a space in which the child has the possibility of learning to argue in favour of his products, to thinking if he agrees or not with the results of his classmates, to taking the interesting ideas of others and to checking his own ideas.

The communication, that is not exclusive of mathematics, forces to returning on the own thinking to specify it, to justify it, to clarify it. The communication in mathematics includes the own expression, in a variety of ways, of a problem with a mathematical content, so much in oral as written form.

To communicate a resolution allows to make explicit what was implicit and there makes possible the recognition of this knowledge by the individual. To report on the produced necessarily implies the reconstruction of the realized action (Ressia, 2003).

Defined in our curriculum, the mathematical communication is one of the capacities of the area that acquires a special meaning in the mathematical education because it allows to express, to share and to clarify the ideas, which get to be an object of reflection, development, discussion, analysis and readjustment. The process of communication helps to, also, give meaning and permanency to the ideas, to express them, as also to listen to the explanations of the others, giving opportunities to develop the comprehension, that is to say, to establish the mathematical connections between the ideas. We also consider that this capacity contributes to the development of the language itself, which allows to express the mathematical ideas accurately.

Due to the fact that the mathematics is expressed through symbols, the oral and written communication of the mathematical ideas is an important part of the mathematical education.

As the degrees of education go on, the communication increases its levels of complexity.

It is necessary to consider the autonomy of the mathematical language in relation with the daily language. For example the "equal" term in mathematical language means that two different expressions designate the same mathematical object; this way in the equality " 3+4 = 9-2 ", both

"3+4" and "9-2" represent the number "7", and for that we say that " 3+4 equal 9-2 "; whereas in the Castilian language we use daily, "equal" means "similarly", "familiar".

To understand and to use the mathematical ideas is fundamental the way in which they are represented. Many of the representations that currently seem natural to us, such as the numbers expressed in the decimal system or in the binary one, the fractions, the algebraic expressions and the equations, the graphs and the spreadsheets, are the result of a cultural process developed throughout many years.

The term representation refers both to the process as to the product (result), this is, to the act of catching a mathematical concept or a relation in a certain way and to the way itself, for example, the student who writes his age using his own symbols, uses a representation. On the other hand, the term is applied to the processes and to the externally observable products and also, to those that take place "internally", in the mind of those who are doing mathematics. Nevertheless, it is important to consider that the students who speak an original language and they do not have the Spanish as a mother language, need additional help to understand and to communicate their mathematical ideas.

The forms of representation, as the graphs and the symbolic expressions, must not be considered as goals of the learning itself, because of being ways of mathematical communication and not either capacities or contents.

By fault, they must be treated as essential elements to sustain the comprehension of the concepts and mathematical relations, to communicate approaches, arguments and knowledge, to recognize connections between mathematical concepts and to apply the mathematics to real problems.

The reading of the mathematical language helps the students to develop their skills to formulate convincing arguments and to represent mathematical ideas in a verbal, graphical or symbolic way. It also refers to the aptitude to obtain and to cross information from different sources (texts, maps, graphs, etc.) to be able to:

- To organize and to consolidate their mathematical thinking to communicate.
- To express mathematical ideas in a coherent and clear way to their classmates, teachers and others.
- To extend their mathematical knowledge together with the thinking and strategies to other areas.
- To use the mathematical language as an economic and precise way of expression.

2. What are your components of mathematical communication to develop?

The Mathematics has its symbolic and formal language, possesses linguistic forms that express operations or transformations and refers to certain reasoning that must be motivated by specific concepts.

In its condition of matter of study, the Mathematics establishes itself since the first years of education, with a series of codes that invade all the spaces of the language; the child acceds to the meeting of laws and procedures that indicate him mathematical behaviors very defined to find solutions that get to be simple goals of the commonness and that includes numbering, counting, ordering, classifying and even inferring, and there is where the verbal communication represents the most effective way to explain the mathematical ideas orientated to the comprehension of the concepts. In this respect, it is important to accentuate that because of the complexity of the Formal Language, constituted by the incorporation of strange symbols more than words, is what makes the children do efforts to understand the Mathematics, because they do not manage to establish relations between the daily language and the formal one. On the other hand, the widespread utilization of the formal language in the classroom by the teachers has serious consequences, since instead of molding the mathematical uses attending to its informal language, emphasizes this special language of the Mathematics in a confusing way (Pimm, 1999). Also, there are the multiple meanings, typical of many mathematical terms, this is due to the fact that in occasions words of daily use are taken to interpret any symbol, but they do not always they adjust to them accurately.

In this order of ideas, Fennell (mentioned by Ruiz, 2003) indicates that "in the mathematical communication the standardized symbols and the definitions of the terminology are necessary, but the education of the mathematics in very formalized language, sometimes, causes a kind of blockade in the comprehension ". This situation must be handled carefully by the teacher who thinks that the pupil is understanding the mathematical concepts, nevertheless the results obtained in the evaluations applied by him demonstrate the weaknesses in the acquisition and comprehension of Mathematical knowledge. On the other hand, Gonzalez (1998), expresses the negative consequences of the traditional approach of the education of the mathematics where there is no emphasis in the transmission and comprehension of the formalized language, where the communication processes are unilateral and the transmission of the information that is acquired by the pupil mechanically without understanding prevails.

With this preamble, I might identify that, among the componentes of the Mathematical Communication we would have to consider: the acquisition of the oral language, which develops across cognitive processes, but depending on the social and cultural environment; the representation and symbolic interpretation.

It is so the quantity of experiences that the child realizes - consciously of his sensory perception - with himself, in relation with the others and with the objects of the surrounding world, transfer to his mind some facts on which he elaborates a series of ideas that are useful to relate himself with the outside world.

The interpretation of the mathematical knowledge is obtained across experiences in which the intellectual act is constructed through a dynamics of relations, about the quantity and the position of the objects in space and time. The logic - mathematical thinking must be understood since the development of the cognitive (internal) processes that imply:

- Aptitude to generate ideas which expression and interpretation on what it is concluded is: truth for everyone or false for everyone.
- Utilization of the representation or set of representations with which the mathematical language refers to these ideas.
- To understand the environment that surrounds us, with major depth, through the application of the learned concepts.

A). Cognitive development

Attending to the different statements presented by Piaget, the stage of the concrete operations coincides with the principle of the education and is decisive in the intellectual development of the child, that means; in this stage are generated the more sophisticated conducts in relation to the quantity and to the reasoning, especially in the area of the mathematics.

Because of that, it is necessary to have supremely care when starting to teach arithmetical, algebraic and geometric knowledge before the appearance of the operative thinking, otherwise the children would have a very limited comprehension to generalize and to reason correctly.

To order, to classify, to bring together or to separate is fundamental so then raise the thinking. With that said, a long exercise of action is necessary to construct the substructures of the later thinking.

For Vigotsky (1978), the cognitive development does not take place in an isolated form, but side by side with the development of the language, with the social development and even with the physical development. Thus, the comprehension and the acquisition of the language and the concepts by the child are made through the meeting with the physical world and by the interaction between the persons that are around him. The acquisition of the culture with sense and meaning supposes a form of socialization where the teachers, the family and friends, with their mediating of the learning function, facilitate the acquisition of the social culture and uses, both linguistic and cognitive.

B.) Cognitive development in the Mathematical Language

Piaget (1967) supports: " The language can constitute a necessary condition for the development of the logic - mathematics operations without being a sufficient condition of education. " (P. 59).

In this order of ideas, we can conclude that the derivation of principles of constructive learning, of concrete representations, of comprehension of the language, of social response and of interaction between teacher and pupil is what would contribute to the construction of the mathematical thinking. This way, the mathematical language is consolidated and acquires great strenght in the measure that is revealed as an effective representation of certain deep structures; because of that, one way or another, the mathematics appear in each of the manifestations of the culture.

To explain the acquisition of the oral language, the social lingüists consider not only the acquisition of structural rules of the language but also the characteristics of the social group, because both are relevant factors for the process of acquisition and development of the language, besides it is also affected by the individual and group differences (Halliday, 1986).

C.) Social and cultural component of the Mathematical Language

The language and the numerical system are communicated from the early infancy: In what concerns the mathematical field several researches have been done, such as the Saxe (1991) who conceived the number as a system constructed culturally, in the same way as Vygotsky understood the construction of any system of signs. He extended his research to relate the social class and the children's numerical competition in pre-school age, concluding that the social class was a determiner of the numerical environment.

His work considers that some linguistic phenomena that can be found in the context of the mathematic lessons, are about 3 general points of meaning; symbols, symbolized things and syntax. As for the meaning; the pupil uses an important plot of mathematical knowledge to provide consistency to the meaning; as for the symbols and symbolized things; he shows that confusions can arise when the pupil focus on the symbols themselves, instead of what they mean, and as for the syntax in mathematics; it is possible to formulate some transformations in an analogous way, in which case the algebra can be considered as a manipulation of symbols according to certain rules. For such a reason in algebra many mistakes take place, because it could be focused in an abstract way and manipulating symbols without paying attention to the possible meanings.

D). Capacities of the Symbolic language

The school mathematics materializes in a language constructed thanks to the increasing human capacities for the symbolization, not verbally in the first stage and verbally later, wich can be observed in the first infancy. The language of the school mathematics therefore, uses a kind of signs constituted by verbal suitable expressions, for example: four, equal, plus, minus, over, under, etc. And notational signs such as: 4, =, +, -, >, <, etc. The school mathematics is learned, according to our understanding, following the model of symbolically through construction, of wich the major or minor control of the semiotic instruments of mediation, for example: the linguistic and notational signs, will favor or not the advance of this learning. Nevertheless, before the formal education of the mathematics and even in absence of this education, the children through all the cultures have experiences of informal mathematics that are easily articulated by their natural aptitudes to observe and understand the phenomena of the control of the mathematics, for example, the estimation of quantities and the not verbal calculation. To this respect, we understand that the mathematics constitutes a natural control of the human thinking, inherited from million years of evolution of our brain in a cultural context in which the numbers (as the words) are an essential parameter.

E.) The mental representation of the quantity

Recent Experiences allow us to suppose that the two-year and a half-old children possess some aptitudes for the counting. It is therefore possible to think that the aptitude to count or solve simple problems of arithmetic is as natural as the language.

Actually, until the 50's, specialists of the cognitive development of the child thought that the numerical capacities were appearing late in the child, nevertheless, non verbal tests have demonstrated that the one-year-old babies can identify small quantities, add them and reduce them (Gelman. 1983). We will begin wondering if the sense of the quantity is specifically human. There has been demonstrated in research with animals that the monkeys, the dolphins and the birds have an elementary sense of the quantity, similar to the babies one. The aptitude for the numerical perception in the animals and the human beings is observed in simple tasks of discrimination (1) as the comparison of two quantities, on having changed the height (size effect) and on having changed the distance that separates them (distance effect). For example, the human beings of 6 months-old and the animals can differentiate 8 from 16 or 16 from 32 but they cannot differentiate 9 from 10. These numerical capacities shared between human beings and animals support the hypothesis that they are the result of a long evolutionary history. The researches in surmise, on the other hand, the evolutionary advantage that contributes to the perception of the quantities when it is a question of estimating a quantity of food or the size of a group of compatriots.

F. The capacity for the non verbal calculation

On the other hand, the neuropsychology based on the research of the functional nets of the brain cortex allows to advance the hypothesis that there would be at least two cerebral systems involved in the mental calculation (Dehaene, S., Molko, N. and Wilson, A. 2004): a non verbal system, based on the sense of the numbers and the manipulation of the quantities, and another one, verbal, based on the memorization of calculations independent from the perception of the numbers, for example, simple addings: 2+2, 20+20 and tables of multiplication. The first one of these systems (intraparietal) is activated in both cerebral hemispheres (according to observations in the cerebral imagery) in all the tasks that require manipulation of quantities, for example, in the visual presentation of quantities and the numerical estimation of groups of objects. In effect, the calculation does not always need the verbal memory; many operations as the adding, the subtraction or the comparison need to manipulate only quantities without resorting to the memorization of tables. Nevertheless, though we share the visual estimation of quantities with other animals, the learning of the language and the writing allow our human intraparietal region to be activated equally by effect of the symbolic notation of quantities or numbers (For example the Arabic numbers, the marks) in sticks or ropes for the count). This changes radically the process of symbolization and mathematical abstraction that takes as a base the human capacities constructed in the course of the evolution.

So that, the language of the mathematics has a non verbal original stage that we must not scorn because in normal conditions the children from a very early age can estimate quantities, manipulate groups, compare, add up, etc. This is confirmed by the researches that demonstrate that up to the age of five, the simple problems of adding and subtraction are solved better when they appear in a non verbal form. For example, an experience carried out by Starkey, P. (1983) allowed to study the adding and the subtraction with 4-year-old children. The child had to place from 2 to 4 identical objects in a box, one after another one. Later, the experimenter in front of the child extracts an object, adds other one or does not do anything. Later, he asks the children to extract the objects of the box one at a time. It is a question of knowing how many times the child puts the hand in the box. He observed that 80 % of the children introduces the hand the times corresponding to the number of objects that remain in the box. Nevertheless, some of the same children failed when the operation was explained to them verbally in the shape of terms of reference: in the shape of history or numbers of objects to add up. Only at the age of 5 years and more, independently of the type of presentation (visual or verbal) of the operation of adding, the results are positive.

G) The use of symbolic materials

With the entry in the school education the mathematical symbolism reaches the first stage of the learning of the mathematics. Before this happens, the apprentice has used environmental materials to count (stones, small sticks), has learned to use his fingers as symbolic material. Nevertheless, in the school, to construct informal experiences of calculation and to favor his learning, we resort to the direct education guided by the adult, with objects or symbolic resources as the representations (abstractions) of the quantities or of the numbers. These representations can be analogical, that is to say, those which are similar to the represented (constellations of points, dices, blocks) and are used as representations of quantities or representations of the numbers themselves. The other representations are conventional, that is to say, they are not similar to the represented quantity, they are the notations written as the notational symbolism of figures and related signs. In the school, therefore, the formal education is favoured to get to operate with, to understand, to handle other abstractions: the numbers. And the most important thing is that the action itself is mediatized by the symbolic used instruments: the verbal language, the symbolic material (environmental or constructed), the graphical representations of quantities. To operate with symbolic instruments, to graphically represent quantities is a necessary experience of mathematical abstraction in the way towards the exclusively notational.

H) The notational symbolism

The use of the language and symbolic materials are two instruments, two ways of representation of the number that lead to another one: the notations and related signs. In effect, the figures refer to other previous symbolizations and acquire meant as a result of previous learnings: verbal knowledge of figures, count and manipulation of materials, graphical representations. Now then, the learning of the notational expression of the number is a new way of expressing the already possessed knowledge on quantities. In effect, the evolution of this learning is ending in the possibility of formalizing, of expressing with arithmetical writing, increases and decreases of quantities: the decomposition and numerical alteration and other operations of increasing complexity.

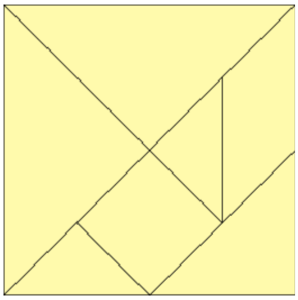
A constant process of symbolization is necessary for the full construction of the mathematical language, process that begins, as we have seen, with the perceptive estimation of quantities and it continues with a meaningful learning of the first numerals), of the notations (figures) and of the related signs. In effect, the school learning of the mathematics is done properly when the children are capable of assimilating the mathematical language, that is to say, when is possible for them the assimilation of symbols: the use of symbols and of symbolic structures that is done most of the time with the words (for example, the ones that refer to quantities: two, three, etc., or the relations between quantities, for example, over, under, equal, etc.) because the words are the expressive support of the mathematical meanings that are constructed.

What kinds of approach will you prefer to develop the communication in classroom?

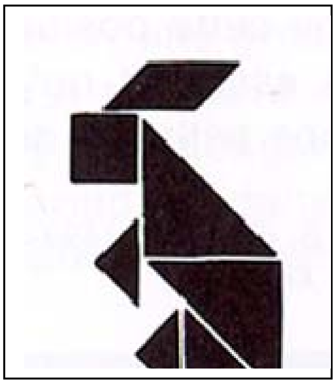
The use of oral and written communication as a tool that allows the pupils to reflect their comprehension of the mathematics, will help them to personalize and realize connections between mathematical concepts. When the pupils communicate mathematical information, they remember it, understand it and use it to discover and to find more information (Perkins 1992).

The teachers need to know how to help the pupils to turn into competent mathematical communicators who can describe their process of thinking in a clear way. The teachers must help them to make their thinking visible to others, stimulating them to speak and write about the process that they follow when they solve a problem.

It could be an activity in the classroom in which each of the children possesses a complete game of Tangram:



A child, whom we will call A, after constructing a mathematical figure with all or some of the pieces of the tangram (for example, a triangle or a parallelogram), explains to his companions how he has done that without showing them the constructed figure at any moment.



The others will have to reproduce the figure that there has been formed by the child A through the indications.
The children will be able to do the explanatory questions in case of being necessary.
Once achieved the goal, another child will be able to expose his work.

The quantity of questions that the children ask will be registered. Finally, we will be able to identify the degree of mathematical communication of the children, analizing if they have received more or less questions to solve the figure.

1. Approach of a situation.

- The children listen to the explanation of the activity.

- They listen to the indications of the participant who describes the construction of the figure.
- The children, mentally or in writting, re-formulate the indications with their own words and propositions.
- They form their ensambles of figures.

1. Formulation of inducting questions for the analysis of the situation.

- He/She identifies the compositions from the indications.
- They do explanatory questions:
- A: " two triangles (a big one, a medium one) are put together by their bases, remaining the biggest below and the smallest above ".
- B Asks: how can we know which are the bases that are joined?

- He relates graphs, drawings and verbalizes the relations found.

3. Comparison and relation of the registered information.

- He relates written records.
- Construction and use of graphical schemes to read and interpret the symbolic propositions.

4. Construction and use of oral and written records to elaborate propositions and to communicate results, decisions, opinions. They show the formed figure and explain the process that they followed to achieve it.

Approach of a situation.
- Recognition of the meaning of words and symbols.
- Ordained formulation of the steps to obtain information

To verbalize, to annotate known terms, previous knowledge, relations, characteristics.

To explain with propositions or their own words, to show the object, to verbalize, to explain the situation and goal. To represent graphically

Formulation of inducting questions for the analysis of the situation.
- Use of oral records to transmit their ideas.
- Uses written records to transmit their ideas.
- Graphical or ideographical to represent their ideas.
- Use of graphical and symbolic records to transmit their ideas.

Selection of the relevant information: He identifies propositions, variables, information.

Relates words and symbols.

He relates graphs, drawings. He verbalizes the relations found.

Comparison and relation of the registered information.
- He relates written records.
- Construction and use of graphical schemes to highlight the existing relations (tables, graphs, conceptual maps).
- Construction and use of symbolic propositions to establish relations.

He completes pictures of similarities and differences, using symbols and graphs.
He formulates symbolic propositions: equations, operations.
He reads and interprets the symbolic propositions.

He elaborates oral and written records: tables, numbers, symbols.

Construction and use of oral and written records to elaborate propositions and to communicate results, decisions, opinions.

Graph, uses numerical or graphical schemes to communicate ideas.

They explain the process step by step to sustain an answer (they argue their answer)