Paper presented at APEC-TSUKUBA International Conference III
December 9-14, 2007, Tokyo and Kanazawa/Japan

Development Mathematical Communication in the Classroom

Thi Thi Khaing, Kunihiko Hamaguchi, Minoru Ohtani
Kanazawa University, Japan

In relation to the three questions for the specialist session, firstly, how mathematical communication is enhanced in the overall objectives of Japanese mathematics school curriculum by JSME. Secondly, the expression on how logical thinking and reasoning enhances deductive proof in lower secondary geometrical problem by learning from teaching learning classes. Thirdly, overview on open process approach in problem solving base on teaching learning activities.

Answering on question 1: Why do we focus on Mathematical Communication? On national curriculum document, how it enhances communication or mathematical communication for students?

(Before answering these questions, I would like to greeting that I am a scholar and came to Japan to study under the program of *In-service Teacher Training (eighteen months*) *Program.* However, my paper will be written about what I see and learn from Japanese Education.)

As I am scholar who do not (or could not) understand Japanese Mathematics Curriculum well, so I was in difficult to answer to this question. Although I tried to understand and write out how Japanese national mathematics curriculum (including the general document of whole curriculum) enhances mathematical communication for students, I could not imagine. However, after had a discussion with my Academic Processor (Mr. OHTANI MINORU) I was understood how Japanese Curriculum is. In the document that edited and published by Editorial Department of Japan Society of Mathematical Education (JSME), the “Overall Objectives” for Mathematics in school levels are stated as follow:

1) For Elementary School

Through mathematical activities concerning numbers, quantities and geometrical figures, children should get basic knowledge and skills, should get abilities to think logically and to think with good perspectives, should notice the pleasure of doing activities and appreciate the value of mathematical methods, and should get attitudes to make use of mathematics in daily life situations.

2) For Lower Secondary School

For the students to understand deeply the fundamental concepts, principles, and rules relating to numbers, quantities, figures and so forth. For students to acquire methods of mathematical expressions and strategies, and to improve their ability to relate phenomena mathematically. For students to enjoy mathematical activities, to appreciate the importance of mathematical approaches and ways of thinking, and to inculcate in them the right attitudes necessary to make use of mathematics.

3) For Upper Secondary School

To help students deepen their understanding of the basic concepts, principles and laws of mathematics, and to develop their abilities to think and deal mathematically with various phenomena, and thereby to cultivate their basic creativity through mathematical activities, and to help students appreciate mathematical ways of observing and thinking, and thereby to foster attitudes which seek positively to apply the qualities and abilities mentioned above.

By learning those overall objectives and other general document of whole curriculum, it does not express to focus mathematical communication. And also in the national curriculum and the textbooks, mathematical communication is not described out and emphasized to write down. However, when we learn deeply on Japanese Mathematics Education there is a place that mathematical communication is strongly enhanced and emphasized. That place is in the teaching classrooms.
In Japanese teaching learning classes, usually they are teaching and learning through observation, manipulation, and experimentation. And when they have to solve the some challenging problems students are being solved together as a group and work mathematical activities collaboratively. More other one is teachers let students explain their solutions in the class. Therefore, mathematical communication is emphasized to develop in mathematical teaching classroom by students' participation. In addition, Editorial Department of Japan Society of Mathematical Education (JSME) states *the main features of mathematics curricula* as follow:

Term	Periods	Main Feature of Mathematics Curricula
I	The second half of the 1940's	Children Centered
II	The first half of the 1950's	Unit Learning
III	The second half of the 1950's	Mathematical Ways of Thinking
IV	1960's	Systematic Learning
V	1970's	Mathematical Modernization
VI	1980's	Basics, Problem Solving
VII	1990's	Individualization, Informatization
VIII	**2000's**	**Mathematical Activities**

By this table, the main feature of mathematics curricula around 2000's is emphasizing on Mathematical Activities. In Japan, It could be assumed that mathematical communication is to not to be described in curriculum document but to be emphasized, developed and used well by teaching and learning activities. By emphasizing mathematical activities, the term to focus mathematical communication had already done. And many educators agree that learning opportunities are enhanced when students do most of the mathematics work during the lesson. It is observed that In Japanese mathematics lessons, the students work most of the mathematics work. From this point of view, the more students do most of the mathematics work the more they take mathematical activities. And the more they act the mathematical activities the more they communicate using mathematical communication.

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Answering on question 2: What are your components of mathematical communication to develop?

By reading the first announcement of "Third APEC- Tsukuba International Conference", I understood the components of mathematical communication to develop in our classroom communication. Some of them are having; 1) dialectic feature of mathematical communication, 2) the features of mathematical way of explanation for sharing ideas and understanding, 3) the feature to use representation (which is developed for mathematics), 4) the feature of both competitive and sympathetic attitudes (sharing and sympathetic). In other words, communication involves expressing oneself, in a variety of ways, on matters with a mathematical content, in oral as well as in written form, and understanding others' written of oral statements about such matters. And in standard by NCTM, "Representation" and "Proof and Reasoning" are also process standard s as well as "Communication".
Among such components of communication I would like to focus on "Logical Reasoning" and "Deductive Proof". To be able to polishing these components I would like to state the actual teaching classes that I have observed.
Before expression these teaching classes, first I would like to introduce the background information of these classes. To have an experience of classroom observation of Japanese Teacher's Teaching Classes I went to Junior Secondary School attached to Kanazawa University. (At that time I had no sense to be able to have a chance to participate this APEC Conference.) They let me to observe Mr. Hamaguchi's teaching classes, 2^{nd} Grade of Lower Secondary Level. I observed two lessons which were about teaching "Finding the measurement of an angle" and "Congruent Triangles." These lessons were from Chapter 4: Parallel and Congruence", and its sub-title are; 1) interior and exterior angle of polygon, 2) Parallel Lines and angles and 3) interior and exterior angle of a triangle. These classes were the normal teaching classes those were observed immediately (not a special research lesson). The flows of two teaching classes are the same as follow:

- Teacher reviewing the previous lesson, poses the problem and hands out the worksheets
- Students solve the problem individually
- Students present the solutions and methods and teacher summarizes
- Teacher poses the problem again
- Students solve and formulate the problem
- Students present the solutions and teacher summarizes students' solution again
- Ending the class with brief announcement

Teaching class 1: Finding the measurement of exterior angle of the polygon
The students' background knowledge: In this chapter they have learnt "interior and exterior angles of a polygon and total measurement of angles of plane figures", "The properties of parallel lines and its angles forming by the transversals" and other previous knowledge.

- Teacher reviewing the previous lesson, poses the problem and hands out the worksheets
  In reviewing, the teacher just said that they have already learnt some properties and the knowledge which concerning to this problem, so try to solve as various ways as they can. Then teacher handed out the worksheets and drew figures on the board. The problem was to find the measurement of angle x and the worksheet was as in Appendix A.
- Students solve the problem individually

Students were solving through their own ways by reasoning and applying their previous knowledge. Although some students stopped trying to solve when they got one solution method, some students were trying to solve more than one way. Then some students were trying to get the solution method which has higher standard than other students or tried to apply high mathematical concept.

- Students present the solution methods and teacher summarize
  Teacher instructed students to present one by one who have the different solution method to others. Students presented by drawing and giving explanations on their solution methods. Teacher summarized and gave suggestion after each student' presentation. During about fifteen minutes, students explained and presented out six solution methods for problem as shown in figures of Appendix B.
- Teacher poses the problem again
  As the second part of the lesson, teacher posed the problem that related to the first problem. Teacher drew the first figure of Appendix C, and told students by using the previous mathematical concept, to prove that "Is angle x = angle (a + b + c)?"
- Students present the solution method again
  By reasoning the previous concept, students presented out two solution methods of this proof. These solution methods were shown in Appendix D.

Teaching class 2: To draw the congruent triangle of the given triangle in various ways.
The students' background knowledge: They know symmetry figures and the conditions of congruent triangle (the corresponding angles and sides of congruent triangles are equal).

- Teacher reviewing the previous lesson, poses the problem and hands out the worksheets
  After reviewing the teacher posed the problem and handed out the worksheet (Fig. 2, Appendix C).
  Question: How many ways can we draw the congruent figures of the given triangle?

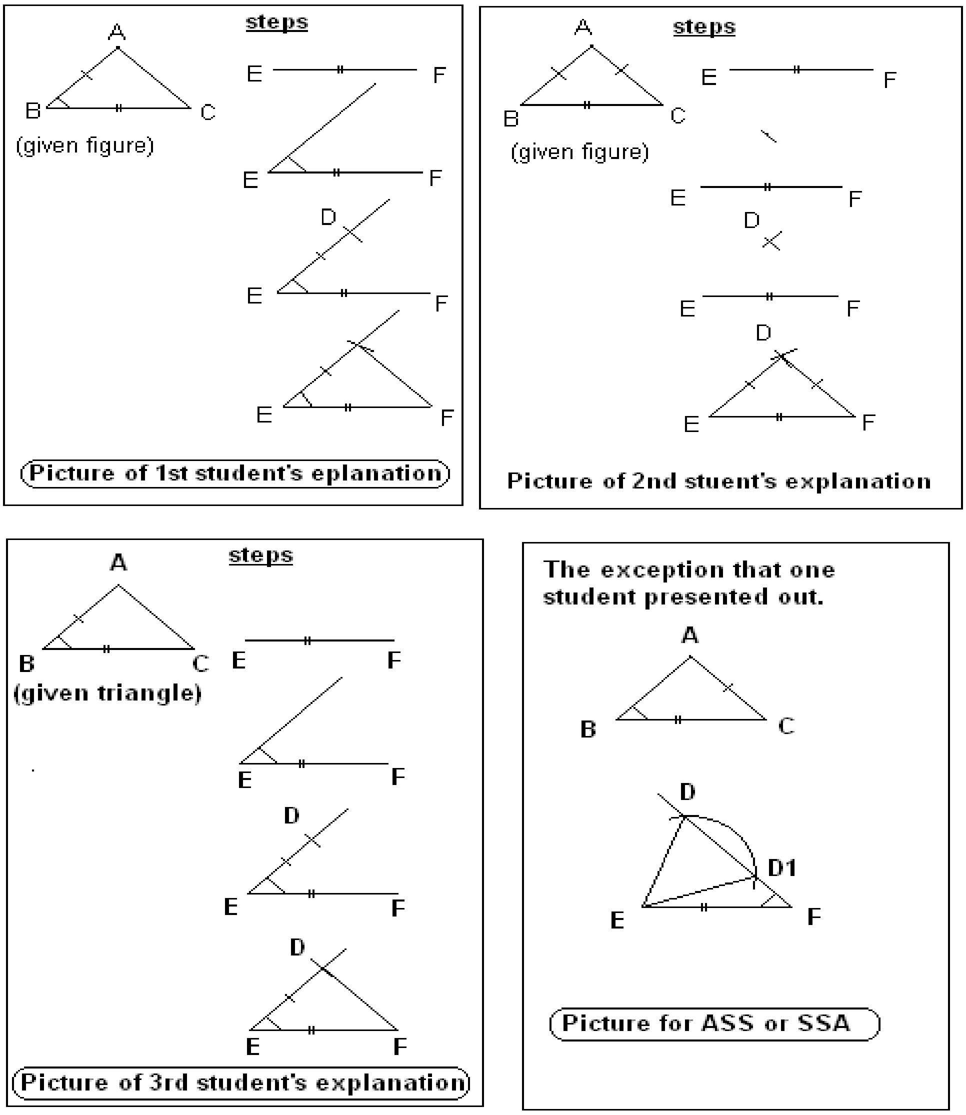
- Students solve the problem individually
  At first, students were silent and trying to imagine the appropriate way for the problem. Then they started to draw; some students measuring the angles of the given triangle and drew at the new place and some students were using the compass to measure the sides and drew the congruent triangles. Some students were trying again and again to be able to draw the congruent triangle or triangles.

- Students present the solution methods and teacher summarize
  First student's presentation and explanation: The student came in front of the class, drawing on the board and explained his solution. He said, first he drew the base line (BC=EF), second he measured angle B and drew at E, then measured side AB by compass and cut at D. Therefore he got that congruent triangle. After his presentation the teacher suggested that because this congruent triangle was drawn by two sides and the angle between them let us say or define (SAS congruent triangle). The second student presented that she drew by measuring all sides of the triangle and third student explained he drew by one side and the two angles at the ends of that side. They explained their solutions step by step like the first student. They used protractor, compass to explain their solutions step by step as shown picture below. Teacher gave suggestions on each student and summarized the congruent methods for triangle SSS and ASA congruent method. The congruent

triangles are drawn by the students and the congruence rules or methods are developed and stated out by them. The first part of the lesson was finished. (The solutions were stated at Fig. A)

- Teacher poses the problem again
  As the second problem, the teacher said students that in SAS congruent method instead of SAS try to think and draw with ASS (or AB=DE to think with AC=DF and) or change the angle, and, instead of ASA congruent method to try to draw with AAS or SAA.
- Students present the solution method again
  After a few minutes, the teacher asked who are found or had the solution method. There were three students held up the hand. And they came and presented one by one. While presenting they were asked by other students. Although two students could not prove and drew out, they showed that it is impossible to draw the congruent triangles by AAS or SAA (they were laughed by others). However, one of the students presented out the exception of SAS congruent method (stated at below).

Figure A: Students' explanation on "Drawing the congruent triangles to the given triangle."



- Teacher summarizes students' solution again
  After students discussed and present, the teacher summarized that in SAS congruent method the angle must be the angle between two sides. Although a triangle was come out, it could not be defined perfectly correct. Therefore, he suggested to take it as a note or exception.

By observing these teaching classes, first students have to think and reason to be able to prove. This is done by the teacher posing the question before the class thus requiring the students to first of all think about the necessary information to pick out from the question, reason how that information can help him/her come to the answer and use that information to solve the problem and there after present the results before the class discussion. During the time for presentation can be used to prove students' answer since they have to explain how they got to the answer while others check for the necessities and inefficiencies through to the answer.

It is clear that in Japanese mathematics classrooms mathematical communication is enhanced by doing mathematical work and activities. And it is strongly emphasized in teaching lessons all around in Japan and it is the main thing to express understanding on mathematics. Therefore, I am difficult to answer to the question asking that why we focus on mathematical communication. Because we want to know how students make do understanding on mathematics, how they think on mathematics and what they learn by mathematics or what they learn for mathematics, it should be focus and it has already focused since many years in Japan. To express these features of students mathematical communication plays as the core component.

Because teaching is a system and also a complexity of teaching activities and teaching methods and every component is useful and good to focus, it is difficult to decide or to choose the focusing on mathematical communication. However, by observing the above teaching classes I would to focus develop logical argumentation of thinking, reasoning and deductive reasoning on proof. Then to sharing the ideas and understanding concept on mathematical way, explanation is also needed to focus.

Answering Question 3: What kinds of approach will you prefer to develop the communication in classroom?

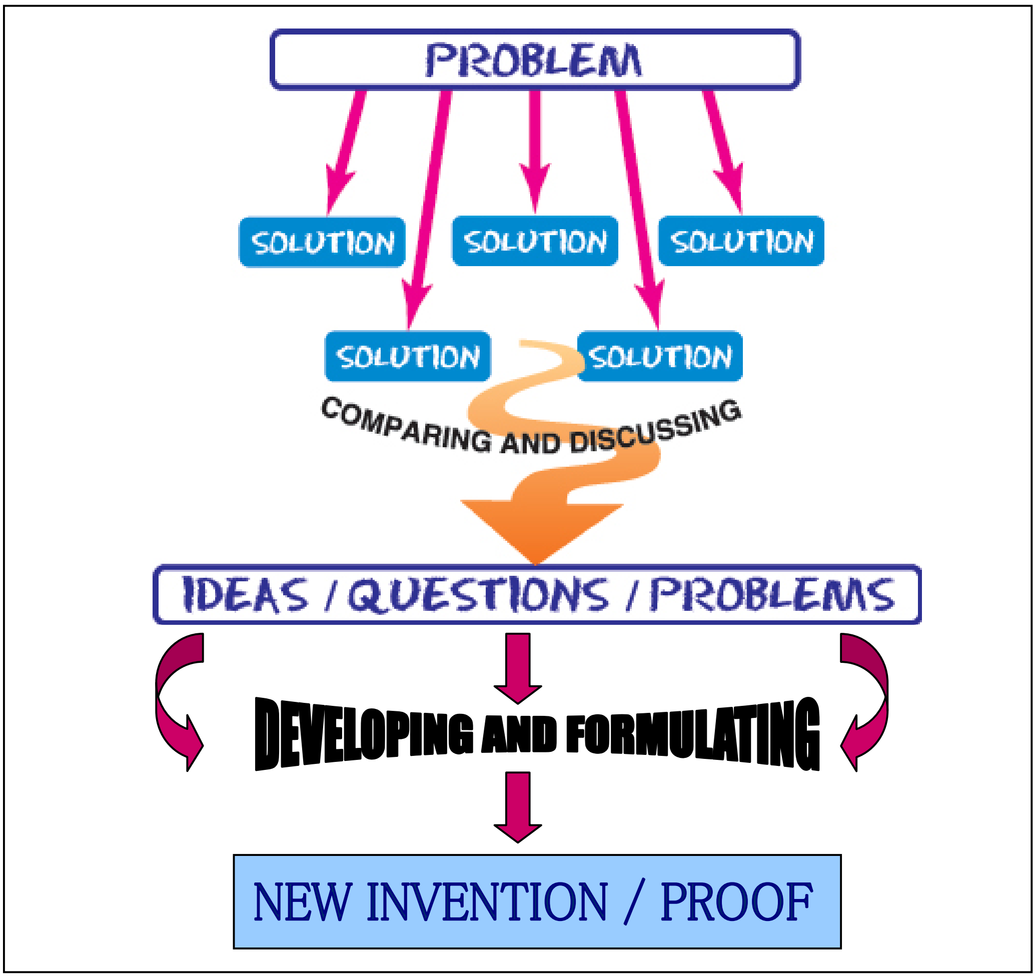
In Japan, problem solving is often viewed as a powerful approach for developing mathematical concepts and skills. Japanese mathematics teaching lessons are structure problem- solving. It was been researched by TIMSS vides study and stated in "The Teaching Gap: Best ideas from the World's Teachers for Improving Education in the Classroom". It is seen that after working with problems, students bring to classroom discussion several different approaches and solutions. And many educators agree that, it is sure that learning opportunities are enhanced when students do most of the mathematics work during the lesson. In other word, when the teacher presents a problem to students without giving a procedure, it is natural that several different approaches to the solution will come out from students. In addition, "Teaching mathematics through lectures may be an easy instructional method for teachers. When students are passively listening to teachers, however, their opportunities to understand mathematical concepts and procedures are not maximized. Rather than just listening to teachers' talk, students need to be actively involved in mathematics and to do mathematical activities (Brown, 1994).

In "Learning across Boundaries: U.S. – Japan Collaboration in Mathematics, Science and Technology Education", Japanese problem-solving oriented often have the following structure:

- Understanding the problem
- Problem-solving by students
- Comparing and discussing (students put their solutions on the board)
- Summing up by the teacher

In this structure, the stage Comparing and discussing which is called *Neriage* (in Japanese) is a critical part of the lesson because during this stage, students examine and compare proposed solutions under the guidance of the teacher and often would criticized by peers. And this also the place that will express teacher's expectation on how students understand and what will they invent. The structure of Japanese lessons is open for students, let them think and solve in their own ways and sharing mathematical concepts or approaches by several mathematical communication.

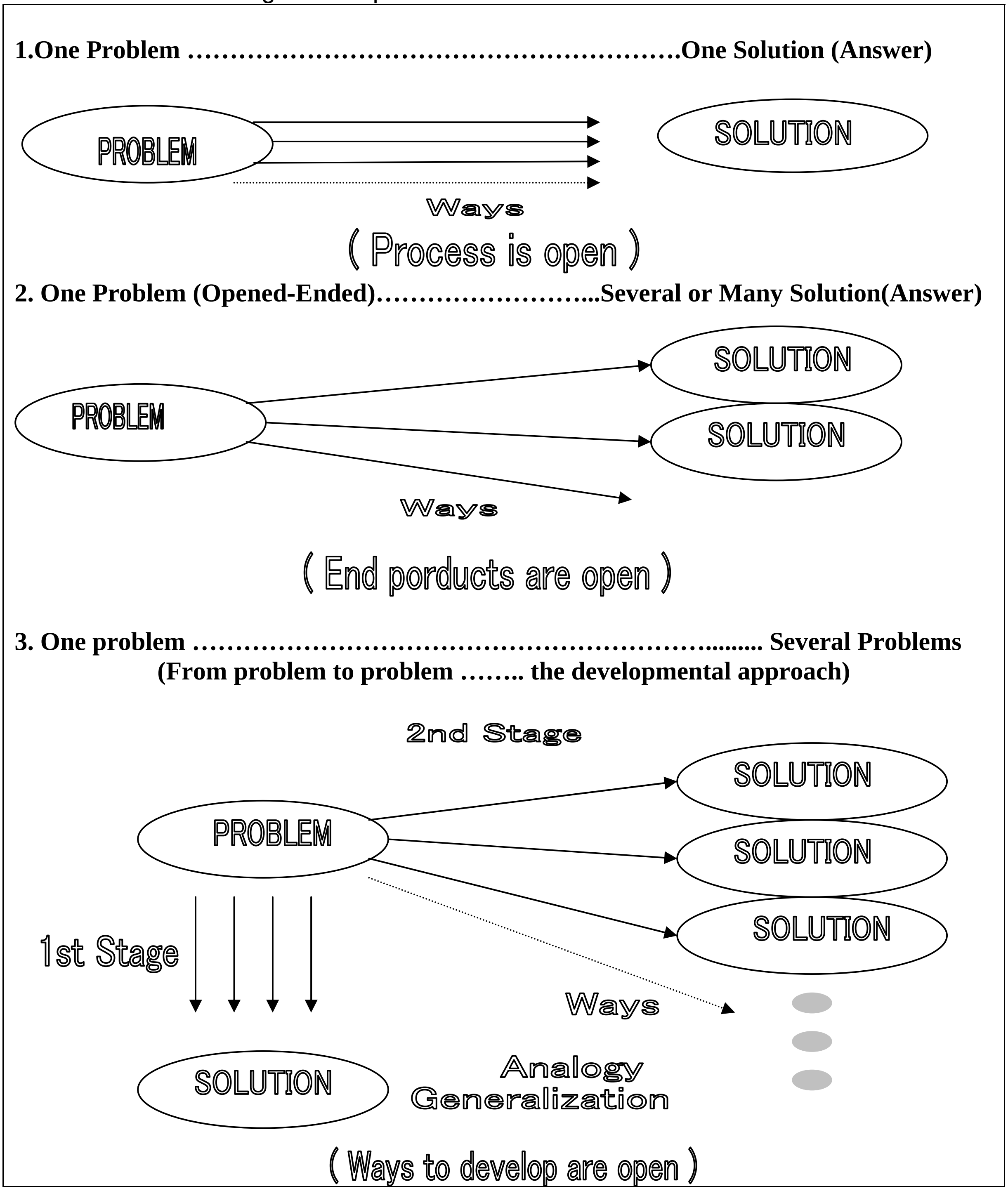
Figure B: Structure of Japanese Problem Solving



1) By the above mentions on Japanese teaching classes and Structure of Japanese mathematics lessons I see that the "Open Process" or "The Openness" enhances student

to think and create much. And many educators agree that when students do most of the works teaching learning is enhanced. The Openness of problem solving gives not only the mathematical thinking to students but also gives much opportunity to our students understanding on mathematics. I would like to focus "The Openness" approach of mathematics problem solving. The structure of Openness is stated in

Figure C: Openness in mathematics education



Note: Resource from "Reflection on U.S.-JP Collaborative Research in Mathematics Education"

Three aspect of Openness that stated in "Learning Across Boundaries: U.S. – Japan Collaboration in Mathematics, Science and technology Education" from *"The Open-Ended Approach: A New Proposal for Teaching School Mathematics"* (Becker and Shimada, 1997) are:

1) Open Process: There is more than one way to arrive at the solution to a problem
2) Open-Ended problems: A single problem may have several or many different solution.
3) From problem to problem or the developmental approach: Students draw on their own resources to solve the problem, and then guide by their teacher, compare and discuss their solutions. It is also sometimes referred to as problem formulation. It begins with a problem, which may or may not have a unique answer, but it does not stop here. Initially the students solve the problem using their own natural ways of thinking, and then discuss their solution. In the next stage, the students are asked to formulate problems o their own

Conclusion

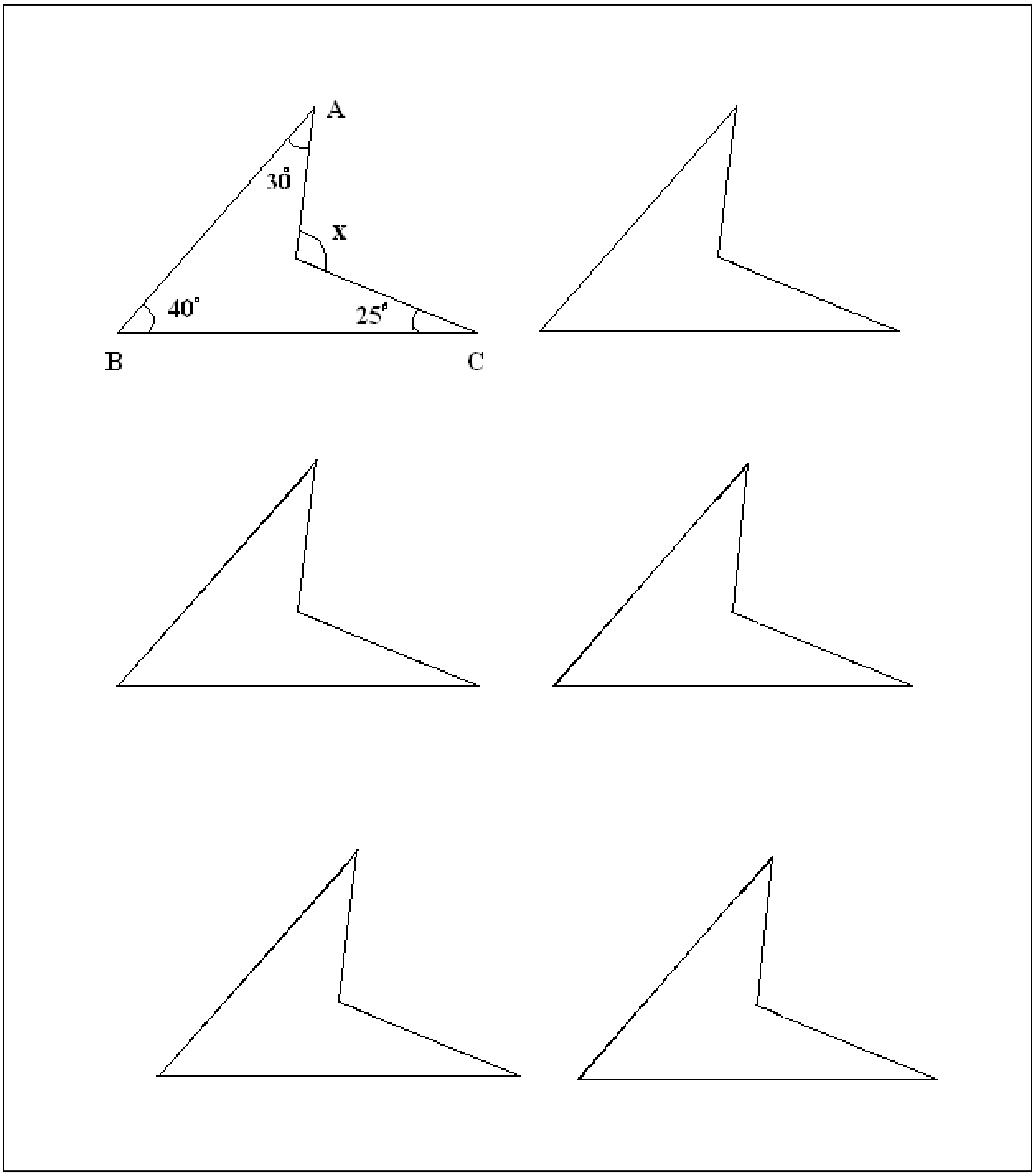
In Japanese Education, the teacher is expected to behave as a guide and to facilitate students' teaching learning activities. They are strong emphasizing on students' activities to develop all round components of mathematical education. It can be learnt how Japanese mathematics teaching class were enhancing and developing mathematical communication by doing mathematical activities in the class, not only in every teaching class but also in every research report or book.

References;

- Japanese Lower Secondary School Mathematics Textbooks
- Mr. Hamaguchi's teaching classes, Lower Secondary School attached to Kanazawa University
- Mathematics Program in Japan (Elementary, Lower Secondary and Secondary Schools) by Japanese Society of Mathematical Education (JSME, 2000)
- The Teaching Gap: Best ideas from the World's Teachers for Improving Education in the Classroom by James W. Stigler and James Heibert
- Learning across Boundaries: U.S. – Japan Collaboration in Mathematics, Science and Technology Education
- http://www.globaledresourses.com
- http://www.apecknowledgebank.org

Appendix A

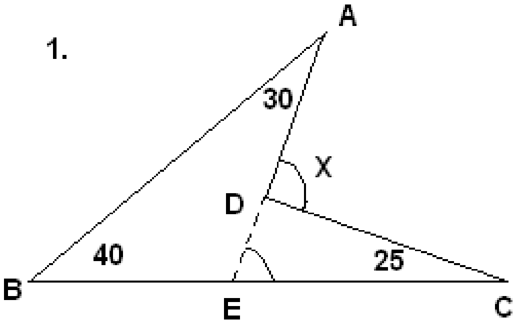
The first worksheet (B4 size paper) that teacher handed out to every student.
Q: Let’s find the degree measurement of an angle x of the given figure. Try find in several way.



Appendix B

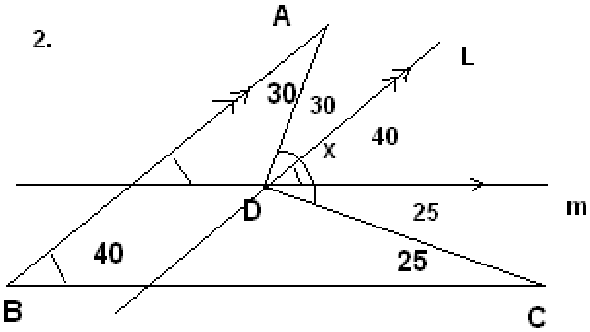
Figures: The solutions those students carried out for Appendix A in discussion stage.

1.



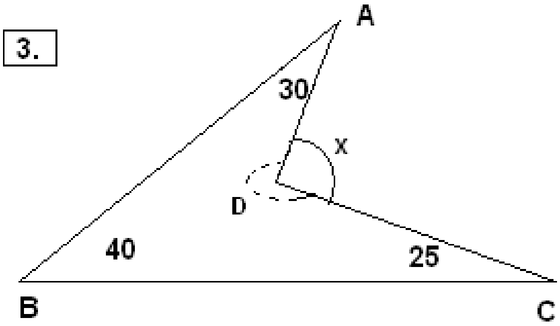
∠DEC is exterior angle of △ ABC.
∠DEC = 40+30
x =70+25=95

2.



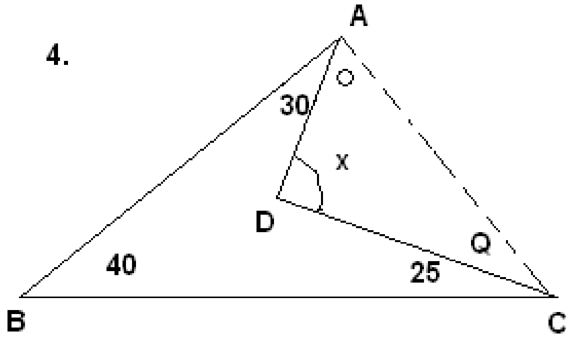
By properties of parallel lines,
angle x = 30+40+25 = 95

3.



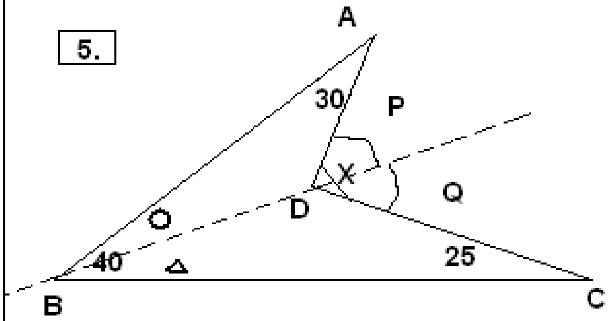
∠ x = 360-{ 360-(40+30+25)}
=360-{265}
=95

4.



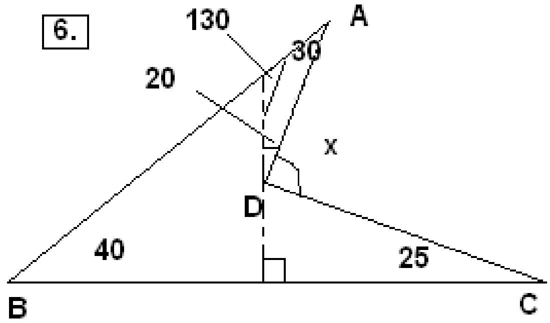
∠ x = 180 - (0 + Q)
(0 + Q) = 180 - (40+30+25)= 85
=95

5.



∠P and Q are exterior angles
of ABP and BCQ.
∠ x = ○ +30+△ +25 (○ + △ =40)
∠ x = 95

6.



(130 = 90+40), then 20 = 180 - (130+ 30)
x + 20 = 90 + 25
x = 95

Appendix C

Figure 1: Hand out for the second part of the first class (extended problem for Appendix A)

Is ∠x=∠a+∠b+∠c ? and Is ∠x+∠y = ∠a+∠b+∠c+∠d ?

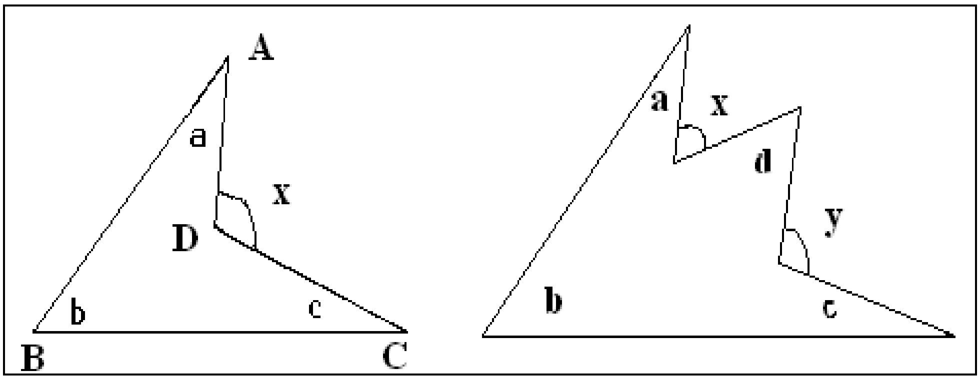
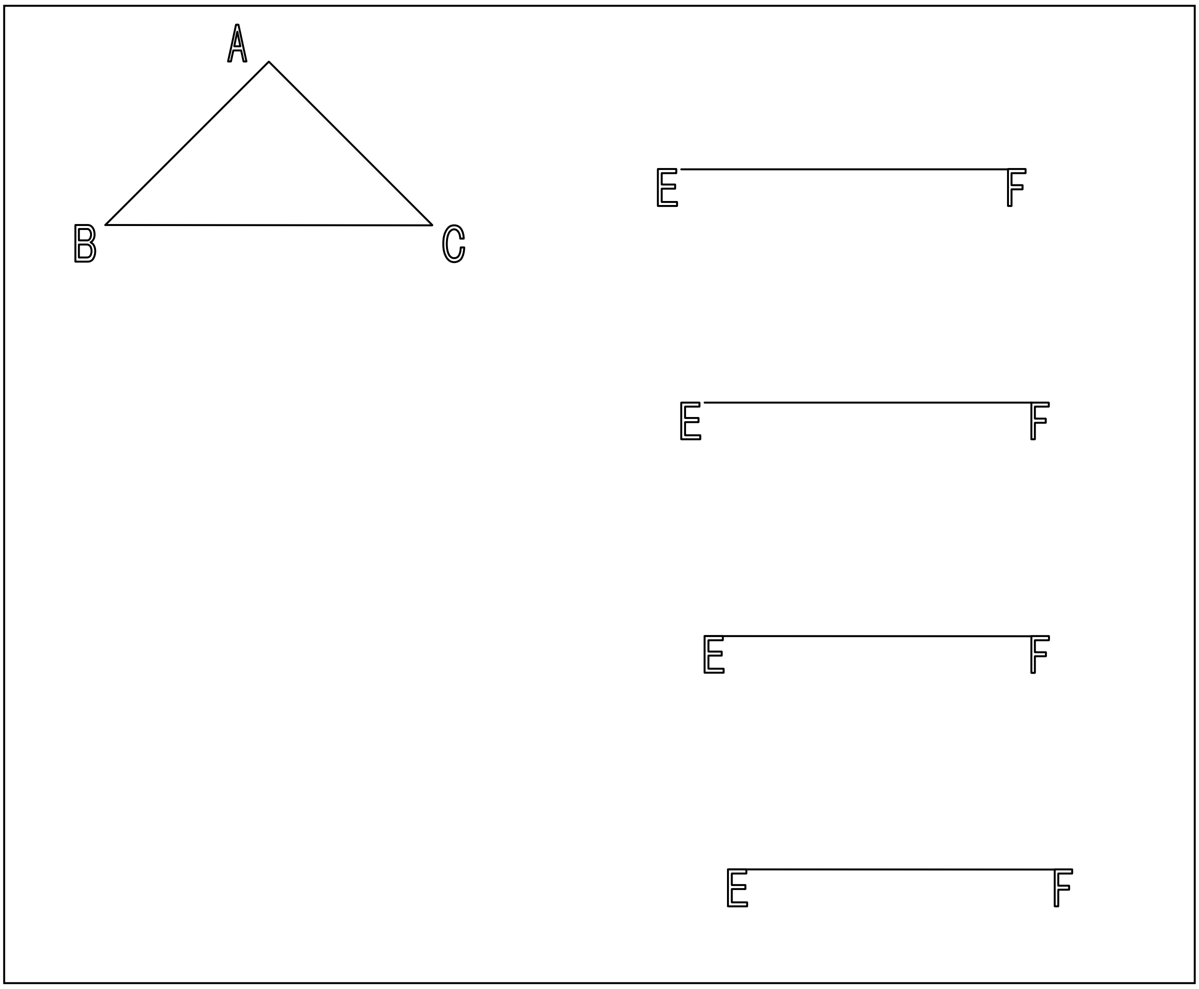


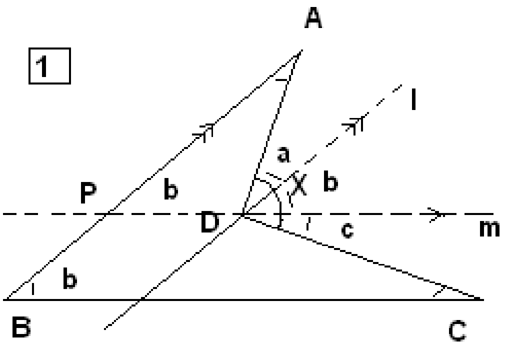
Figure 2: The hand out for the second lesson (A4 size paper)

*** How many ways are there to draw the congruent triangles? Given: If BC=EF.



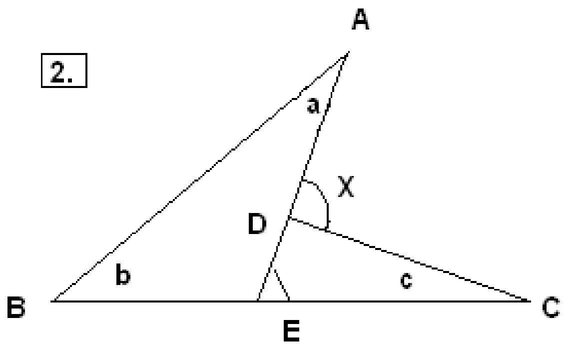
Appendix D

Figure: The solution or Proof those students carry out in the 2nd part of second lesson. The solution for figure 1, Appendix C



1. First student's explanation

In polygon ABCD,
Angle APD= angle b (corresponding angles)
Angle LDM= angle b (~)
Angle ADL = angle a (Alternate angles)
Angle CDM = angle a (~)
Therefore, angle x = angle (a + b + c).



2. Second student' explanation

Draw AD to BC and meet at E.
Angle DEC = angle a + angle b ……..(1)
(Exterior angle of triangle ABE)
Then angle x is exterior angle of triangle DEC,
Therefore, angle = angle DEC + angle c
By substituting an equation (1),
Angle x = angle (a + b + c)

Note: Those Appendix figures were I took and drew by myself.