APEC-Tsukuba Intrenational Conference
Developing mathematical thinking through reflective experience
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Mathematics Education and Reflective Experience

: The Significance of "Unlearning" in Mathematics Education

College of Education , Ibaraki University
Hiroshi Nemoto

1 Acquisition of Viable Knowledge

1-1 Flexible Understanding and Viable Knowledge: Thinking Like a Human

1-2 Presentiments of the Future and Mathematical Activities
: Transcending the Sensory Domain

2 Reflective Experience and Mathematical Activity

2-1 Mathematical Activity and Insight: Observing That Which Keep out of Sight

2-2 The Importance of Reflective Experience: Connecting with the Future

3 Suggestions for Improving the Teaching of Arithmetic/Mathematic

3-1 The Importance of Meaningful Experience: Reflect on oneself (Introspection)

3-2 Improving Arithmetic/Mathematics Teaching
: Encouragement of Unlearning

The Course of Study (*Gakusyusidouyouryo* in Japanese) for Lower Secondary School

Overall Objectives

For the students to understanding deeply the fundamental concepts, principles, and rules relating to numbers, quantities, figures, and so forth. For students to acquire methods of mathematical expressions and strategies, and to improve their ability to relate phenomena mathematically. For students to enjoy mathematical activities, to appreciate the importance of mathematical approaches and ways of thinking, and to inculcate in them the right attitudes necessary to make use of mathematics.

Chap.1.

Problem : Which is larger, 5/13 or 11/19?

Both fractions are approximately 0.5 (1/2).
5/13 → if the numerator were 6, 6/13 < 6/12 (0.5).
(∵ the denominator is larger.)
And since 5 is one less than 6, 5/13 < 6/13.
Therefore, 5/13 < 0.5.
11/19 → if the numerator were 9, 9/19 ≒ 0.5.
11/19 > 9/19 (≒ 0.5). (∵ 11 is two greater than 9.)
Therefore, 11/19 > 0.5.
∴ 5/13 < 0.5 <11/19, and therefore, 5/13 < 11/19.

Both fractions are approximately 0.5 (1/2).
5/13 → If the fraction were 5/10, 5/10＝1/2. 5/13 < 1/2.
(∵ the denominator is 13.)
11/19 → 10/20 is also equal to 1/2, 11/19 > 1/2.
(∵ the numerator is larger than 10 and its denominator is smaller.)
Therefore, 5/13 < 1/2 < 11/19. Consequently, 5/13 < 11/19.

This shows just how flexible human thought is.

fundamentals/basics in life

Common to both reasoning

appropriate standards to judge

… the action of setting some sort of a standard is simplistic, even in our daily lives, but it is happening naturally.

viable knowledge

… the setting of a certain standard (viewpoint) for making a judgment, which is used when pursuing ordinary thoughts, is fundamental/basic viable knowledge, and even in learning the size relation of fractions, it is important to be aware of the fact that you are acquiring such knowledge.

evolvability

Knowledge is what prompts the subject (person) into acting according to the situation. Knowledge can be thought of as something having built-in evolvability, in the sense that it spurs new actions.

Generally speaking, the methods and concepts that are taught are alive in the minds of students, and therein lies evolvability. … a close relationship is forged between concepts and between methods, and those concepts and methods take root deep in the mind, resulting in the potential for self-evolvability. Ryoichiro Sato

Mathematical facts can be conveyed through language, and students can probably learn that. However, it is wrong to readily think that mathematics learned this way is transformed in their knowledge. Furthermore, it must be noted that this is nothing more than ready-made knowledge prepared by adults. If it does not have any impact on the future action of students, then it cannot necessarily be called *viable knowledge*.

Cognitive Conflict

The real problem is not one of "form" (i.e. the fraction x/y), but rather it is a matter of entering into a world of numbers that have a different relational structure that what was known thus far.

One difficulty, no doubt, is that mathematics requires such refined and complicated symbolic techniques for its expression. The teacher who becomes so utterly engrossed in mastering techniques for operating with symbols that he forgets *their conceptual background* is fated to lose the interest of his pupils and will also leave them with some of the misconceptions of mathematics that are so current today. (R. L. Wilder)

the initial teaching of fractions

magnificence of mathematical activity that ventures into unknown worlds

Problem

1. Compare the size of fractions that have the same denominators.

(2/9 5/9) , (4/5 3/5)

2. Compare the size of fractions that have the same numerators.

(1/2 1/3) , (2/5 2/9)

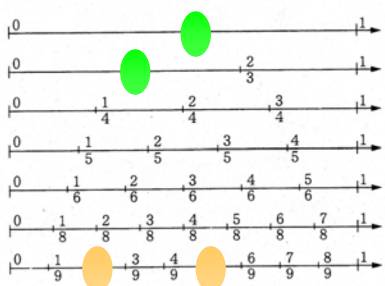


Fig.1.

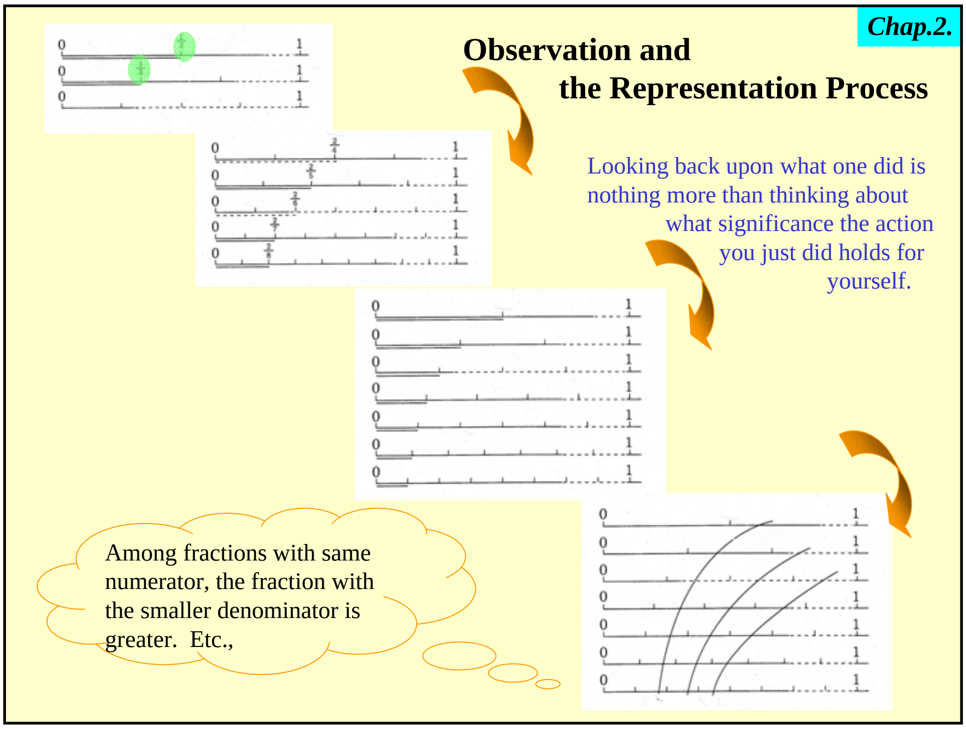
The generality of *a)* and *b)* is not thought to be recognizable simply with a passing look at Figure 1. !!

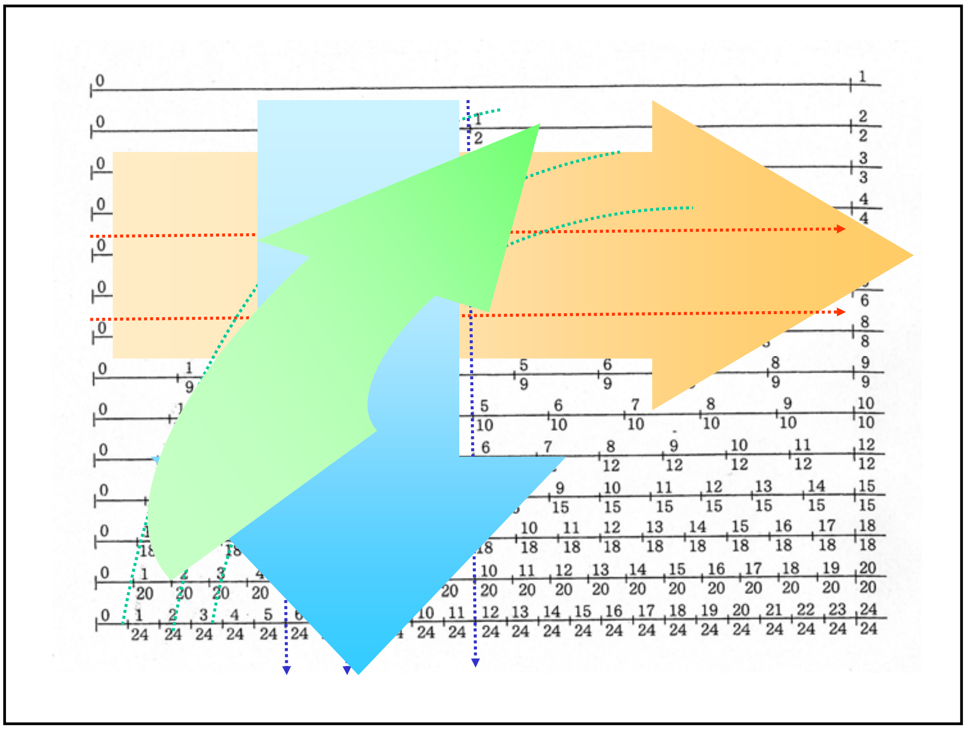
linguistic knowledge

●Among fractions with same denominator, the fraction with the larger numerator is greater. …… *a)*

●Among fractions with same numerator, the fraction with the smaller denominator is greater. …… *b)*

●There are many that are the same size even though they have different numerators and denominators. …… *c)*


Chap.2.
Observation and
the Representation Process
Looking back upon what one did is nothing more than thinking about what significance the action you just did holds for yourself.
Among fractions with same numerator, the fraction with the smaller denominator is greater. Etc.,



Mathematical activity &. Knowledge

Knowledge is organized within the individual (student)

the careful observation of a fact
actions and mental activity

Human beings have the ability to engage in action within the mind and action transcending facts.

(1) The success or failure of problem solving does not necessarily depend only on the memorizing of algorithm-like knowledge.

(2) The recognition of a general mathematical fact (prior knowledge) accompanies the non-verbal aspect of knowledge in that area.

(3) The organization of knowledge signifies an overall systematic understanding, and the representation process is crucial therein. It is mathematical behavior that underpins this. In particular, mental behavior is key.

reflective experience &. physical experience

… carefully observing an object and perceiving the mathematic relation, universal nature, and generality that lie behind it

… the partial and sensory relation to an event

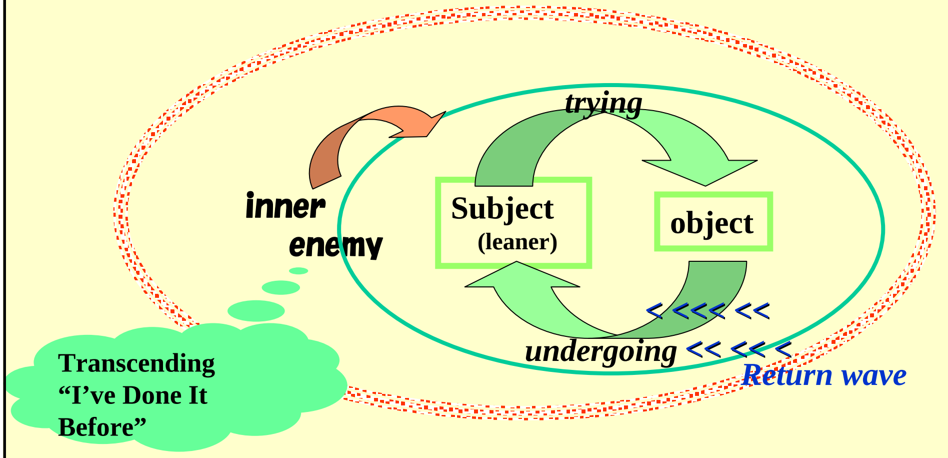
the verbal memory of *a*) and *b*)

- **You should focus on either the numerator or denominator to judge fraction size. (Re1)**
- **You can judge size by transforming fractions into the same numerator or denominator. (Re2)**

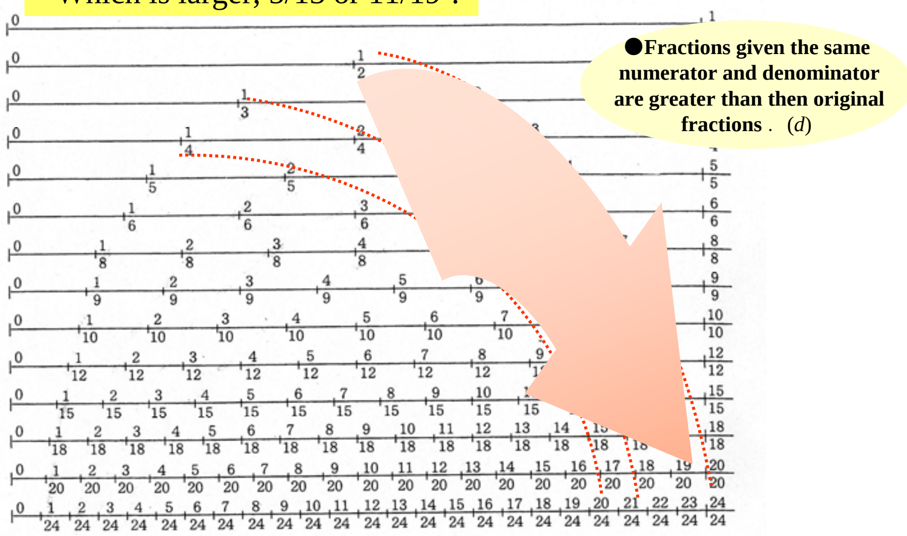
In terms of mathematics education, reflective experience is an experience where, by looking back on one's actions, the stimulus source transcends the range of sensory reception and thereby we can expect an experience in which we can operate and process as if we can see with our eyes and an understanding in which partial visual facts can be perceived within a comprehensive dynamic understanding. This can also be called a leap. *Mathematical activity can also be called a mathematical translation of this reflective experience.*

Reflective experience

(1) What constitutes experience?
(2) What decides the value of experience?
(3) What is the difference between experience and mere activity?
(4) What is meaningful experience?



Which is larger, 5/13 or 11/19 ?



$$\frac{a}{b} \leqq \frac{a+m}{b+m} \quad (b > a > 0,\ m \geqq 0)$$

$$0 < \frac{a}{b} < 1$$

Chap.3.

Suggestions for Improving the Teaching of Arithmetic/Mathematics

Meaningful Experience

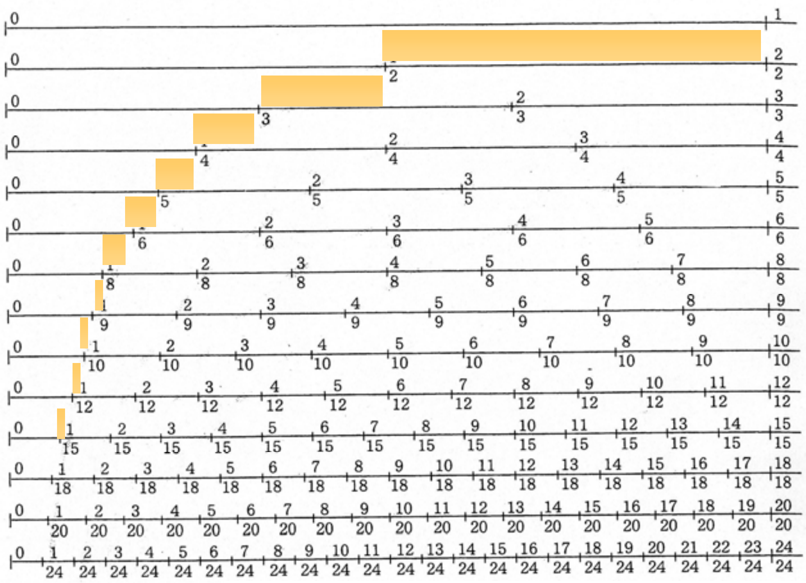
Mathematics instruction is focused too much on mathematical content and not enough on mathematical behavior. If we want our students to become active learners and doers of mathematics rather than mere knower of mathematical facts and procedures, we must design our instruction to help develop their metacognition. Leroy G. Callahan and J. Garofalo

- make sense that links to the future
- action where a conscious effort is made to discover a rule

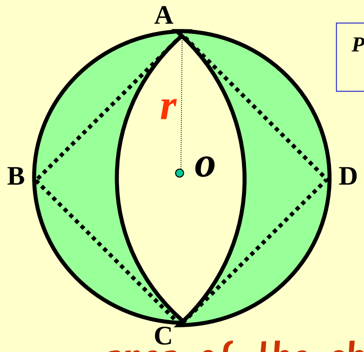
Symbolic Initiative

Man possesses what we might call symbolic initiative; that is, he can assign symbols to stand for objects or ideas, set up relationships between them and operate with them on a conceptual level. …… However, much of our mathematical behavior that was originally of the symbolic initiative type drops to the symbolic reflex level. … R.L.Wilder

- … a place must be provided for experience that promises high-level standards, and behavior that can foresee leaps to the future.



$$\lim_{n\to\infty}\sum_{k=1}^{n}\left\{\frac{1}{k(k+1)}\right\} = \lim_{n\to\infty}\sum_{k=1}^{n}\left\{\frac{1}{k}-\frac{1}{k+1}\right\} = 1$$



Problem : Show that the area of square ABCD in the following diagram is equal to the shaded area.

r ; radius of the circle

area of the inscribed square

$2r^2 \quad (= (\sqrt{2} \cdot r)^2)$

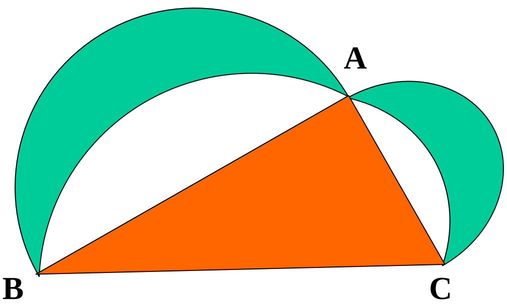
area of the shaded

$$S = 2 \cdot \left\{ \frac{\pi r^2}{2} - \left(\frac{\pi(\sqrt{2}\,r)^2}{4} - \frac{(\sqrt{2}\,r)^2}{2} \right) \right\}$$

$$= 2 \cdot \left\{ \frac{\pi r^2}{2} - \left(\frac{2\pi r^2}{4} - \frac{2r^2}{2} \right) \right\} = 2r^2$$

a result will be obtained after carrying out the proper calculation. …

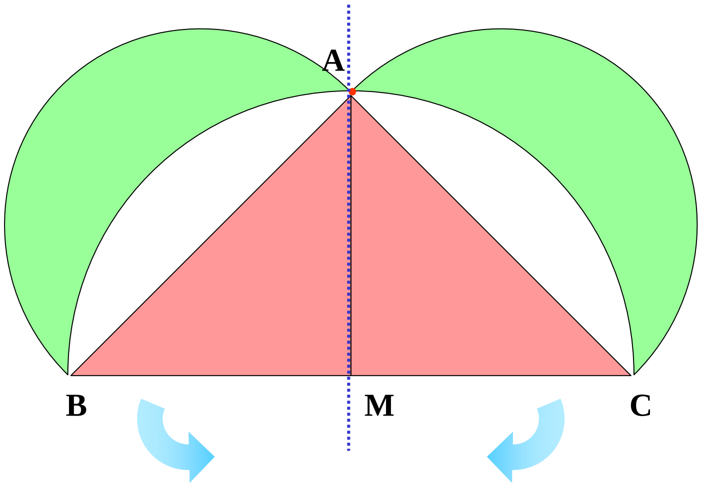
Hippocrates' Lunes



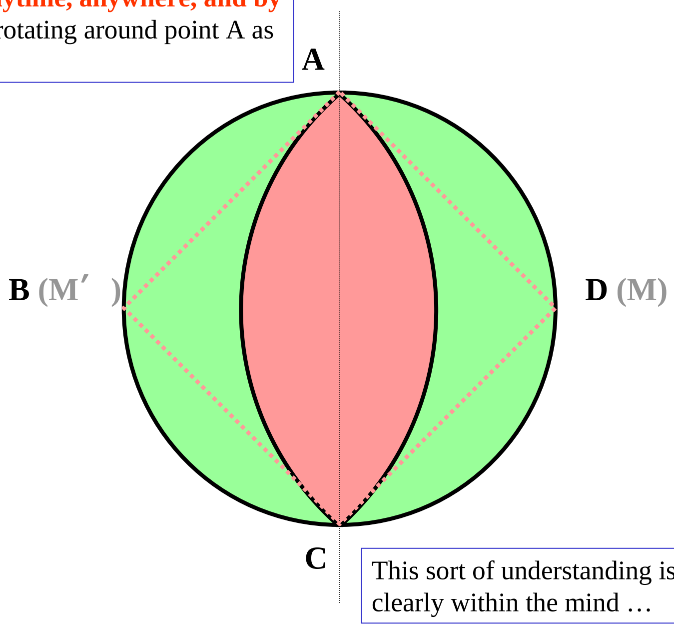
right triangle ABC

"The area of the figure that is enclosed by the half circle that forms the diameter of the hypotenuse and the half circle that forms the diameter of the two sides that are between the right angle of the right triangle is equal to the area of that right triangle."

This fact should also hold true even for right isosceles triangles.



The figure in the problem will be achieved **anytime, anywhere, and by anyone** by rotating around point A as the center.



This sort of understanding is formed clearly within the mind …

Improving Arithmetic/Mathematics Teaching

Encouragement of Unlearning

Actively Monitoring One's Own Progress

Trying to understand What You Are

- the labor of making sense in one's own way for self-action.

Questioning Yourself About What You Are Doing

- It is also about questioning yourself about what you are doing while enabling yourself to think.

Trying to Put What You Know to Work

- In short, the meaning of the phrase "It is better to know than not to know" is slighting different. What is important is how that knowledge was come to be known.

The Importance of the Word "Oneself"

Reflect on Oneself (Introspection)

The point of evaluation is self-evaluation. … Taking responsibility for what you have done is a requirement for living in society, but appropriate self-evaluation is required to ensure this. The learning of mathematics can be thought of as something that gives students the ability to do this and further refine it themselves.

self-negotiation

Let him know that he is expected to be making discoveries all the time; not merely that the best established law is not complete, but that is the very simplest things it is not so much what he is told by a teacher, but what he discovers for himself, that is of real value to him, that becomes permanently part of his mental machinery. Educate through the experience already possessed by a boy; look at things from his point of view --- that is, ***lead him to educate himself***. J. Perry

I learned many things at the university,
but I had to ***unlearn*** them later. (Helen Keller)

Tsurumi said he came to understand her words as follows.

I imagined I had knit a sweater in a conventional manner and then unraveled it into the yarn that comprised it and knitted it again while holding it up to my own body. The knowledge learned at universities is, of course, necessary. However, it is of no use if it is simply memorized. That which results from unlearning such knowledge becomes flesh and blood.

…… the spirit of this unlearning and the reflective experience for making what you have learned a part of you are one in the same.

END