Math Lessons That Teach Creating Lessons That Focus on "Starting Phrases" Children to Use "For Example…," "But Then Again…," "If…," and "In That Case…"

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- 1.Expression through formulas, tables, and symbols
  2.Expression through diagrams (line segment diagrams, tape diagrams, and proportional number lines)
  3.Expression through pictures or operations
  4.Expression through words

- Types 1 and 2 are more highly abstract, completed methods of expression than types 3 and 4.

- However, for children to be able to effectively use methods 1 and 2, which are specific to mathematics, they need to be able to specifically express their understanding of a phenomenon using methods 3 and 4.

Paying Attention to Students' "Starting Phrases"

When a child really wants to convey what he/she is thinking to his/her peers, that child spontaneously uses a variety of expressions. It would be a mistake to think that all of the phrases that children use to diligently express their ideas in class are of the same type.

When categorizing these phrases by focusing on "starting phrases," the following types are identified:

"For example…"

This phrase is used by children who want to replace their ideas with more specific examples. The ability to replace one's thoughts with a demonstrative example is not something a child can do unless he/she has a fairly solid understanding of the material

“But then again...” and “But...”

These words are used by children when they try proactively to connect to what is being said by their teacher or peers. Their purpose may change depending on the situation, as sometimes they try to connect by offering a counterexample and sometimes by providing an explanation.

“First…. Then…”

Children who start expressing themselves with these phrases are capable of examining their own ideas and then separating and organizing their thought processes into segments.

“In that case...”

This phrase can be viewed as a benchmark for evaluating a lesson, as it is the phrase students will use when they have become self-motivated. When classroom activities progress along a natural flow, the children can begin to work with an expectation of what will happen next. If the lesson consists of a series of fragmented questions, the children cannot get a sense of the flow of the activities and remain in a passive state.

“If...”

This is a very convenient phrase and may be the phrase that students most need to learn in math. Phrases that start with “if” are also used when trying to organize a phenomenon. It is like trying to explain a pentagram, which tends to be harder to understand, by using triangles, which tend to be easier to understand. It is used when trying to understand something by converting it into something that is easier to understand, and in that way, it is similar to the use of “for example.”

Lessons Using "Children's Misunderstandings" Derived from "For Example"

- "What number would have to go in the box for you to be able to solve this problem?"

$\square \div 1/4$

$\square \div 1/4$

- First, I entered 1/4. The problem 1/4 ÷ 1/4 is easy to solve. A number divided by itself is always 1.

□ ÷ 1/4

- when I tried to enter an integer, there were some misunderstandings. Some children wanted to enter 4 while others wanted to enter 1. Since these were integers, I thought their reasons for choosing one or the other might be fairly similar, but they were quite different. Those who wanted to enter 4 thought the answer would be 1. They thought that ÷ 1/4 meant finding the value of one piece of an item that was divided into four pieces and, thus, that one portion of 4 that was divided into four pieces would be 1.
- Other students disagreed.
- "The term 1/4 means that one is broken into four pieces. In that case, if 4 is entered, and four items were broken into four pieces, there would be 16 pieces. So the answer should be 16," they argued.

□ ÷ 1/4

- 4 ÷ 1/4 = 16/4 ÷ ¼
- "But what if the denominator weren't 4?" they questioned, their enthusiasm subsiding.
- Other students explained that perhaps the fractions could be reduced to a common denominator, just as it is done when trying to add fractions with different denominators, reigniting their excitement. They concluded that fractional division could also be performed by finding a common denominator.
- Example: 3/5 ÷ 1/4 = 12/20 ÷ 5/20 = 12÷5