Fifth Grade Math Lesson Plan
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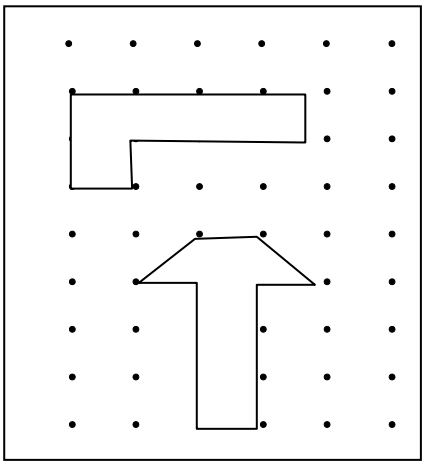
Research Topic
Create Lessons That Helps Students Learn to Think Developmentally

1. Topic: The Secret to 5 cm^2 Shapes (Estimation of area through *Functional thinking* which means that the observer sees that the students are reasoning using the idea of a function without explicitly saying that they know what a function is.)

2. Preferred Approach to This Lesson
I give the students a piece of graph paper, and instruct them to draw a shape with an area of 5 cm^2. The students try to draw various shapes by connecting the dots on the paper. The students spontaneously produce squares, rectangles, triangles, a variety of quadrilaterals, and other complex shapes. First they will try to find the area of the shape using the formula for finding area as well as by using the method of counting the number of 1 cm^2 sections. Because the students are fifth graders, these activities are important themselves, but you should focus on the children who go a step further by trying to find the area using a function as shown below.



If you look closely at the figures drawn by the students, you will find that many of them are made by connecting 12 points. You will also notice that none of these shapes contain points inside the shape. The students who notice this will try to figure out if it holds true for the other shapes as well. Eventually they will discover a shape that contains one point in the middle. At first they will think this is a special case, but after looking closely, they will notice that this shape connects only 10 perimeter points.

Hopefully you will start to hear the students say things like, “Hey, that’s interesting.” And also hopefully some of the students will start to notice that if 12 points means 0 interior points, and 10 points means 1 interior point, then perhaps a shape connecting 8 points will have 2 interior points. Following this clue, they will start to try to find other shapes.

The students will start to try to create shapes that connect eight perimeter points and contain two interior points. Then they will confirm whether the shape they end up with actually has an area of 5 cm^2. In the first part of the lesson, the students tried to draw shapes with a certain area. In the second part, they will work from the opposite direction that is by first creating a shape based on what they’ve learned and later checking the area. This lesson is designed to help students who have already learned about area to look at shapes from a new and different perspective. A lot of students may think that studying area is all about using formula, but I want to help them realize that they can make new discoveries if they look at things using functions.

This experience can help students learn to look at everyday phenomena from a new perspective.

3. Development of the current lesson
(1) Purpose
To use an activity in which students draw shapes of 5 cm^2 on graph paper to make them realize that when creating shapes of the same size, the number of perimeter points and the number of interior points vary in relationship to one another. And to teach them to try to find new shapes based on their expectations regarding that variance.

(2) Development plan

Lesson Activity	Teacher's Role
1. Students try to draw a 5 cm^2 shape on graph paper. 2. They present their shapes to the class. - Check whether the area is actually 5 cm^2. - Look at the completed drawings lined up beside one another. The teacher tries to suggest, based only on a few special cases, that "All of the shapes seem to be made by connecting 12 points, don't they?" Students respond • "Wait, mine only connects 10 points." • "But here's an exception." • "This has the same area, but the number of points is different." 3. The students compare the shapes connecting 10 points with shapes connecting 12 points and notice the difference that: - Even though they have a different number of perimeter points, they all have an area of 5 cm^2. - The 12-point shapes do not contain any interior points, but the 10-point shapes contain a single interior point. - In that case, maybe an 8-point shape would contain two interior points. - And it would be interesting if a 6-point shape contained three interior points. Let's find out. 4. Students try to find new shapes. - They confirm whether the new shape has the correct area.	- The teacher should let the students know that their shapes can be simple, but should encourage them to make a variety of shapes by suggesting that they try to come up with shapes that are different from those created by their classmates. - The teacher should deliberately comment on the number of perimeter points. Later the students will find counter-examples, but this will help them realize that they cannot make judgments based on a few special cases. - Here the teacher creates a scenario in which the area of the newly created figures needs to be checked. - The teacher should let the students find the similarities between the two groups of shapes. - Sort the shapes by using the number of connecting points and the number of interior points and wait to see how the students start to notice the pattern. Connecting points 12 Interior points 0 Connecting points 10 Interior points 1 Connecting points 8 Interior points 2 - The level of difficulty of this activity will change once the students make this discovery. The students will conclude that a figure with 4 connecting points will have 4 interior points, and the teacher will have to explain this.