An Analysis on the Preparation of Elementary Mathematics Teacher in Mainland China

Yunpeng Ma

School of Education，Northeast Normal University, P. R. China

Introduction

Dramatic changes have taken place in China to upgrade its elementary teacher preparation system since 1999. The preparation of elementary school teachers has changed from what was dominated by teacher preparation normal schools to multiple pathways. Now, junior normal colleges and normal or comprehensive universities are dominant institutions. At a time when this change has occurred, do those prospective elementary mathematics teachers get enough training before they teach elementary mathematics? How about their mathematical knowledge for teaching? Much still remains unknown.

From an international perspective, it is new trend to investigate pre-service teachers' preparation, such as MT21 (Schmidt et al., 2007) and TEDS-M (IEA, 2008). Investigating pre-service elementary mathematics teachers' preparation is one part of TEDS-M, but Mainland China is not the participant country. In this article, we will introduce the status of preparation of pre-service elementary mathematics teachers' in Mainland China. This article contains three parts. In the first part, we will describe program features of elementary teacher preparation in China. Special attention is given to curriculum requirements in mathematics content training. Secondly, we will present some results of a survey on prospective elementary teachers' competence. In the last part, we will give a reflection on the elementary mathematics teacher preparation and draw some implications for its future.

Features of elementary teacher preparation program and its curriculum

In 1999, the Ministry of Education of China (hereafter MOE) issued a policy *three opinions on restructuring teacher education system*. One of the main ideas of this policy is to gradually change teacher education system from 3-level (normal school, junior college, and normal university) into 2-level (junior college, and normal university). Ever since then, the number of normal school was decreasing sharply. Now, junior normal colleges and normal or comprehensive universities are dominant institutions. The programs in different institution levels always differ. Different programs also exist in institutions on the same level. The following will describe features of each type of preparation program and its curriculum requirements.

Programs in junior normal colleges

There are two types of programs in junior normal colleges: 5-year program and 3-year program. 5-year program admits junior secondary school graduates. It includes two stages: the first stage is 3 years and the second stage is 2 years. So, 5-year program is also named "3+2" program. As far as curriculum of this program is concerned, the quantity and quality of courses in former 3 years is mostly equal to those of normal school; the courses in later 2 years are focus on one specific field of study. According to *The guideline for 5-year program of elementary teacher training (experimental)*(MOE, 1995), there are eight specific fields for choosing, such as Chinese, Mathematics, English, Music, Physical Education, Art, Science, and Society.

3-year program admits senior secondary graduates. Prospective teachers need to choose one special field of study to get relative deep and systematic training for elementary school teaching. According to *Three-year program curriculum plan for preparing elementary school teachers* (MOE, 2003), there are six special fields for prospective teachers choosing. They are Chinese and Society, Mathematics and Science, Music, Arts, ICT, and English.

Table 1 summarizes program structure and requirements for preparing elementary teachers in these two types of programs.

Table 1. Curriculum requirements of “3+2” program and 3-year program

Type of program & specific field of study	5-year program Mathematics		3-year program Mathematics and Science	
	Course hours	percentage	Course hours	percentage
Gen. education	3070	57.4%	702	26.1%
Edu. major	730	13.7%	672	25.0%
Content subjects	1046	19.6%	324 (Mathematics)	12.1%
			602(Science)	22.4%
Selectives	500	9.4%	385	14.3%
Total	5346	100.0%	2685	100.0%
Edu. practice	10 weeks		15 weeks	

In Table 1, we adapted a course structure that can roughly accommodate different type of elementary teacher preparation programs. Both 5-year program and 3-year programs offer subject specifications beyond the general education curriculum for all prospective teachers. In the 5-year program, prospective teachers typically need to take five mathematics content courses (e.g., algebra, geometry, mathematical analysis) and one “mathematics teaching methods” course in their first three year of study. For those majoring in mathematics education, they need to take three more courses in mathematics, such as probability and statistics, mathematical thinking and method, and analytical geometry, in later two years. In the 3-year program, for those majoring in mathematics and science, prospective teachers typically need to take 4 courses relevant to mathematics and mathematics education (see Table 2).

Table 2. mathematics courses for those majoring in mathematics and science

Course name	Course hours
Foundation of higher mathematics	180
An Introduction to Modern Mathematics	72
Mathematics Practice and scientific experiments	72

Primary Mathematics & Science Teaching and Research	72
Total	396

Programs in normal or comprehensive universities

In 1999, MOE approved to establish 4-year B.A. or B.Sc. program for elementary school teachers preparation. This program is offered by normal or comprehensive universities that admit senior secondary school graduates. The Chinese MOE has not developed unified curriculum guidelines for this program. Three types of program structures can be identified and named as (1) integrated, (2) focus-area specified, and (3) middle-ground.

Table 3 presents the curriculum structure and average course requirements obtained from several normal universities representing each of the three sub-categories: integrated, middle ground, having a focus area.

Table 3. curriculum of three different 4-year preparation programs

	Integrated	Mid ground	Focus area
Gen. education	30.6%	29. 4%	28.7%
Edu. major	21.7%	20.2%	22.7%
Content subjects	17.3%	25.9%	24.5%
(Mathematics courses)	*(6.7%)*	*(13.3%)*	*(18.4%)*
Edu. activities	10.4%	12.4%	11.9%

Table 3 shows that programs in these three sub-categories have a similar curriculum structure. The main difference between three sub-categories is in content subjects courses. We will take curriculum of three universities as examples to give a glimpse of course requirement in different type of programs.

(1) Integrated program

Northeast Normal University is one of the universities that use integrated program. In its integrated program, students are required to obtain a minimum of 150 credit hours. Among all courses, mathematics courses contain advanced mathematics (4 credit hours), theories of elementary mathematics teaching and learning (4 credit hours), methods of mathematical thinking (2 credit hours), and psychology of mathematics learning (2 credit hours), amounting to 12 credit hours. While the first two mathematics courses (8 credit hours in total) are required courses, the other two (4 credit hours in total) are part of the program electives. Apparently, mathematics is a small part of this type of elementary teacher preparation program. Moreover, there are limited courses in mathematics available for prospective elementary teachers to choose.

(2) Focus-area specified program

Let take Capital Normal University as an example. There are seven different subject specifications offered in this university’s program for elementary school teacher preparation, including Chinese, mathematics, English, science, information technology, arts, and music. The mathematics specification requires a minimum of

197 course credit hours. Many more courses and credit hours in content subject are required. For prospective teachers majoring in mathematics, Table 4 shows the list of required and elective mathematics courses offered in this normal university. In particular, a total of nine courses in mathematics (29 credit hours) are provided in the category of required courses (44 credit hours in total).

Table 4. Required and elective courses in mathematics – Capital Normal University

Required courses in mathematics (credit hours required: 29)			
Courses	**Credits**	**Courses**	**Credits**
Theories of elementary math curriculum and instruction	3	Mathematical analysis II	4
Mathematical analysis I	4	Advanced algebra II	3
Advanced algebra I	3	Probability and statistics	3
Analytical spatial geometry	3	Mathematical thinking methods	3
Elementary number theory	3		
Elective courses in mathematics (credit hours required: 13)			
Mathematics pedagogy courses	**Credits**	**Mathematics content courses**	**Credits**
Problem solving studies in elementary mathematics	2	Multi-variable calculus	3
Case studies of elementary mathematics teaching	2	Algebraic structure	3
Psychology of elementary mathematics learning	2	Statistical analysis	3
Comparative studies on elementary math education	2	Mathematical experiment	3
Educational studies on Olympic elementary mathematics	2	Introduction to fuzzy mathematics	3
		Ancient Greek mathematics thinking	2
		Mathematics paradox	2
		Non-Euclidean geometry	2
		Affine geometry	2
Credit hours required	4	Credit hours required	9

(3) Middle-ground program

Taking South China Normal University as an example, the required minimum credit hours are 156. The program offers three general focus areas in elementary teacher education: Chinese and liberal arts, mathematics and science, and English and liberal arts. For prospective teachers majoring in mathematics and science, they are expected to take at least six mathematics courses and one teaching method course. See Table 5.

Table 5. Required courses in mathematics - South China Normal University

Course name	Credits
analytical geometry	4
mathematical analysis	10
linear algebra	4
probability and statistics	4
mathematical thinking methods	4
mathematical game and competition	4
elementary mathematics curriculum and instruction	3
Total	33

Above are three examples of each type of program. Readers should be aware that the three sample curricula are used only to illustrate possible differences across different types of programs, but not to represent these three types of programs. In fact, curricular differences also exist across institutions with the same type of preparation program in nature.

The status of prospective elementary mathematics teachers' preparation

In this section, we will report some results of a survey on the status of prospective elementary mathematics teachers' preparation. This survey was conducted in 2007. We choose 9 institutions that cultivate pre-service elementary teachers (5 normal universities and 4 junior normal colleges) in Mainland China. A total of 314 soon-to-graduate prospective elementary teachers participated in this study, of which, 178 are from normal universities, and 136 are from junior normal colleges.

The survey investigates prospective elementary mathematics teachers' preparation in four aspects: prospective teachers' (1) understanding of mathematics syllabus; (2) self-rating of their readiness to teach elementary school mathematics; (3) mathematics content knowledge; and (4) pedagogical content knowledge. According to above aspects, we developed a questionnaire containing many items. Most of items were focus on the topic of division of fractions. Some of them were taken from Li & Smith (2007)'s study; some were adapted from school mathematics textbooks and others' studies (e.g., Hill, Schilling, & Ball, 2004; Tirosh, 2000). Given the limited page space we have here, only a few items and prospective teachers' responses are presenting as followed.

Understanding of mathematics standard

Item 1: How would you rate yourself in terms of the degree of your understanding of the National Mathematics Standard?

Table 6. Percent of sampled prospective teachers' self-rating of their understanding

	High	Proficient	Limited	Low
J. N. C.	5.1%	12.5%	57.4%	25.0%
N. U.	2.2%	13.5%	73.0%	11.2%

Total	3.5%	13.1%	66.2%	17.2%

Note 1: "J. N. C." means "junior normal college". "N. U." means "normal university".

Note 2: The results presented in the tables may not add to 100% due to rounding errors.

Table 6 shows that many sampled prospective teachers did not think that they know their national mathematics standard well. In particular, there are only a 16.6% self-rated with either "high" or "proficient". Consequently, 83.4% self-rated with either "limited" or "low".

To deeply investigate prospective teachers' understanding of mathematics sandard, we asked sampled teachers to answer the following item.

Item 2: "Multiplication and division of fractions" is one important topic that is included in school mathematics. Choose the response that best describes whether primary school students have been taught each topic.

(a) Mostly taught before grade 5 (b) Mostly taught during grades 5-6

(c) Not yet taught or just introduced during grades 5-6

(d) Not included in the National Mathematics Syllabus (e) Not sure

According to *the National Mathematics Sandard for 9-year compulsory education*, Choice (b) is correct. The results show that the majority (64.6%) of sampled prospective teachers know that the topic of "multiplication and division of fractions" is mostly taught during grades 5-6 in elementary mathematics curriculum. 67.6% of J.N.C. prospective teachers and 62.4% of N.U. prospective teachers chose the correct answer. There is no significant difference between J.N.C. prospective teachers and N.U. prospective teachers（$\chi^2=0.94$，df=1，$p>0.05$）.

Readiness to teach elementary school mathematics

Item 3: Considering your training and experience in both mathematics and instruction, how ready do you feel you are to teach the following topics?

(a) Very ready (b) Ready (c) Not ready

( ) Topic 1: Primary school mathematics in general

( ) Topic 2: Number – Representing decimals and fractions using words, numbers, or models

( ) Topic 3: Number – Representing and explaining computations with fractions using words, numbers, or models

Table 7. Percent of sampled prospective teachers' choices of their readiness to teach elementary school mathematics

		Very ready	Ready	Not ready
Topic 1	J. N. C.	16.2%	72.1%	11.8%
	N. U.	16.3%	70.8%	12.9%
	Total	16.2%	71.3%	12.4%
Topic 2	J. N. C.	14.7%	52.9%	32.4%
	N. U.	14.6%	52.2%	33.1%
	Total	14.6%	52.5%	32.8%
Topic 3	J. N. C.	13.2%	45.6%	41.2%

	N. U.	15.7%	44.9%	39.3%
	Total	14.6%	45.2%	40.1%

Table 7 shows that about 15% of prospective teachers felt that they are very ready to teach these three topics (16.2%, 14.6%, and 14.6% respectively). However, with only about 12.4% felt that they are not ready to teach primary school mathematics in general, relatively high percentages of sampled prospective teachers felt that they are not ready to teach the other two topics with the use of different representations (32.8% and 40.1%, respectively). In general, prospective teachers do not feel over-confident with their knowledge and readiness to teach specific topic about fraction. There is no significant difference in topic 1, 2 and 3 between J.N.C. prospective teachers and N.U. prospective teachers（topic 1：$\chi^2=0.10$，df=2，$p>0.05$. topic 2：$\chi^2=0.02$，df=2，$p>0.05$. topic 3：$\chi^2=0.40$，df=2，$p>0.05$）.

Prospective teachers' competence in solving fraction problems

The following are four of items to investigate prospective teachers' mathematics content knowledge. The percentages of sampled teachers' correct responses are presented in table 8.

Item 4: $5\frac{1}{4} \div 3\frac{1}{2}$ =?

Item 5: How many $\frac{1}{2}$ does $\frac{1}{3}$ have?

Item 6: If $\frac{14}{15} \div \frac{?}{9} = \frac{3}{10}$, what is ?

Item 7: Ligang's candy store sells sorts of moon cakes. One day, he sold 24 boxes of ham moon cakes, and the quantity of the nut moon cake he sold is $\frac{3}{4}$ of the former, and $\frac{2}{3}$ of double egg yolk moon cake. Please answer how many boxes of double egg yolk moon cake did his store sell?

Table 8. The percentages of sampled teachers gave correct answers in item 4-7.

	Total	J. N. C.	N. U.
Item 4	94.3%	93.4%	94.9%
Item 5	86.6%	83.8%	88.8%
Item 6	72.6%	71.3%	73.6%
Item 7	68.2%	59.6%	74.7%

By the 4 items, we can see the understand of mathematics content knowledge of the example. Table 8 shows that，the percentages of who gave correct answers in item 4, 5, and 6 are all more than 70.%. J.N.C. prospective teachers' performance was similar to N.U. prospective teachers'. There is no significant difference between J.N.C. prospective teachers and N.U. prospective teachers in these three items. Comparing with item 4-6, item 7 received relative low correct answers. This is resulted in J.N.C. prospective teachers' low correctness. Only 59.6% of J.N.C. prospective teachers gave correct answers in item 7. There is significant difference in item 7 between J.N.C. prospective teachers and N.U. prospective teachers （χ^2=8.16，df=1，$p<0.01$）.

Prospective teachers' competence in solving context-based teaching problem

Item A: How would you explain to your students why $\frac{2}{3} \div 2 = \frac{1}{3}$? (item adapted from Tirosh, 2000)

90.4% of sampled prospective teachers provided valid explanations. The main strategies used are as followed.

S1: to present the process of operating by using the algorithm, i.e., "dividing a number equals to multiplying its reciprocal".

S2: to present the algorithm by using words.

S3: to explain the meaning of fraction. such as "dividing a whole into three equal parts, each part should be 1/3, so two parts should be 2/3. 2/3 ÷ 2 means to equally divide 2/3 into 2 pieces, thus one piece should be 1/3."

S4: to use manipulative materials such as apple or cake; or to use real-life problems, such as "You have a cake, cut it into 3 equal parts. One pare is for you, the other two parts are for your two friends. If you divide the two parts to two friends equally, how many parts will one friend have?"

S5: to draw picture or use number line. (See examples in appendix 1&2)

S6: to multiply 1/3 with 2, then get 2/3. Take this as a proof.

From diagram 1 we can see that, among those who provided valid explanations, most of sampled teachers used strategy 1, 2, or 3, while, strategy 4 and 5 are used by few. Most sampled teachers tend to use abstract algorithm, few tend to use intuitive strategy. From diagram 2 we can find that, the total percentage of J.N.C. prospective teachers who used algorithm is much more than those of N.U. prospective teachers. While, the percentage of N.U. prospective teachers who used intuitive strategy is more that those of J.N.C. prospective teachers.

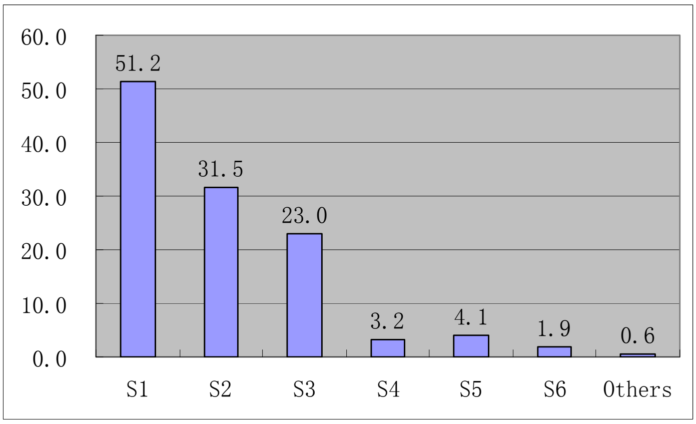


Diagram 1. Valid strategies used by prospective teachers [1]

[1] Some sampled teachers used more than one strategies, so the total percentage is more than 1.

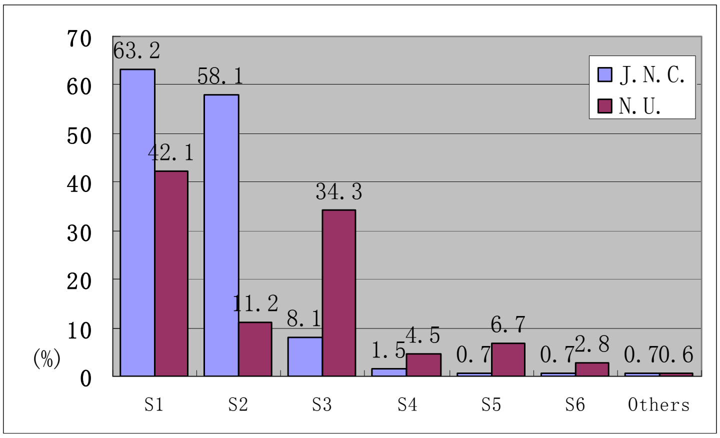


Diagram 2. Comparing the strategies used in item A by J.N.C. and N.U.

Item B: When you are teaching fraction division rule （that is $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ ）in your class, your pupils ask why we change "division" into "multiplication" and reverse numerator and denominator of the second fraction. How do you explain it to your students?

23.6% of sampled teachers provided valid explanations. The percentage of N.U. prospective teachers who provided valid explanations is 30.9%, which is more than that of J.N.C. prospective teachers (14.0%). There is significant difference between J.N.C. prospective teachers and N.U. prospective teachers ($\chi2=12.27$，$df=1$，$p<0.01$). N.U. prospective teachers performed better than J.N.C. prospective teachers in item B.

Two strategies are used by those who provided valid explanations. Strategy one is to use specific numbers to take place a, b, c and d, such as $\frac{4}{8} \div \frac{1}{2} = \frac{4}{8} \times \frac{2}{1}$, to prove the algorithm. Strategy two is to prove the algorithm by using the meaning of fraction, the nature of division computing, and the definition of the reciprocal. (See examples in appendix 3). Among those provided valid explanations, 94.7% of J.N.C. prospective teachers and 38.2% of N.U. prospective teachers used strategy one, and 5.3% of J.N.C. prospective teachers and 61.8% of N.U. prospective teachers used strategy two.

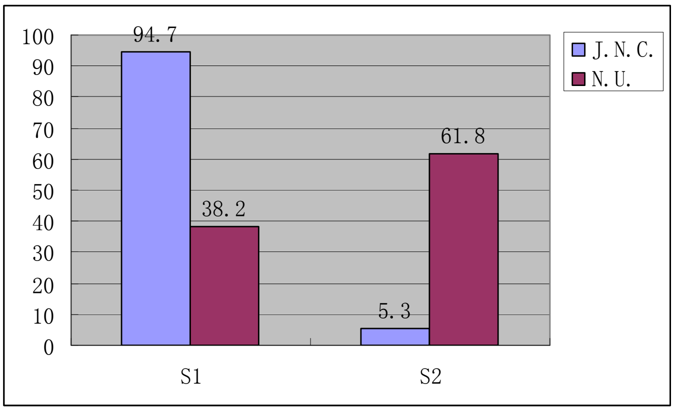


Diagram 3. Comparing the strategies used in item B by J.N.C. and N.U.

By the 2 items, we want to know the pedagogical content knowledge of the example. As item A is concerned, several valid strategies were used by the prospective teachers. From a perspective of mathematics, these strategies are all valid. However, considering the psychological characteristics of elementary school students, teachers should explain not only from a mathematical perspective, but also from a pedagogical perspective. It is necessary to try some intuitive representation appropriately when teacher teach abstract mathematics to elementary school students.

It can be found that most of prospective teachers tend to use algorithm in their responses to item A. Does this mean they have a profound understanding of algorithm? Form their responses to item B, it is proved that most of them have a limited conceptual understanding of the fraction division algorithm. They know how to use algorithm, but they do not know why it is.

A summary of above findings

In general, it is showed that a limited number of sampled prospective elementary mathematics teachers perceived themselves to have sufficient preparations for teaching fraction division. Consistently, the majority of sampled prospective teachers mastered the computation of fraction division well but had limited conceptual understanding of the fraction division algorithm. Moreover, it is showed that prospective elementary teachers had insufficient pedagogical content knowledge in providing adequate explanation about the fraction division algorithm to elementary students. Comparing with N.U. prospective teachers, J.N.C. prospective teachers' inadequate preparation in conceptual understanding of algorithm and pedagogical content knowledge is more obvious.

Prospective elementary school mathematics teachers' preparation: reflection and implications

What is the most effective program for training elementary mathematics teachers?

In this article, we have introduced five types of program for training elementary mathematics teachers. They are 5-year program and 3-year program in junior normal colleges and integrated type program, focus-area specified program, and middle-ground type of program in normal/comprehensive universities. Now, multiple types of elementary teacher preparation programs co-exist in the Chinese teacher education system. It seems that we have multiple roads to choose in training elementary mathematics teachers, but it is not. In fact, which is the most effective program for training elementary mathematics teachers? Which is suitable for the mathematics education in elementary school? Much still remains unknown. Although we have investigate the preparation status of prospective elementary mathematics teachers and made a comparison between J.N.C. and N.U., we have not enough proof yet to distinguish which program is more effective. Viewing in this point, we are indeed facing many roads, but we are standing at a crossroads. How shall we decide? We need more empirical research to provide references for our decision. Maybe the research ideas of TEDS-M (IEA, 2008) can be used for our further research on preparation of prospective elementary mathematics teachers.

Do Chinese prospective elementary school mathematics teachers have profound understanding of mathematics?

The results of our survey have shown that majority of sampled prospective teachers mastered the computation of fraction division well but had limited conceptual understanding of the fraction division algorithm. These prospective teachers' tacit understanding of algorithm is probably enough for them to solving mathematics problems, but it is surely not enough to teach mathematics in elementary school. It is often said that if you want to give a glass of water to your students, you should have a bucket of water. As mathematics teaching is concerned, a teacher who only knows an algorithm only has "a glass of water". If she/he wants to have a bucket of water, she/he should know why the algorithm is it, she/he should know how to help her/his students understand the algorithm easily too. The findings of our survey remind the Chinese mathematics educator and mathematics teacher educator reflecting our mathematic teacher education and carefully estimating mathematics education in elementary school in future.

A further question is "Do Chinese in-service elementary school mathematics teachers have profound understanding of mathematics?" Ma (1999)'s study was conducted ten years before. We can not use Ma's findings to infer the present status of mathematics teachers' profession. It is necessary to investigate the status of Chinese mathematics teachers' professional development and give judgments from our insiders.

How to explain algorithm to elementary school students? Mathematically, or pedagogically?

Cai & Lester (2005)'s study shows that Chinese students tend to use symbolic representations in problem solving, while U.S. students tend to use concrete representations, and the representations students use are influenced by the representations their teachers use. Our survey finds that Chinese prospective elementary school mathematics teachers tend to use symbolic representations when they explain fraction division algorithm to students, while concrete representations are less used. Of course, each type of representation has its merits and limitations. As Cai (2007) suggested, at the beginning of teaching mathematics concept, algorithm and relationship, a teacher should use concrete representations or encourage students to use their own strategies to solve problem or understand mathematics. With the development students' mathematics concept, students should be encouraged to develop the ability to use symbolic representations, rather than to rely on concrete ones. Considering concrete representation is necessary in the early stage of students' mathematics learning, we suggest prospective teachers give concrete representations more concern, and use concrete representations in appropriate teaching context, which could help students understand abstract mathematics content.

Some implications to mathematics teacher education reform in Mainland China

From the second part of this article we can see that prospective elementary mathematics teachers have limited understanding of mathematics syllabus, they lack conceptual understanding of algorithm and profound pedagogical content knowledge. Elementary mathematics teacher education institutions should draw lessons from above findings. Following implications might be referred by them: (1) Offer prospective teachers opportunities to know and understand National Mathematics Syllabus. The new round curriculum reform of basic education is ongoing in Mainland China now. The new curriculum has changed much in many aspects. Helping prospective teachers get familiar with the new curriculum will benefit their future teaching. (2) Attach importance to those courses closely relevant to elementary school mathematics education. Those courses about advance mathematics, such as advanced algebra, analytical geometry, and mathematical analysis, are emphasized by most programs in training elementary school mathematics teachers. These courses might help a teacher to develop good mathematics literacy. however, they can not help teachers to develop conceptual understanding of elementary school mathematics. In future, courses relevant to elementary school mathematics should be strengthened to help teachers deeply understand the common concept, algorithm, and theorem in elementary school mathematics. (3) Promote the quality of courses about mathematics teaching. Those courses about mathematics teaching should integrate general mathematics educational theory with the practice of elementary school mathematics education. Meanwhile, more practice opportunity should be offered to prospective teachers. Prospective teachers might benefit from these opportunities to develop better pedagogical content knowledge.

Acknowledgement: Most of data in this article is base on findings of two researches conducted in 2007 in China by the author, Yeping Li, Dongchen Zhao, and Shu Xie. The author of this article would like to thank the other 3 co-researchers for their efforts.

References:

Cai, J. & Lester, F. A. (2005). Solution and pedagogical representations in Chinese and U.S. mathematics classroom. *Journal of Mathematical Behavior, 24* (3-4), 221-237.

Cai, J. (2007). *Series of empirical research on students' mathematics learning in Chinese and U.S.*. Beijign: Educational Science Publishing House. [in Chinese]

Fan, L., & Zhu, Y. (2004). How have Chinese students performed in mathematics? A perspective from large-scale international comparations. In L. Fan, N-Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 3-26). Singapore: World Scientific.

Hill, H.C., Schilling, S. G., & Ball, D.L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *Elementary School Journal*, *105*, 11-30.

IEA. (2008). *IEA teacher education study: A cross-national study of primary and secondary mathematics teacher preparation*. Retrieved November 21, 2008, from http://teds.educ.msu.edu/documents/tedsSummary.pdf.

Li, Y., & Smith, D. (2007). Prospective middle school teachers' knowledge in mathematics and pedagogy for teaching – The case of fraction division. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the*

Psychology of Mathematics Education, Vol. 3, pp. 185-192. Seoul, The Republic of Korea: PME.

Ma, Liping. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum Associates.

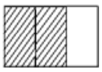
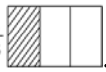
Ministry of Education, Mainland China. (1995). *The guideline for 5-year program of elementary teacher training (experimental)*. Beijing, China: Author. [in Chinese]

Ministry of Education, Mainland China. (2003). *Three-year program curriculum plan for preparing elementary school teachers*. Beijing, China: Author. [in Chinese]

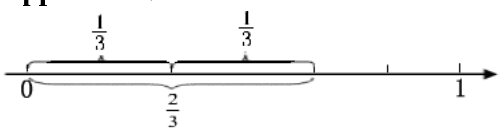
Schmidt, W., Tatto, M. T., Bankov, K., et al. (2007). *The preparation gap: Teacher education for middle school mathematics in six countries*. East Lansing: Michigan State University.

Tirosh, D. (2000). Enhancing prospective teachers; Knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31* (1), 5-25.

Appendix 1:

"Divide $\frac{2}{3}$  into 2 parts, then we get $\frac{1}{3}$ ."

Appendix 2:



Appendix 3:

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\div\frac{c}{d}\times(\frac{c}{d}\times\frac{d}{c})=\frac{a}{b}\div\frac{c}{d}\times\frac{c}{d}\times\frac{d}{c}=\frac{a}{b}\times1\times\frac{d}{c}=\frac{a}{b}\times\frac{d}{c};$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\div(1\div\frac{d}{c})=\frac{a}{b}\div1\times\frac{d}{c}=\frac{a}{b}\times\frac{d}{c};$$

$$\frac{a}{b}\div\frac{c}{d}=(\frac{a}{b}\times bd)\div(\frac{c}{d}\times bd)=ad\div cb=\frac{ad}{bc}=\frac{a}{b}\times\frac{d}{c};$$

$$\frac{a}{b}\div\frac{c}{d}=a\div b\div(c\div d)=a\div b\div c\times d=\frac{a}{b}\times\frac{d}{c}。$$