The Lesson Study as a strategy to change the paradigm of teaching mathematics:
A Brazilian experience

Yuriko Yamamoto Baldin
yuriko@dm.ufscar.br
Universidade Federal de São Carlos
Brazil

Abstract: In this note for APEC – University of Tsukuba Conference 2010 we first comment on some cultural barriers in Brazil that prevent the straightforward implementation of Lesson Study in basic school classrooms, but next we present reasons to justify its introduction into teacher education programs as well as into actual teaching practices. We comment a recent experience in which to follow the principles of Lesson Study in a 7th grade classroom has changed the paradigm in teaching mathematics of a public school teacher in Brazil.

Introduction: The Lesson Study is a powerful asset to the improvement of the process of teaching mathematics, especially at basic school level, for it allows the teacher to be creative as to propose refreshed lessons to stimulate the students, to evaluate the efficiency of the lesson, and to get profits of receiving external critics and suggestions from participants/observers ([3]). This process could sound natural to a culture that assimilates it as part of continuing education of teachers as well as an effective way to introduce investigative activities in the teaching profession. However, as we know, the teaching culture differs among countries, and the lack of the mechanisms to improve the teaching practices beyond the small details of methodologies constitutes a teaching gap and is one of the causes of differences among students’ achievements across different countries. Stigler - Hiebert ([4]) have pointed out the teaching gap in the U.S. and argued about teaching as a cultural activity. According to them, this fact explains why teaching is resistant to change, because “the common attitudes largely accepted within a culture make the teachers unaware of widespread attributes” (op.cit.p.11). Nonetheless, they say that the recognition of the cultural nature of teaching would give insights into what is needed to improve the teaching.

In this note we present some considerations about the cultural background of basic education system in Brazil and why we believe that the Lesson Study principles could help change the paradigm of teaching mathematics in this context. An earlier text [1] has already started our discussion on the difficulties of introducing Lesson Study in Brazil and many considerations of this note is taken from this text. In this note, we comment a field experience that is an attempt to attack the problem of teaching gap in Brazil.

Cultural background: The school system in Brazil has two levels of instruction: Fundamental (1st to 9th grades) and Secondary levels. Each State of Federation is autonomous to organize the school system in the level of its competency. The Fundamental level starts at 6 years of age, and from 1st to 5th grades constitutes, in the State of São Paulo, the First Cycle of Fundamental School, while 6th to 9th grades belong to Second Cycle that correspond to Lower Middle School. Kindergarten is not mandatory in Brazil, but the late educational policy tends to strengthen the importance of the investment of government in this direction. The Secondary level corresponds to Upper Middle School, and it is regulated by the Secretary of Education of each State of Federation. The National Curriculum Standard Documents (Parâmetros Curriculares Nacionais - PCN) are issued by the Ministry of Education, and the local educational authorities refer to them as they organize the school system under their responsibility.

Kindergarten and the First Cycle of fundamental level are the responsibility of each town/city, with expectations that in few years they will be responsible for the whole fundamental school system. At the moment, the Second Cycle of fundamental level (Lower Middle School) may
be the responsibility of each local county or of the State, being in any case administrated differently from of the First Cycle.

In [1], the authors said about this system:

- This disruption of educational policy in the middle of elementary schooling is felt as one of the causes of the difficulties to follow a sensible pedagogical planning for mathematics education at this level. Besides being a political issue, the serious problem is that the education of teachers suffers also a discontinuity.

In fact, the courses that prepare teachers for 1st to 5th grades are essentially Pedagogy with almost no mathematical content as well as practical teaching methodologies for mathematics, in their curriculum. On the other hand, the courses that prepare teachers for 6th to 9th grades (and also Secondary School) usually do not provide opportunities for prospective teachers in their undergraduate programs to make connections among content knowledge of mathematics and the process of teaching/learning practices in actual classrooms. The lack of connections in content knowledge is observed among course units of advanced content as well as in bridging the advanced knowledge to the curriculum content of Fundamental School level. Also, the curriculum of these preparation programs do not focus on the actual learning stage of students coming from a different school system that is deficient in both content knowledge and mathematical reasoning/attitudes, thus the transition from elementary to middle years of fundamental school is troublesome, especially felt in 6th grade.

The Lesson Study turns out to be one strategy to overcome this disruption in Brazilian context, since it could help the teachers to understand the educational problems caused by insufficient professional preparation that is not their fault, but to make them able to improve their teaching in the transition from a system to the next, and to actually promote the students’ learning in their classrooms. We believe that turning a teacher into a researcher of own classroom practices, through a carefully planned lesson followed by reviews of actually happened classroom events and of assessment of students achievements, is an effective way to fulfill the gaps of formal knowledge as well as to learn about different approaches and teaching materials for different subjects and levels of instruction. From this perspective, Lesson Study could be a powerful concept that would help closing the teaching gap in Brazil.

Nevertheless, the cultural background in the Brazilian educational system causes barriers to a straightforward introduction of Lesson Study. The essential parts of Lesson Study consists of the planning a lesson on a defined subject; the execution of the lesson to be investigated; and the review session with reflections and critics to improve the lesson itself as well as the perception of the teacher as investigator of own practice. The strength of this methodology is grounded on collaborative team-work that involves observers: teachers, pedagogical coordinators, administrators, educators, researchers, that is, it demands the participation of whole school community. ([3])

However, in Brazil there is no tradition in visiting classrooms even by pedagogical coordinators or school principals, with the possible exception in selected schools of the supervised visits of students of teacher preparation courses, as part of requirements of their curriculum. In these cases, the prospective teachers restrict themselves as silent observers and do not participate in the actual dynamics of the lesson. The notes taken from the observation to write reports are not shared or worked out with the actual teacher. Some graduate programs in mathematics education encourage the visits of researchers to schools and to conduct polls with teachers, but these programs do not aim, in general, at the changing the teaching culture of schools, for their focuses are predominantly education theories.

The pedagogical planning is frequently a list of contents scheduled for a lesson with time allocated to each topic, and it is not an instrument of reflection to contribute to further improvement
of the teaching. A careful study of teaching plan with expectations of students’ reactions as well as the analysis of the chosen teaching materials is not, as a general rule, either seen or expected.

The analysis and evaluation of the lessons would never happen in Brazilian schools. The pedagogical orientations about the content and the teaching methodologies given by Secretary of Education of each state are developed outside the classroom context, so the effects on actual teaching practices are not clear.

In [1], the authors wrote:

In order to introduce Lesson Study as educational concept that could aid the improvement of teachers’ practices and the learning of students, it is necessary to break the inertial state and to convince all the agents of the educational system to see this methodology as helpful to change the cultural tradition and that change of culture could actually contribute to the improvement of the education globally.

Therefore, it turns out to be extremely important to study careful adaptation of Lesson Study to forms that can be integrated into the cultural context, in order to convince the educational authorities about its efficiency in promoting the quality improvement of school education. In [1], the first efforts to disclose the Lesson Study in Brazil were informed. In the following, we describe one of field experiences implemented in a public school of State of São Paulo with the main objective of breaking the inertial state of teaching mathematics, in which the lessons are traditionally teacher-centered exposition of content even for problem-solving, and the non satisfactory results about students’ achievements are blamed on teacher or just left unfairly as students’ incapacity.

A case experience: Because of the cultural background at Brazilian schools, it is impossible to start any actions regarding a Lesson Study as team work that involves all the school, but developing a project based in the school teaching plan, following the principles of Lesson Study ([3]), was taken as strategy to change the paradigm of teaching mathematics.

The adapted principles were: a) careful planning of lessons on selected topics from curriculum determined by the State, including the choice of methodology, teaching materials and establishment of expectations regarding students´ reactions and the development of activities; b) execution of teaching plan based on problem-solving, promoting the active participation of students in each phase of resolution, instigating the critical inquiry about solving strategies and the results; c) the review session after lesson, based on observations of students’ performance and of own practice, annotated as a diary, and on critics of eventual observers.

We can notice that each step above signalizes a rupture with the traditional way of teaching in Brazil, pointed out in the previous section.

More precisely:
- The inclusion in the teaching plan of the justification about the choice of methodology and materials, as well as the expectations about the possible reactions at the classrooms, implies the change of attitude of teacher towards a student-centered lesson. Also, a careful plan would help greatly the teacher to evaluate own practice and the efficiency of the lesson itself, after its execution.
- The use of problem-solving methodology, even in those lessons that introduce new concepts or results in mathematics, breaks the inertia of teacher-centered exposition and involves the students as they participate in the construction of own knowledge. Also, the teacher is allowed to observe and to know better each of his students during the lesson, so it contributes to a quality improvement of assessment with accurate diagnosis of each student’s difficulties.
The practice of reviewing the lesson after the real execution is definitely a change to the tradition. This activity developed upon critics from external observers (in this experience, the author of this note attended several lessons during the execution of the project) combined with the observations annotated as a diary is a powerful material to promote the teachers’ investigations about own practices and to provide the necessary improvements to plan the following classroom activities.

The results of this experience have been elaborated as Master Dissertation presented to the Graduate Program in Exact Sciences Teaching, of Universidade Federal de São Carlos, Brazil ([2]).

The experience was developed in 7th grade classrooms of the public school, Escola Estadual Major Telmo Martino, in Osasco, a town in the vicinity of Great São Paulo city. The lessons were about following topics: i) fractions as division of numerator by denominator; ii) comparing fractions; iii) plane figures as projection of spatial objects; iv) compass-ruler construction and the triangle inequality; v) Theorem on the sum of internal angles of a triangle.

To illustrate the experience, we comment briefly one of the Lessons

**Lesson about compass-ruler geometric construction and the triangle inequality. (execution on May 11, 2009).**

**Class:** 7th grade (12 years old) with 35 students. **Time slot:** 45 minutes.

**General Objective:** promotion of a collective work by students, their participation in acquiring ability to use geometric construction tools and in understanding the meaning of triangle inequality through inquiry and making conjectures.

**Specific Objective:** Understand the possibilities of the construction of a triangle, given measures for its three sides.

**Tools:** Ruler and Compass. **Assumed previous knowledge:** the students know how to use the tools and what is a triangle.

**Comments on teaching plan:** The teaching plan was made upon the topic “triangle” recommended for the 1st academic semester (February to June) to 7th grade by the curriculum of Secretary of Education of State of São Paulo. The official recommendation to develop topics on geometry reflects the efforts to reverse the current negligence of school education about this area of mathematics content. The topics on geometry are often avoided by school teachers (caused by their weak preparation at university level), and the treatment of geometry, when it is done, is not connected to the rest of mathematics abilities. The geometry activities are frequently limited to geometric setting for algebraic calculations of measurements, without exploring the education of deductive reasoning so appropriated as a role of geometry inside the curriculum.

The main Problem proposed in the lesson was:

“Given any three segments, what is the condition that makes it possible to construct a triangle which sides have respectively the measures of the segments?”

The planning of the lesson started with the reflection of the teacher about what he knew about the learning stage of his students, and the need to explore the compass-ruler constructions to study geometric properties, since this practice has almost disappeared from Brazilian classrooms in late years. So, the plan started proposing a preliminary situation, with teacher drawing three segments on the board with measures: 5 cm, 6 cm and 7 cm, and asking the students to construct a triangle with these measures on its sides. This activity has meant a warm-up of the students that would manipulate their tools and remember how to transport measures with compass. After five minutes, the teacher would ask the same problem with the measures: 3 cm, 4 cm and 8cm. This is the catch to instigate the students to participate with their thoughts and reasoning. It is important to point out that traditionally in Brazilian classrooms, the teacher would construct himself the triangles
on the board, he will ask and will induce answers from students about the phenomenon that comes up with some measures and will quickly resume the lesson with teacher’s words. Therefore, the problem-solving methodology that makes the students to understand the problem and think about the reasons of the possibility (or impossibility) of construction method, before trying to justify a systematized mathematics result, has been adopted as important strategy to break the inertia of teacher-centered lessons. The teacher expected to conduct the lesson in a way that the students would reach the conclusion by themselves, establishing as final outcome the triangle inequality. Since he was trying the lesson as an experience to understand better the influences of Lesson Study on his own practice, he was anxious to see the results at the real classroom.

Comments about the execution of the lesson: It has been quite interesting the change of attitudes of students observed during this lesson. They have turned from passive listeners of teacher’s explanations to question raisers. At this part, the teacher has succeeded in keeping a dialogue with students that established a good relationship among them and stimulated them to participate in the discussions. It is also important to notice that there are always those saying they have succeeded in constructing a triangle with 3, 4 and 8 cm on its sides. So, it is a rich opportunity for teacher to know deeply his students and their level of knowledge and errors as well as their attitudes towards learning.

![Figure 1: Student manipulating a compass.](image1)

![Figure 2: Student trying different measures to construct triangles.](image2)

After having the attention of the class, the teacher proposed the main problem on board and suggested to try some measures chosen by the class. And then, he asked the case: 3, 4 and 7 cm. With these measures, there were always students asserting they had actually constructed a triangle, caused by the natural imprecision in measurements and inappropriate use of the tools by students. It
is the moment to call for the logical reasoning to rule this case out, and connect the geometric setting to the sum of numbers in line of real numbers. The teacher has to be clever to conduct a dialogue with students in order to make them understand the situation and the power of mathematics based reasoning.

Figure 3 – Construction with 3, 4 and 7 cm.

After working some possible and impossible constructions, the teacher asked if it is possible to elaborate any condition about three given numbers that implies the possibility of construction. In this phase, the students reviewed what they had learned so far to make a strategy to work on this problem. The discussions about how to justify a possible answer is a gain to proceed to the final stage of problem-solving methodology, which is the validation and investigation of their answers and other conjectures. In the sequence, the discussion occurred among students has been resumed in a table with some of the cases they considered:

<table>
<thead>
<tr>
<th>Example 1</th>
<th>Example 2</th>
<th>Example 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 + 6 &gt; 7</td>
<td>3 + 4 &lt; 8</td>
<td>3 + 4 = 7</td>
</tr>
<tr>
<td>6 + 7 &gt; 5</td>
<td>4 + 6 &gt; 3</td>
<td>4 + 7 &gt; 3</td>
</tr>
<tr>
<td>7 + 5 &gt; 6</td>
<td>6 + 3 &gt; 4</td>
<td>7 + 3 &gt; 4</td>
</tr>
<tr>
<td>Construction: Possible</td>
<td>Construction: Impossible</td>
<td>Construction: Impossible</td>
</tr>
</tbody>
</table>

Table 1: A table with examples of some cases considered by students ([2]).

The teacher then asked the students to analyse the table and make conclusions with their own words. The statement of the property of triangle inequality came out as the effective product of their reasoning. As part of homework, each student was asked to write in a sheet of paper a phrase that starts with “What I learned today:” to be turned in next day.

During the execution of this lesson, the teacher was surprised with many findings, more than expected in his plan, because the dynamics of the lesson as it developed has revealed the prompt response of students to a new form of classroom activity, allowing him to observe better his students. They were more interested mainly for using tools, as well as they showed being glad to participate in the activities, for being heard and having their considerations shared by whole class. Also, the exercises, done with their own data, not copied from teacher’s models, permitted them to be more self-confident about the activity and the sought results.

Comments about reflections after lesson: After the lesson, the teacher reviewed the happenings and heard the critics of the observer, and this principle adopted from Lesson Study was determinant to improve his next mathematics lessons on geometry as well as on other topics. He learned about his own attitude during the lesson that he never took attention at before, because the traditional role of teacher as transmitter of the models of thinking prevents the active participation of each student. Reading the reports written by students also helped him to evaluate the levels of difficulty shown by each student, so the assessment of the class has gained a shift in quality. He realized that, in spite of the enthusiasm of students with new form of learning, the objective of the lesson has not been accomplished thoroughly by some students. Then, the next lesson was dedicated to a review of the procedures of construction with the use of DGS when the dynamic feature of the software caught the attention of students that participated by following their own thinking through manipulations.
The use of technology has magnified the results already got through previous problem-solving activity.

The lesson about geometry deals with a subject that the students show resistance to follow, as consequence of the lack of habit in reasoning and deducing, that is, the exercise of abstraction and logical thinking in earlier years. It is our opinion that the gap between first and second cycles in fundamental level of schooling is caused also by the abrupt change of methodology between these cycles. In first cycle, the geometry is worked out almost exclusively by drawing, knowing the names of figures and elements, without problem-solving that would enable the geometric perception and skills to deduce properties but the calculation of perimeters/areas of basic figures or ludic activities. After moving to second cycle, the students are surprised when asked to think about “possibilities”, they hardly understand what is being solicited. In Brazil, students (and many teachers too…) often think of mathematics as a “finished object” to be memorized as it is delivered by teachers and they get used to just follow the models without critical reflection or comprehension.

Therefore, the planning of student-centered lessons conducted through inquiry and discovery should be a change of paradigm of teaching mathematics in Brazil. Especially, the reflection session after the lessons could show to the teacher the advantages of Lesson Study to empower the whole process of teaching/learning inside the school system, whatever the cultural background is behind. The teacher of this case experience has noticed the individual struggling of his students, has understood many of the errors they continuously do and their causes. Then, it facilitated the subsequent work of planning new lessons as well as to help students to overcome the difficulties and to get better performance in formal evaluations.

**Concluding remarks:** We commented only one lesson from the case experience of the project that aims at the recognition by teachers and educators of the ways opened by Lesson Study to change the paradigm of teaching mathematics in Brazilian reality. The experience project is in its beginning but it is systematized as a research project within a broader collaborative work of LIMC (Laboratory of Research and Development of Materials for Teaching Mathematics and Sciences), a Research Center located in Rio de Janeiro, Brazil, in which the author and some collaborators are studying the necessary adaptations of Lesson Study to be understood and accepted by Brazilian school community. Collaborations between LIMC and the Ministry of Education of Brasil has opened in late years the possibility of large scale projects for the professional development of school teachers, especially at elementary level (1st to 5th grades), when the stress is on the specific knowledge of mathematics content combined with appropriate methodologies, in order to smooth the way between the grades and the levels of school systems. About 50,000 teachers have been attended since 2006, and we believe that the Lesson Study principles should also be included into these courses.

**References:**


