FROM AN ACTIVITY IN A TEXTBOOK TO AN OPEN-ENDED PROBLEM: DEVELOPING STUDENTS' MATHEMATICAL THINKING AND COMMUNICATION

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This paper reports how textbooks, blackboards, computers, and overhead projectors are used in mathematics classes for teaching and learning mathematics in a secondary school in Metro Manila. In this school, all the mathematics teachers in each grade level are engaged in lesson study. The paper also presents an example of how an activity from a mathematics textbook can be made into an open-ended problem which students can explore further by using computers. Solving then exploring the problem further can deepen the students' understanding of the concept which the activity is trying to develop and enable them to appreciate the connections of the new concept that they are learning with those that they have learned before.

PROVISIONS OF TEXTBOOKS IN PUBLIC ELEMENTARY AND SECONDARY SCHOOLS IN THE PHILIPPINES

Textbooks in public elementary and secondary schools in the Philippines are provided by the Department of Education (DepED). Every student is lent a copy at the start of the school year. This is returned to the school at the end of the school year. The textbooks have an accompanying teacher's manual. In most cases particularly in the rural areas where access to bookstores is limited or does not exist, these textbooks are the only resources that teachers have for preparing their lessons (S. Pilor, personal communication, January 28, 2010).

Mathematics textbooks are written in English. The national language is Filipino. Mathematics is taught and assessed in English.

HOW ARE EDUCATIONAL TOOLS USED IN MATHEMATICS CLASSES

The practices of the mathematics classes in Sta. Lucia High School (SLHS) in Pasig City in using educational tools are described here. These are based on the responses to a survey on this of the 16 grades 7 to 10 mathematics teachers who are participating in lesson study in the school.

Current Uses of Textbooks

The students of SLHS have two different mathematics textbooks. One is what the DepED provides to all schools. The other is what some teachers of Pasig City developed for use by schools in the city. The latter has a teacher's manual and limited copies of student's workbook. The teachers bring the workbooks to class only when they will be used. Each copy is shared by two students. They do not write on the

workbook. The teachers use the student's textbooks in different ways, namely: (1) to determine the topics or lessons that they will teach, (2) to determine the teaching strategy that they will use to teach the topics or lessons, (3) as a source of examples for explaining the topics or lessons, (4) as a source of problems, activities, and exercises that they will ask students to do in class in groups or individually, and (5) as a source of items for the daily quizzes and homework. The teachers refer to the teacher's manual to get ideas for the objectives of their lessons and suggestions on how to teach them. They modify these based on the ability of their students. They also use this manual to verify the correctness of the students' answers to the exercises in the textbooks and workbook. The teachers use other reference materials. The students use their textbook as (1) a reference material for the topics or lessons discussed in class, (2) a source of seatwork and homework that the teachers assign, and (3) a review material for their examinations.

When asked what characteristics they want the student's mathematics textbooks to have, the teachers gave the following responses: (1) they should have more examples and exercises, (2) the examples should be realistic, (3) the exercises should have different levels of difficulty, (3) they should have activities, and (4) they should include strategies.

Current Uses of Blackboards, Overhead Projectors, and Computers

Among others, the teachers use blackboards when presenting their lesson. They are where they write the examples of what they will ask students to do later. They are also where they post their visual aids and manila paper or cartolina on which notes such as definitions of mathematical terms or steps in a mathematical algorithm are written.

The students write on the blackboards their solutions and answers to exercises or problems they do in class or as homework. They also post on the blackboards manila paper or cartolina on which they have written their group work outputs.

There are three overhead projectors (OHP) in the school. However, the teachers do not use an OHP to teach mathematics. This is because it is not convenient to bring this heavy equipment from one classroom to another especially when the classrooms are located on different floors of the six-storey school building. There are teachers who also find preparing the transparencies time-consuming.

The school has 40 computers located in the Computer Room. By arranging schedules, classes of the different subject areas can use them. However, only the grades 9 and 10 Technology and Livelihood Education classes use them. In very rare cases, a few mathematics teachers had used these computers for demonstration teaching. All the other mathematics teachers do not use these computers to teach mathematics because according to them, these are not available for use in mathematics (perhaps they mean, during the regular schedules of their mathematics classes). Likewise, the students do not use the computers in the school to learn mathematics. Interestingly, despite this situation, all the teachers comment that it will be good to use computers in mathematics classes. Such a remark may indicate that they are willing to teach mathematics using

computers if these are available for their and their students' use.

Summary

To a large extent, the teachers use the student's textbooks in teaching mathematics although they also use other reference materials to supplement them. Besides the textbooks, the blackboard is a main educational tool in mathematics teaching and learning. Although most of the teachers do not use computers in teaching mathematics, they express a positive attitude in using these in their classes if these are available.

HOW CAN TEXTBOOKS BE USED MEANINGFULLY: SOME INNOVATIVE APPROACHES

To develop mathematical thinking and communication among students, textbooks should reflect learner-centered approaches in learning mathematics. This can be done if textbooks include a variety of inquiry-oriented activities. That is, the content and presentations in textbooks should be aligned with the emphasis of inquiry-oriented classes. For instance, practical work activities should not only help students solve problems but should also challenge them to make further mathematical explorations. Doing so can deepen and broaden their understanding and appreciation of mathematics. Mathematical explorations often requires much time if done without the help of technology. Using computers does not only save a lot of time but can also make the analysis of results of explorations easier. Students can then spend more time analysing mathematical relationships, making and testing conjectures, and justifying obtained results.

From an Activity in a Textbook to an Open-ended Problem: Introducing Polynomial Function

To develop the concept of polynomial function, the grade 10 mathematics textbook used in public schools before, utilized a box-making activity (Pascua, et al 1992). It provided a real-life context for introducing this concept to which students could relate. With visual illustrations, it showed how a box with an open top could be made by cutting a side of a square at each corner of a rectangular sheet of paper with given dimensions. It asked students to make different boxes with an open top using the same procedure and the same size of paper. Then it asked them if the volumes of the boxes formed were the same. Finally, it presented the mathematical results if the activity was done.

To introduce the concept of polynomial function, the lesson study group of grade 10 mathematics teachers of SLHS formulated an open-ended problem on making boxes with an open top. The problem is shown on the next page. Unlike the activity mentioned earlier, the problem required the students to devise their own ways of making the boxes. They were only given pointers to follow.

Problem:

Christmas is fast approaching. Lucy wants to give a personalized gift to her friends. She plans to make a box with an open top where she can store the gifts. Before making the actual box, she wants to try it first by using a plain sheet of grid paper measuring 10 cm by 16 cm. If you were Lucy, what possible boxes can you make? Among the boxes that you made, which do you prefer and why?

Pointers to follow:

1. *Construct an open-top box of different sizes using sheets of grid paper, pair of scissors, and tape.*
2. *Use one sheet of grid paper for each box.*
3. *Do not remove any part of the sheet of grid paper. You can cut the paper but not cut off any part.*
4. *Avoid any folds on the top part of the box.*
5. *Do not waste the sheets of grid paper so that you can make many boxes.*

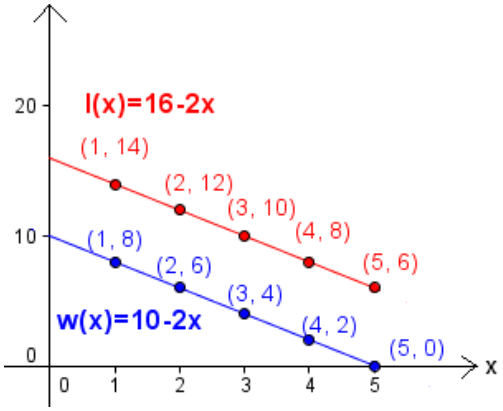
Developing Students' Mathematical Thinking and Communication

To solve the problem, the students can discover that if they fold along the sides of a rectangular sheet of grid paper such that the rectangles formed have the same width then the folds will create a square at each corner of the paper. If they cut one side of each square, then each corner will have a flap and a box with an open-top can be formed. This is one way of making the boxes. Using this method, the students can make boxes of different sizes. Then they can justify their box preferences.

If the length of a side of a square that is cut at each corner of a sheet of grid paper is represented by x, then the height of the box formed is also x, its length is 16 – 2x, and its width is 10 - 2x. The students should be able to provide reasons for the allowable values of x.

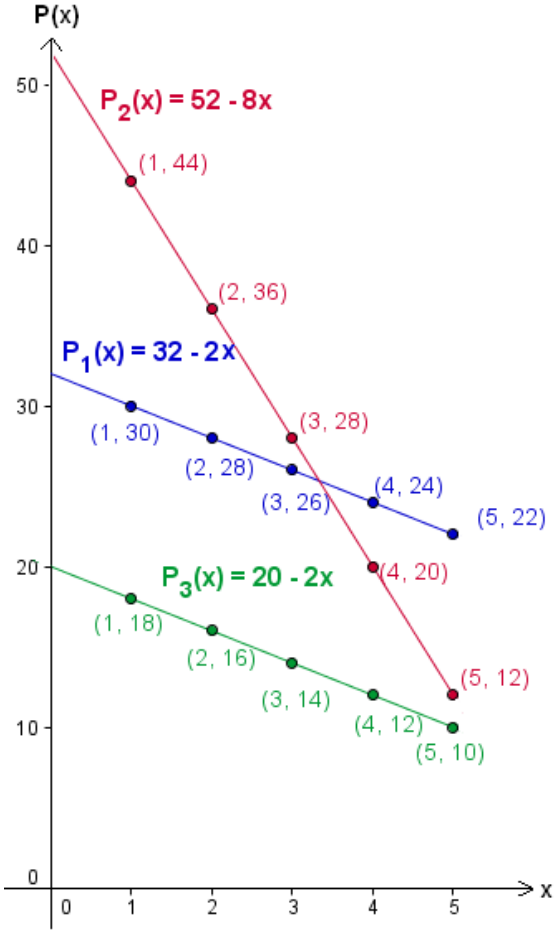
After answering the problem, if the students examine the different boxes that they made, they can observe that there are quantities that change as the height x of a box changes. These quantities are: (1) the length and width of a box, (2) the perimeter of each face of a box, (3) the area of each face of a box, (4) the volume of a box, and (5) the surface area of a box. So, they can write the equation representing each quantity in terms of the height of the box and explore their relationship by graphing their equation. The graphs of the relationship between the height and each of these quantities are shown below using Geogebra. A dynamic geometry software, Geogebra can show the graph, equation, and specific points on the graph.

1. The dimensions, length l(x) and width w(x), of a box
   $l(x) = 16 – 2x$
   $w(x) = 10 – 2x$
   height = x



2. The perimeters P(x) of different faces of a box

$P_1(x) = 32 – 2x$
$P_2(x) = 52 – 8x$
$P_3(x) = 20 - 2x$
height = x



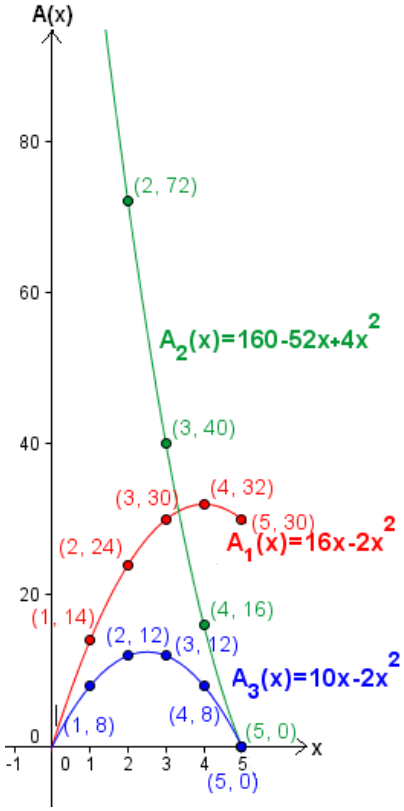
3. The areas A(x) of different faces of a box

$A_1(x) = 16x - 2x^2$

$A_2(x) = 160 - 52x + 4x^2$

$A_3(x) = 10x - 2x^2$

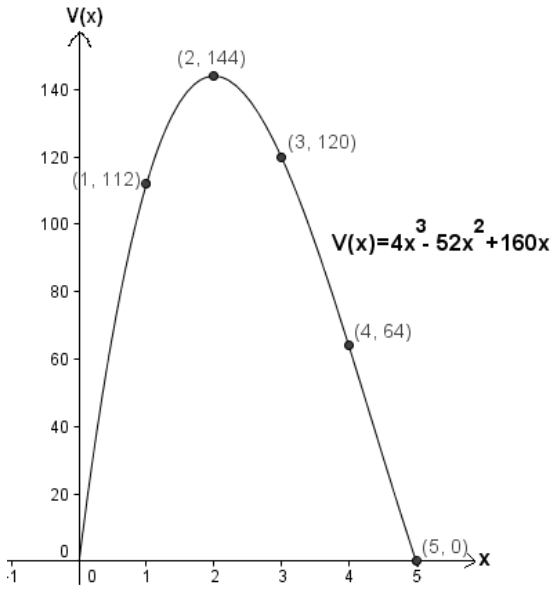
height = x



4. Volume V(x) of a box

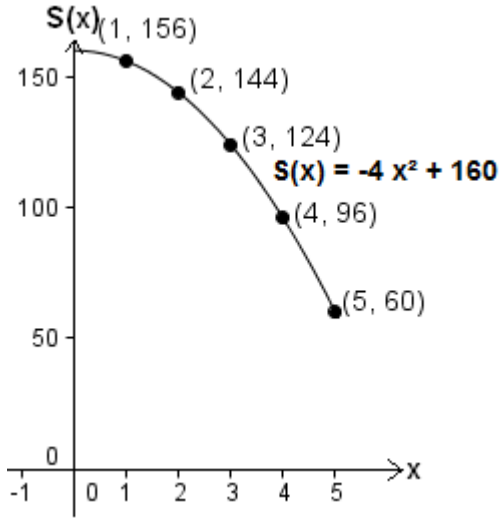
$V(x) = 4x^3 - 52x^2 + 160x$

height = x



5. Surface area of a box:

S(x) = $-4x^2 + 160$

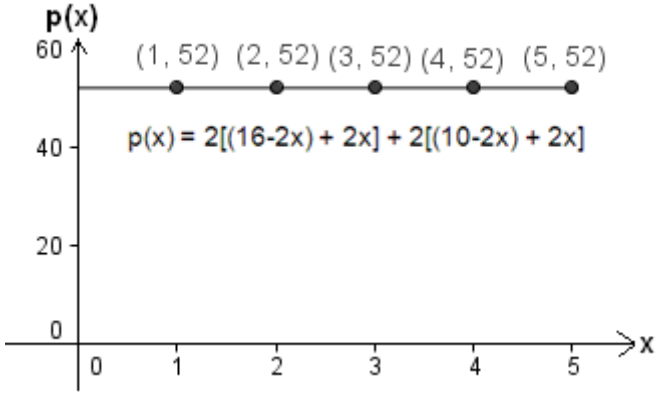


The SLHS mathematics teachers can make arrangements so that their classes can use the Computer Room. Since in a class there are more students than computers, then a computer will have to be shared by at least two students. This situation demands that the students discuss with each other so that they can have a common understanding of what they will explore and of the results of their explorations. The teacher may first ask them to predict the visual representations of the relationships of the changing quantities by sketching their graphs and to justify their predictions. Then they can verify if their predictions are correct by actually graphing using Geogebra.

The students can interpret the graphs and explain their behaviors. Mathematical interpretation of phenomena just like mathematical representations of solutions to problems is also a characteristic of inquiry-oriented mathematical activities (Shimizu 2010). Moreover, students can analyse the graphs and their corresponding equations and realize that these represent different functions. They can classify these functions because except for the cubic function, they have already learned them before. Five of them represent a linear function; four represent a quadratic function; and one represents a cubic function. The teacher can then relate all these functions by introducing polynomial function based on what students already know about polynomials and functions. Thus, the mathematical explorations lead to mathematical connections which can reinforce what students have learned in the past by building on them to learn a new concept.

The students can do many more explorations. For instance, they can find out how the graphs of the different equations will look like if the context of the problem is disregarded. They can also explore other relationships such as that between the length x of a side of a square that is cut at each corner of a rectangular sheet of paper and the perimeter of the net of a box that will be formed. The graph is shown below.

Perimeter of the net of a box



Hence, by modifying a practical work activity from a textbook into an open-ended problem that can challenge students to make further explorations using computers, students' mathematical communication and thinking can be developed.

References

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