

Title: **Exploring Linkages with Geometry**

Lesson Presenter: Masami Isoda (Univ. of Tsukuba), At Junior High School

Theme for Study: How can we develop students' ability to apply geometric proof for problem solving in the real world? (*It is the theme of discussion: Study lesson is a sample and strategy to enquire*)

Objective of the Class: *Through exploring* the van Schooten (1646) textbook problem with various representations, *students learn* how to adopt geometric reasoning into mechanism/kinematics and appreciate the significance of proof which they learned.

(Through A, students learn B' is recommended to involve the process into the outcome)

Explanation of Subject Matter in Relation to Theme, Objective, Curriculum and Students:

In Japan, students study geometric proof at Junior High School such as congruence and similarity of figures, and circle and Pythagorean theorems. One third of students in ordinary schools enable to prove the simple problems in geometry, one third may follows the explained proof by others, and one third try to understand the explanations by others, approximately. At the Pythagorean theorems, students usually learn how to apply its algebraic representation at the real world situation. Except the construction problems with situations, they do not have a chance to apply the ideas of proving to the situation. Linkages such as Pantograph are good examples to learn how to apply the idea of similarity and congruence. However, even teachers, in these days, do not have experience to see and use it.

'How to draw a straight line: a lecture on linkages' by Kempe, A. B. (1877) focused on how to transfer the circle motion into the line motion. It is a major topic of kinematics and used for essential structure in mechanical engineering. 'De organica conicarum sectionum constructione' by Schooten, F. V. (1646) is known a first book which described the conic section as plane figures from the beginning (figure 1). It described based on Elements by Euclid (4th~3rd BC) and some notion of Conic Section by Apollonius (3rd BC). It also appeared in Japanese Textbook (1943), figure 2.



Figure 1. Schooten 1646

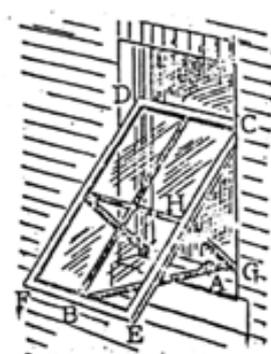
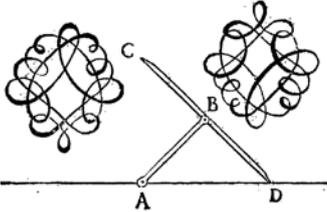


Figure 2. Monbusyo 1943

In Schooten's textbook, for proving the locus is a line on figure 1, we use conditions and propositions what we already knew. And then, the proved proposition on figure 1 is used for basic proposition to prove other propositions. For proving the same proposition by analytic geometry, we have to use the proposition which should be proven by elementary (Euclidian) geometry. It is a good example to know the difference of systems in mathematics. Actually, if students re-learn this problem at senior high school with analytic geometry, it is good chance for them to learn the difference of systems in mathematics.

Application for real world situation enhanced in new curriculum standards. Knowing the difference of systems enhanced in new curriculum standards for senior high school.

Activities	Notes
<p>Read the textbook on the screen.</p> <p>Problem:</p> <p>Rod CD connected with rod AB at B and $AB=CB=BD$. When A fixed on the line and D slides on the line, how does C move?</p> <p>T: Let's expect how C moves.</p> <p>Ss: Curve?, Line?, or Circle?</p> <p>T: Yes, there are many possibilities. In my previous teaching experience, there are three types of answers. Which one is most closest for your imagination?</p> <p>T: Then, how can we find what it is?</p> <p>Ss: Drawing picture?, Using Model?, A4 paper?, Proving?</p> <p>T: Yes, there are various approaches. Let's solve it by drawing and developing model with A4 paper, at first and if you have good imagination, then prove it.</p> <p>< Solving with neighbors ></p> <p>T: Let's share your activities.</p> <p>S: (Present: Line)</p> <p>T: Have you ever seen this kind of figure or mechanism?</p> <p>S: Rectangular!</p> <p>S: Magic hands?</p> <p>T: Then, let's explain your proof!</p> <p>S: Proof 1. Using idea of Rectangular property.</p> <p>S: Proof 2. Using idea of Isosceles triangle theorem.</p> <p>S: Proof 3. Using idea of Circle theorem.</p> <p>T: Which one do you prefer and why?</p> <p>T: Where do you want to apply this mechanism?</p>	<p>Teacher distributes a work sheet and A4 papers.</p> <p>Try to read Schooten's textbook which is written by Latin and guess the problem itself.</p>  <p>Push free talk for imagining and expecting.</p> <p>They knew the line, circle, hyperbola and parabola, teachers enlarge their imagination even if line and circle are preferred.</p> <p>Push paper folding using two sheets of A4 papers with their neighbor for enhancing communication to confirm the conditions.</p> <p>Allow students to prove.</p> <p>Using camera, samples of drawing are shown.</p> <p>Paper folded group will present.</p> <p>Using Camera and Projector</p> <p>Just check the idea of proof because this is the end of junior high school.</p> <p>Ask then to write down why.</p> <p>There will be no time for presentation, thus teacher explains. Proof 3 used the result of Proof 2 on the textbook. Proof 1 is possible to understand by elementary school students.</p> <p>If we still have time, the example of window will be shown and ask students what it is.</p>

Reference:

Bartolini Bussi, M. G., Taimina, D., Isoda, M. (2010). Concrete models and dynamic instruments as early technology tools in classrooms at the dawn of ICMI: from Felix Klein to present applications in mathematics classrooms in different parts of the world. *ZDM International Journal*. 42(1).19-31

Isoda, M. & Inprasitha, M. (to appear) Mathematical Activity with Innovation of Technology in Japan: Twentieth Century World Mathematics Education Reforms in Relation to van Schooten, T. (1646), von Sanden, H. (1914), and Shimada, S. et al. (1943). *East West Journal Mathematics*

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