Lesson Study: Preparing for Evidence of Learning

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APEC - Tsukuba International Conference X
Innovation of Mathematics Education through Lesson Study:
Challenges to Energy Efficiency on STEM and Cross-border Education
Defining Formative Assessment

- Using evidence about student understanding
- By teachers, learners, and peers
- Make decisions on next step based on the evidence rather than just following a prescribed plan

(Black & Wiliam, 2009)
## Focus:
**Short-cycle Formative Assessment**

<table>
<thead>
<tr>
<th>Type</th>
<th>Focus</th>
<th>Time Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Cycle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minute by minute</td>
<td>During instruction</td>
<td>A few seconds to a few minutes</td>
</tr>
</tbody>
</table>
Development of an Observation Protocol for Short-cycle Formative Assessment

AssessToday (Eddy & Harrell 2013)
Purpose:
Close achievement gap

Learning Target → Elicit evidence of learning → Close the Gap

- Scaffolding in the ZPD
- Interpreting the evidence
- Identifying the gap
- Feedback
- Responding to Learning Needs

Adapted from the “Process of Formative Assessment” (Heritage, 2010, p. 11)
Outcomes

• Improve student self-regulation

• Increase student success

• Support teachers developing sustaining formative assessment practices
Seven Components

- Learning Target
- Question Quality
- Nature of Questioning
- Self-evaluation
- Observation of Student Affect
- Instructional Adjustment
- Evidence of Learning

Stages of Teacher Growth

- Novice
- Apprentice
- Practitioner
- Master
What does it mean to prepare for Evidence of Learning?

- Planning that includes
  - Research theme
  - Objective(s) aligned with the standards
  - Selecting a high cognitive task (activity/problem)
  - Anticipating how students will respond
  - Determining what and how student responses will facilitate understanding by all students
  - Preparing questions to facilitate student learning and guide them to mastery of the objective
Selecting (Smith & Stein 2011)

- **Research theme**: for students to develop critical thinking skills relating to their mathematical knowledge and potential to allow them to communicate and engage in their future endeavors with confidence.

- **Objective**: Students will be able to work and characterize with infinite sets of numbers.

- **The activity/problem**: Which numbers can be expressed as the difference of two perfect squares? (from Driscoll 1999)

- **Texas Essential Knowledge and Skills (2012)**

  Algebra 12.D - Number and algebraic methods.

  The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, and functions.

  The student is expected to write a formula for the $n^{th}$ term of arithmetic and geometric sequences, given the value of several of their terms...
Anticipating
Which numbers can be expressed as the difference of two perfect squares? (from Driscoll 1999)

<table>
<thead>
<tr>
<th>ANTICIPATING: Strategy</th>
<th>SELECTING &amp; SEQUENCING: Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Create a table</td>
<td></td>
</tr>
<tr>
<td>— Of consecutive perfect squares</td>
<td>1</td>
</tr>
<tr>
<td>— Of any two perfect squares</td>
<td></td>
</tr>
<tr>
<td>Write an expression</td>
<td></td>
</tr>
<tr>
<td>— $a^2 - b^2$</td>
<td>2</td>
</tr>
<tr>
<td>Writing a Solution</td>
<td></td>
</tr>
<tr>
<td>— all odd numbers</td>
<td></td>
</tr>
<tr>
<td>— $2n + 1$</td>
<td></td>
</tr>
<tr>
<td>— $(n + 1)^2 - n^2$ (this results in a positive integer)</td>
<td>3</td>
</tr>
<tr>
<td>— $n^2 - (n + 1)^2$ (this results in a negative integer)</td>
<td></td>
</tr>
<tr>
<td>— all even numbers multiples of 4</td>
<td></td>
</tr>
<tr>
<td>— $4n$ (This does include 0 as a solution)</td>
<td></td>
</tr>
<tr>
<td>— $n^2 - (n + 2)^2$ (This does not include 0 as a solution)</td>
<td></td>
</tr>
</tbody>
</table>
Anticipating (con’t)
Which numbers can be expressed as the difference of two perfect squares? (from Driscoll 1999)

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<thead>
<tr>
<th>ANTICIPATING: Strategy</th>
<th>SELECTING &amp; SEQUENCING: Order</th>
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</thead>
<tbody>
<tr>
<td><strong>Numbers not included</strong></td>
<td>include when parallel to the numbers included for the following strategies: create a table, write an expression and writing a solution</td>
</tr>
<tr>
<td>— all even numbers not multiples of 4</td>
<td></td>
</tr>
<tr>
<td>— all odd numbers multiplied by 2</td>
<td></td>
</tr>
<tr>
<td>— up to 20 includes 2, 6, 10, 14, 18</td>
<td></td>
</tr>
<tr>
<td>— 4n – 2</td>
<td></td>
</tr>
</tbody>
</table>

| Other Strategies | |
| — not including zero as a perfect square | |
| — 4n + 4 (This does not include 0 as a solution) | |
| — 2n+ 2 when n > 4 and even | |
Questioning

- What is a perfect square?
- Work on writing an expression to represent the question.
- How do you know the expression represents all the numbers included?
- What do those numbers mean? (set of even differences)
- See if you can write a solution that includes all of the numbers.
- How do you represent all the numbers that fit the problem?
- What patterns are you noticing?
- How do you represent the numbers that don’t fit the problem?
Create a Table

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>$a^2 - b^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>13</td>
</tr>
</tbody>
</table>

For consecutive squares

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>$a^2 - b^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>24</td>
</tr>
</tbody>
</table>

For any two squares
Write an Expression

\[ x^2 - y^2 = (a+b)(a-b) \]

Ex:

\[
\frac{73^2 - 27^2}{5329 - 729} = (73+27)(73-27) = (100)(46)
\]

\[ 4600 = 4600 \]
Included:
ODDS & MULTIPLES OF 4

Expression: \((n+1)^2 - n^2\)

2n+1 when \(n \geq 0\)

4n when \(n \geq 2\)

Verbal Solution
Further refining the solution

Included:

\[
\{ x \mid 2x + 1 \equiv 4x, \ x \text{ is an integer} \}
\]

\[
\{ 2n-1, 4n \mid n \in \mathbb{Z} \}
\]

Not Included:

\[
\{ x \mid 4x - 2, \ x \text{ is an integer} \}
\]

\[
\{ 4n-2 \mid n \in \mathbb{Z} \}
\]

\[
\exists 2(2n-1) \mid n \in \mathbb{Z}^3
\]
AssessToday Observation Instrument
(Eddy & Harrell 2013)

Seven Components
• Learning Target
• Question Quality
• Nature of Questioning
• Self-evaluation
• Observation of Student Affect
• Instructional Adjustment
• Evidence of Learning

Stages of Teacher Growth
• Novice
• Apprentice
• Practitioner
• Master
Questioning Quality
(Eddy & Harrell 2013)

Indicators
- Open ended questions
- Mix of Bloom’s high and low questions
- Connect to students’ prior knowledge

Master - Teacher consistently and appropriately uses multiple, varied, and differentiated questions to scaffold instruction.
  - Open ended questioning - Teacher consistently and appropriately uses open-ended questions that challenge students to think critically during instruction.
  - Bloom’s high and low order questions - Teacher uses appropriate high-level and low-level questioning throughout the entire lesson cycle. The combination of the questions is aligned with the learning target(s). Evidence of planning essential questions related to instructional learning target is present. Teacher consistently and appropriately matches Bloom’s levels to the learning target(s) and/or standards.
  - Prior knowledge - Teacher’s expectation of prior knowledge and student ability to scaffold learning from prior knowledge are appropriately aligned. Learning grain size is appropriate and effective for learning.
Questioning Quality

Indicators
- Open ended questions
- Mix of Bloom’s high and low questions
- Connect to students’ prior knowledge

Examples
- How do you know the expression represents all the numbers included?
- What do those numbers mean? (set of even differences)
- See if you can write a solution that includes all of the numbers.
- How could you represent all the numbers that fit the problem?
- How could you represent the numbers that don’t fit the problem?
- What patterns are you noticing?
- What is a perfect square?
Evidence of Learning Indicators

(Eddy & Harrell 2013)

Indicators

- All student responses
- Individual responses
- Artifacts

Master - There is evidence of learning for almost all of the students according to suggested indicators.

- Individual responses - Teacher collects and uses information from almost all the students through verbal responses related to the lesson.

- Artifacts of learning - Almost all of the students complete an authentic assessment to demonstrate understanding. Traditional assessments are never used as the sole assessment for demonstrating understanding. When used, traditional assessments should always be paired with authentic assessments.
Evidence of Learning
example of consecutive perfect squares

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• What patterns are you noticing?

• Does this represent all the numbers for the difference of two perfect squares?
Evidence of Learning

See if you can write a solution that includes all of the numbers.

\[ 2n+1 \text{ when } n \geq 0 \]

\[ 4n \text{ when } n \geq 2 \]
Evidence of Learning
Con’t

Included:

\[ \{ x \mid 2x + 1 \equiv 4x, x \text{ is an integer} \} \]

\[ \{ 2n-1, 4n \mid n \in \mathbb{Z} \} \]

Not Included:

\[ \{ x \mid 4x-2, x \text{ is an integer} \} \]

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\[ \exists 2(2n-1) \mid n \in \mathbb{Z}^3 \]
Revisiting the Purpose

- **The activity/problem**: Which numbers can be expressed as the difference of two perfect squares? (from Driscoll 1999)

- **Research theme**: for students to develop critical thinking skills relating to their mathematical knowledge and potential to allow them to communicate and engage in their future endeavors with confidence

- **Objective**: Students will be able to work and characterize with infinite sets of numbers.
Questions & Discussion
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Thank you!

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References


Wiliam, D., & Thompson, M. (2007). Integrating assessment with learning: What will make it work?