MATHEMATICS
SYLLABUS
Secondary One to Four
Express Course
Normal (Academic) Course

Implementation starting with
2013 Secondary One Cohort

Ministry of Education
SINGAPORE

Learning Mathematics

A 21st Century Necessity

Learning mathematics is a key fundamental in every education system that aims to prepare its citizens for a productive life in the 21st century.

As a nation, the development of a highly-skilled and well-educated manpower is critical to support an innovation- and technology-driven economy. A strong grounding in mathematics and a talent pool in mathematics are essential to support the wide range of value-added economic activities and innovations. Many countries are paying attention to the quality of their mathematics education. The growing interest in TIMSS and PISA speaks of the global interest and importance placed on mathematics education.

At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making informed decisions about personal finances. It supports learning in many fields of study, whether it is in the sciences or in business. A good understanding of basic mathematics is essential wherever calculations, measurements, graphical interpretations and statistical analysis are necessary. The learning of mathematics also provides an excellent vehicle to train the mind, and to develop the capacity to think logically, abstractly, critically and creatively. These are important 21st century competencies that we must imbue in our students, so that they can lead a productive life and be life-long learners.

Students have different starting points. Not all will have the same interests and natural abilities to learn mathematics. Some will find it enjoyable; others will find it challenging. Some will find the theorems and results intriguing; others will find the formulae and rules bewildering. It is therefore important for the mathematics curriculum to provide differentiated pathways and choices to support every learner in order to maximise their potential. The curriculum must engage the 21st century learners, who are digital natives comfortable with the use of technologies and who work and think differently. The learning of mathematics must take into cognisance the new generation of learners, the innovations in pedagogies as well as the affordances of technologies.

It is the goal of the national mathematics curriculum to ensure that all students will achieve a level of mastery of mathematics that will serve them well in their lives, and for those who have the interest and ability, to pursue mathematics at the highest possible level. Mathematics is an important subject in our national curriculum. Students begin to learn mathematics from the day they start formal schooling, and minimally up to the end of secondary education. This gives every child at least 10 years of meaningful mathematics education.

About this document

This document provides an overview of the curriculum. It explains the design of the curriculum from the primary to the pre-university level, and provides details of the O- & N(A)-Level Mathematics syllabuses, including the aims, content, outcomes and the approaches to teaching and learning.

This document comprises 5 chapters as described below.

Chapter 1 provides an overview of the curriculum review, the goals and aims of the different syllabuses of the entire mathematics curriculum (primary to pre-university) as well as the syllabus design considerations across the levels.

Chapter 2 elaborates on the Mathematics Framework which centres around mathematical problem solving. The framework serves as a guide for mathematics teaching, learning and assessment across the levels.

Chapter 3 focuses on the process of teaching and learning so as to bring about engaged learning in mathematics. It highlights the principles of teaching and phases of learning as well as the learning experiences to influence the way teachers teach and students learn so that the aims of the curriculum can be met. The role of assessment and how it can be integrated to support learning in the classroom is also highlighted in this chapter.

Chapter 4 details the O-Level Mathematics syllabus in terms of its aims, syllabus organisation, mathematical processes, content and learning experiences.

Chapter 5 details the N(A)-Level Mathematics syllabus in terms of its aims, syllabus organisation, mathematical processes, content and learning experiences.

Contents

Chapter 1
Introduction

Background
Goals and Aims
Syllabus Design

Background

Staying Relevant and Forward-Looking

As in all previous reviews, the 2010 full-term review aims to update the syllabuses so that they continue to meet the needs of our students, build a strong foundation in mathematics, and make improvement in the school mathematics education. It takes into consideration the analyses of students' performances in national examinations as well as international studies such as TIMSS and PISA. This review also takes on board the curriculum-wide recommendations from envisaging studies into the overall Singapore curriculum such as seeking a better balance between content and skills, creating opportunities to develop 21st century competencies, promoting self-directed and collaborative learning through ICT-based lessons, and developing assessment to support learning.

It is clear at the start of the review that there is more to be considered than just focusing on the content. While there is a need to constantly review what students learn, the changes in content will not be the key lever. In fact, little has been changed in the content as this has stabilised over the years. Instead, more focus has now been given to skills and competencies that will make a better 21st century learner – the process of learning becomes more important than just what is to be taught and remembered. The syllabuses are therefore written with the view that not only will it inform teachers on what to teach, it will also influence the way teachers teach and students learn. One key feature of this set of syllabuses is the explication of learning experiences, besides the learning outcomes. This gives guidance to teachers on the opportunities that students should be given as part of their learning. Ultimately, how students learn matters.

Curriculum review and design is ongoing work. The quality of the curriculum is as much in its design as it is in its implementation. Teachers, who are the frontline of curriculum delivery, must believe in the value of the changes. Support, resources and training will be provided to build capacity in our teachers. All these will be part of the continuous effort to deliver the best mathematics curriculum for the students.

The O- & N(A)-Level Mathematics syllabuses will be implemented level by level starting from Secondary One in 2013. The implementation schedule is as follows:

Year	2013	2014	2015	2016
Level	Sec 1	Sec 2	Sec 3	Sec 4

This online syllabus document will be updated yearly according to the implementation schedule.

Goals and Aims

Different Syllabuses, Different Aims

The overarching goal of the mathematics curriculum is to ensure that all students will achieve a level of mastery of mathematics that will serve them well in life, and for those who have the interest and ability, to pursue mathematics at the highest possible level.

The broad aims of mathematics education in Singapore are to enable students to:

- acquire and apply mathematical concepts and skills;
- develop cognitive and metacognitive skills through a mathematical approach to problem solving; and
- develop positive attitudes towards mathematics.

The mathematics curriculum comprises a set of syllabuses spanning 12 years, from primary to pre-university, and is compulsory up to the end of secondary education. Each syllabus has its own specific set of aims to guide the design and implementation of the syllabus. The aims also influence the choice of content, skills as well as contexts to meet the specific needs of the students at the given level or course. Each syllabus expands on the three broad aims of mathematics education differently to cater for the different needs and abilities of the students (see table of aims on the next page).

What does it mean to teachers?

Understanding the aims of the syllabus helps teachers stay focused on the larger outcomes of learning and guides teachers when they embark on the school-based curriculum innovations and customisations.

Overview of Aims Across the Levels

Primary
Laying a Strong Foundation

The Primary Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for everyday use and continuous learning in mathematics;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence and foster interest in mathematics.

Secondary
Building Up Strengths

The O- and N(A)-Level Mathematics syllabuses aim to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics; and
- build confidence and foster interest in mathematics.

The O- and N(A)-Level Additional Mathematics syllabuses aim to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences;
- develop thinking, reasoning and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

The N(T)-Level Mathematics syllabus aims to enable students who are bound for post-secondary vocational education to:

- acquire mathematical concepts and skills for real life, to support learning in other subjects, and to prepare for vocational education;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence in using mathematics and appreciate its value in making informed decisions in real life.

Pre-University
Gearing Up for University Education

The H1 Mathematics syllabus aims to enable students who are interested in pursuing tertiary studies in business and the social sciences to:

- acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics; and
- appreciate the value of mathematics in making informed decisions in life.

The H2 Mathematics syllabus aims to enable students who are interested in pursuing tertiary studies in mathematics, sciences and engineering to:

- acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences and engineering;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving and the use of mathematics language;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics; and
- appreciate the beauty of mathematics and its value in making informed decisions in life.

The H3 Mathematics syllabus aims to enable students who have an aptitude and passion for mathematics to:

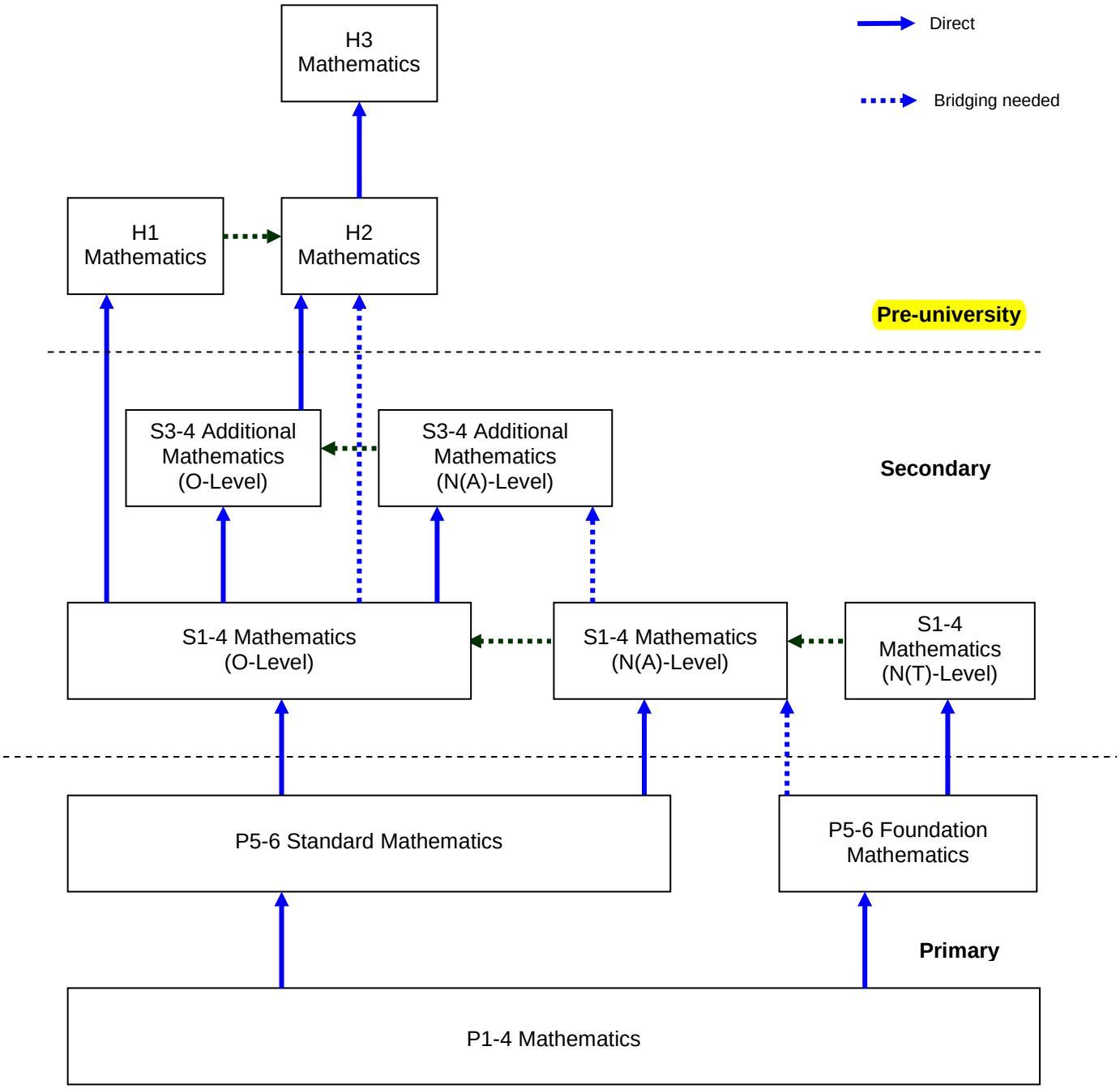
- acquire advanced mathematical concepts and skills to deepen their understanding of mathematics, and to widen the scope of applications of mathematics;
- develop rigorous habits of mind through mathematical reasoning and proof, creative mathematical problem solving, and use of mathematical models;
- connect ideas within mathematics at a higher level and between mathematics and other disciplines through applications of mathematics; and
- appreciate the beauty, rigour and abstraction of mathematics through mathematical proof and applications.

Syllabus Design

Spiral Curriculum, Connected Syllabuses

Mathematics is largely hierarchical in nature. Higher concepts and skills are built upon the more foundational ones and have to be learned in sequence. A spiral approach is adopted in the building up of content across the levels.

The mathematics curriculum consists of a set of connected syllabuses to cater to the different needs and abilities of students. This section gives an overview of the syllabuses and their connections so that teachers are better able to appreciate the mathematics curriculum as a whole.



The Primary Mathematics syllabus assumes no formal learning of mathematics. However, basic pre-numeracy skills such as matching, sorting and comparing are necessary in providing a good grounding for students to begin learning at Primary 1 (P1).

The P1-4 syllabus is common to all students. The P5-6 Standard Mathematics syllabus continues the development of the P1-4 syllabus whereas the P5-6 Foundation Mathematics syllabus re-visits some of the important concepts and skills in the P1-4 syllabus. The new concepts and skills introduced in Foundation Mathematics is a subset of the Standard Mathematics syllabus.

The O-Level Mathematics syllabus builds on the Standard Mathematics syllabus. The N(A)-Level Mathematics syllabus is a subset of O-Level Mathematics, except that it re-visits some of the topics in Standard Mathematics syllabus. The N(T)-Level Mathematics syllabus builds on the Foundation Mathematics syllabus.

The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics content and includes more in-depth treatment of important topics. The N(A)-Level Additional Mathematics syllabus is a subset of O-Level Additional Mathematics syllabus. O-Level Additional Mathematics together with O-Level Mathematics content provides the prerequisite knowledge required for H2 Mathematics at the pre-university level.

At the pre-university level, mathematics is optional. The H1 Mathematics syllabus builds on the O-level Mathematics syllabus. H2 Mathematics assumes some of the O-Level Additional Mathematics content. H3 Mathematics is an extension of H2 Mathematics.

Flexibility and Choice

There are two mathematics syllabuses at the P5-6 level. Most students would offer Standard Mathematics and for students who need more time to learn, they could offer Foundation Mathematics.

There are five mathematics syllabuses in the secondary mathematics curriculum. O-Level Mathematics, N(A)-Level Mathematics and N(T)-Level Mathematics provide students from the respective courses the core mathematics knowledge and skills in the context of a broad-based education. The more mathematically able students from the N(A) course can choose to take O-Level Mathematics in four years instead of five years. Likewise, the more able N(T) course students can also offer N(A)-Level Mathematics. The variation in pace and syllabus adds to the flexibility and choice within the secondary mathematics curriculum. At the upper secondary level, students who are interested in mathematics and are more mathematically inclined may choose to offer Additional Mathematics as an elective at the O-Level or N(A)-Level. This gives them the opportunity to learn more mathematics that would prepare them well for courses of study that require higher mathematics.

For students who wish to study in the Engineering-type courses at the polytechnics, Additional Mathematics will be a good grounding. The N(A)-Level and N(T)-Level Mathematics syllabuses will prepare students well for ITE courses. Students who aspire to study Mathematics or mathematics-related courses at the universities could offer H2 Mathematics, and if possible, H3 Mathematics.

What does it mean to teachers?

Teachers need to have the big picture in mind so that they can better understand the role of each syllabus, the connection it makes with the next level and the dependency relationship between syllabuses. This enables teachers to better understand what they have to do at their level, as well as to plan and advise students in their learning of mathematics. For example, H2 Mathematics assumes some of the O-Level Additional Mathematics content but may be offered by students without Additional Mathematics background as long as effort is made to bridge the gap.

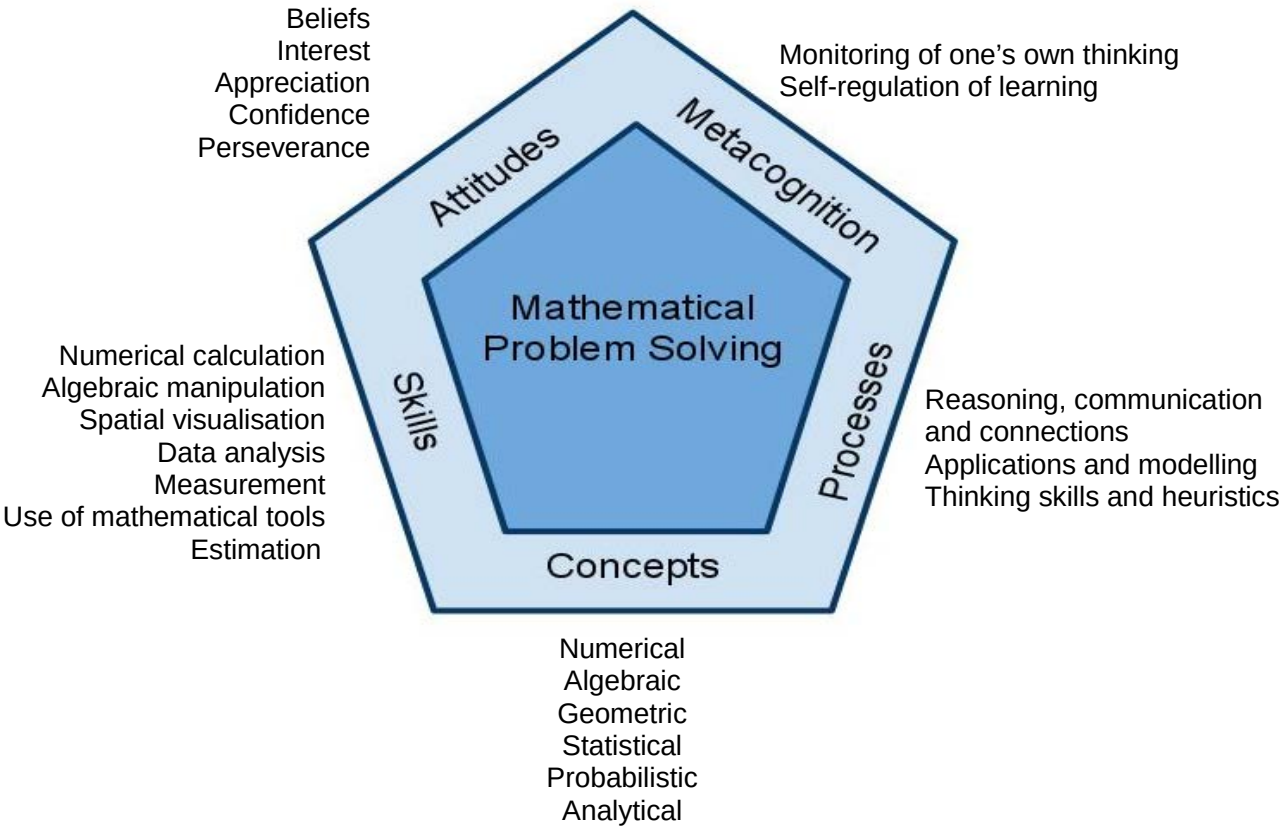
Chapter 2
Mathematics Framework

Problem Solving

Problem Solving

Concepts, Skills, Processes, Metacognition, Attitudes

The Mathematics Framework has been a feature of our mathematics curriculum since 1990, and is still relevant to date. The central focus of the framework is mathematical problem solving, that is, using mathematics to solve problems. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics at all levels, from primary to pre-university. It reflects also the 21st century competencies[1].



The framework stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.

Concepts

Mathematical concepts can be broadly grouped into *numerical*, *algebraic*, *geometric*, *statistical*, *probabilistic*, and *analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

[1] Information on the MOE framework for 21st century competencies and student outcomes can be found on www.moe.gov.sg

To develop a deep understanding of mathematical concepts, and to make sense of various mathematical ideas as well as their connections and applications, students should be exposed to a variety of learning experiences including hands-on activities and use of technological aids to help them relate abstract mathematical concepts with concrete experiences.

Skills

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

To develop proficiencies in mathematics skills, students should have opportunities to use and practise the skills. These skills should be taught with an understanding of the underlying mathematical principles and not merely as procedures.

Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics and beyond.

In the context of mathematics, *reasoning, communication and connections* take on special meanings:

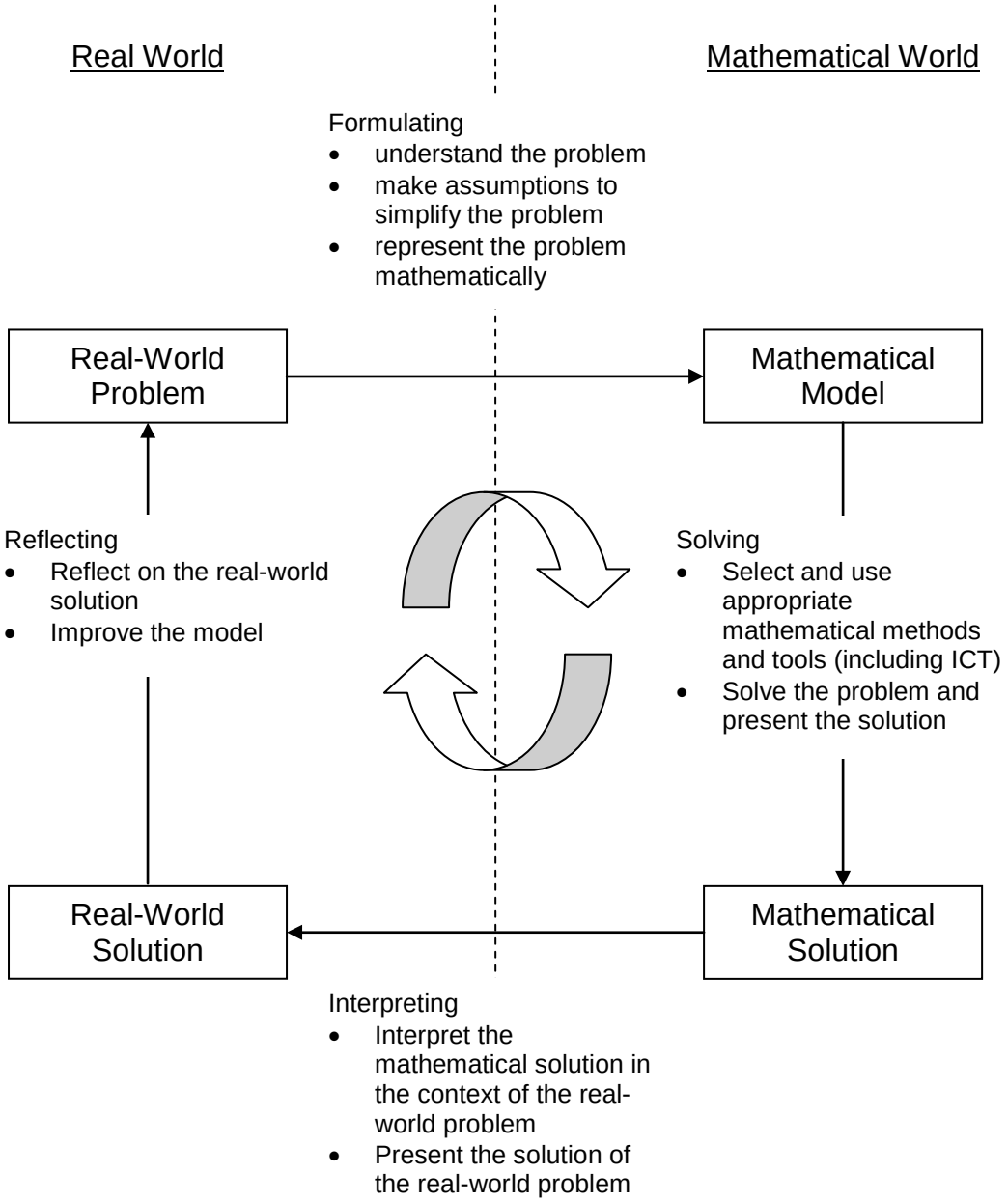
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through application of mathematics in different contexts.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their understanding of mathematics and sharpen their mathematical thinking.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world. This helps students make sense of what they learn in mathematics.

Applications and modelling allow students to connect mathematics that they have learnt to the real world, enhance understanding of key mathematical concepts and methods as well as develop mathematical competencies. Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. Mathematical modelling is the process of formulating and improving a mathematical model[2] to

[2] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

represent and solve real-world problems. Through mathematical modelling, students learn to deal with ambiguity, make connections, select and apply appropriate mathematical concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.

Mathematical Modelling Process (version 2010)



Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, analysing parts and whole, identifying patterns and relationships, induction, deduction, generalising, and spatial visualisation. Heuristics are general

rules of thumb of what students can do to tackle a problem when the solution to the problem is not obvious. These include using a representation (e.g., drawing a diagram, tabulating), making a guess (e.g., trial and error/guess and check, making a supposition), walking through the process (e.g., acting it out, working backwards) and changing the problem (e.g., simplifying the problem, considering special cases).

Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

To develop metacognitive awareness and strategies, and know when and how to use the strategies, students should have opportunities to solve non-routine and open-ended problems, to discuss their solutions, to think aloud and reflect on what they are doing, and to keep track of how things are going and make changes when necessary.

Attitudes

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

Students' attitudes towards mathematics are shaped by their learning experiences. Making the learning of mathematics fun, meaningful and relevant goes a long way to inculcating positive attitudes towards the subject. Care and attention should be given to the design of the learning activities to build confidence in and develop appreciation for the subject. Above all, students' beliefs can influence their attitudes in learning, especially in student-centred learning where students are encouraged to take on more responsibility for their own learning.

What does it mean to teachers?

The five components of the Mathematics Framework are integral parts of mathematics learning and problem solving. The intent of the framework is to help teachers focus on these components in their teaching practice so as to provide a more engaging, student-centred, and technology-enabled learning environment, and to promote greater diversity and creativity in learning.

Chapter 3
Teaching, Learning and Assessment

Learning Experiences
Teaching and Learning
Assessment in the Classroom

Learning Experiences

It matters how students learn

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. For example, to encourage students to be inquisitive, the learning experiences must include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students must be given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students must be given opportunities to set learning goals and work towards them purposefully. A classroom, rich with these opportunities, will provide the platform for students to develop these 21st century competencies.

Learning experiences are stated in the mathematics syllabuses to influence the ways teachers teach and students learn so that the curriculum objectives can be achieved. These statements expressed in the form "students should have opportunities to …" remind teachers of the student-centric nature of these experiences. They describe actions that students will perform and activities that students will go through, with the opportunities created and guidance rendered by teachers. The descriptions are sufficiently specific to provide guidance yet broad enough to give flexibility to the teachers.

For each topic, the learning experiences focus on the mathematical processes and skills that are integral parts of learning of that topic. There are also generic learning experiences that focus on the development of good learning habits and skills such as:

Students should have opportunities to:

- take notes and organise information meaningfully;
- practise basic mathematical skills to achieve mastery;
- use feedback from assessment to improve learning;
- solve novel problems using a repertoire of heuristics;
- discuss, articulate and explain ideas to develop reasoning skills; and
- carry out a modelling project.

These learning experiences, whether they are topical or generic, are not exhaustive. Teachers are encouraged to do more to make learning meaningful and effective.

Teaching and Learning

Principles of Teaching and Phases of Learning

This section outlines three principles of mathematics teaching and the three phases of mathematics learning in the classrooms.

Principles of Teaching

Principle 1

Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

Teaching is an interactive process that is focused on students' learning. In this process, teachers use a range of teaching approaches to engage students in learning; students provide teachers with feedback on what they have learnt through assessment; and teachers in turn provide feedback to students and make decisions about instructions to improve learning.

The learning of mathematics should focus on understanding, not just recall of facts or reproduction of procedures. Understanding is necessary for deep learning and mastery. Only with understanding can students be able to reason mathematically and apply mathematics to solve a range of problems. After all, problem solving is the focus of the mathematics curriculum.

Principle 2

Teaching should build on students' knowledge; take cognisance of students' interests and experiences; and engage them in active and reflective learning.

Mathematics is a hierarchical subject. Without understanding of pre-requisite knowledge, foundation will be weak and learning will be shallow. It is important for teachers to check on students' understanding before introducing new concepts and skills.

Teachers need to be aware of their students' interests and abilities so as to develop learning tasks that are stimulating and challenging. This is important in order to engage students in active and reflective learning where students participate and take ownership of the learning.

Principle 3

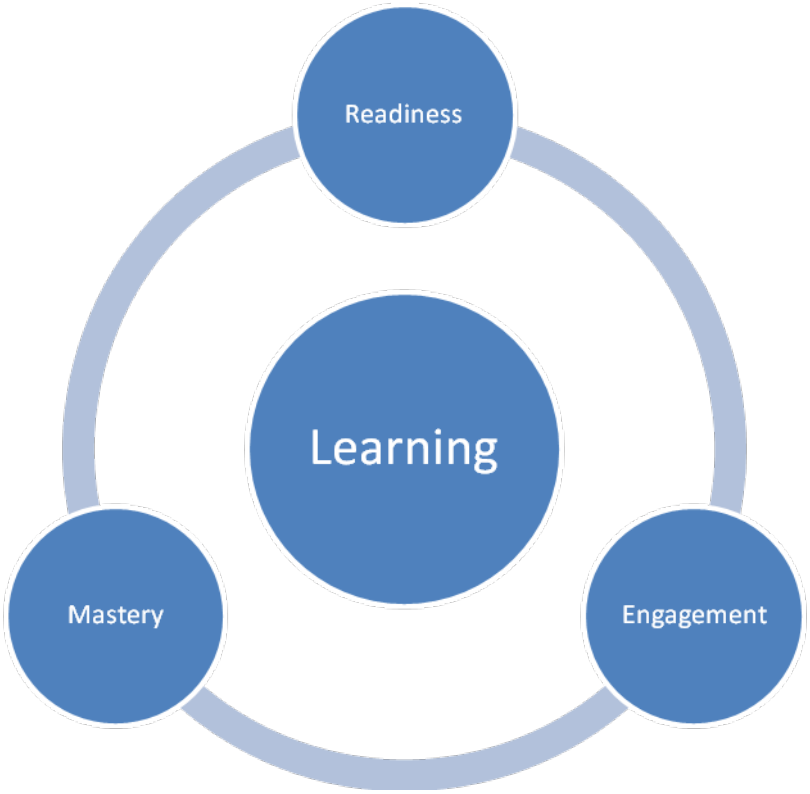
Teaching should connect learning to the real world, harness ICT tools and emphasise 21st century competencies.

There are many applications of mathematics in the real world. Students should have an understanding and appreciation of these applications and how mathematics is used to model and solve problems in real-world contexts. In this way, students will see the meaning and relevance of mathematics.

Teachers should consider the affordances of ICT to help students learn. ICT tools can help students understand mathematical concepts through visualisations, simulations and representations. They can also support exploration and experimentation and extend the range of problems accessible to students. The ability to use ICT tools is part of the 21st century competencies. It is also important to design learning in ways that promote the development of other 21st century competencies such as working collaboratively and thinking critically about the mathematical solution.

Phases of Learning

Effective instruction of a unit typically involves three phases of learning: *Readiness*, *Engagement* and *Mastery*.



Phase 1 - *Readiness*

Student readiness to learn is vital to learning success. In the readiness phase of learning, teachers prepare students so that they are ready to learn. This requires considerations of *prior knowledge*, *motivating contexts*, and *learning environment*.

- ***Prior Knowledge***
  For students to be ready to learn, teachers need to know students' prior knowledge in relation to the new learning. This requires knowing whether students have the pre-requisite concepts and skills. Some form of diagnostic assessment is necessary to check that students are ready to learn.

- ***Motivating Contexts***
  For students to be ready to learn, teachers need to provide motivating contexts for learning. These contexts should be developmentally appropriate. For example, younger students may like contexts such as stories and songs, and play-based activities such as games, whereas older students may appreciate contexts related to everyday life so that they can see the relevance and meaningfulness of mathematics. For the more advanced students, applications in other disciplines can serve as motivation for learning.

- ***Learning Environment***
  Shared rules help promote respectful and emotionally-safe interactions between teacher and students and among students that are necessary for productive and purposeful learning. Established procedures for organising students and managing resources will also facilitate a smooth start and transitions during lessons.

Phase 2 - *Engagement*

This is the main phase of learning where teachers use a repertoire of pedagogies to engage students in learning new concepts and skills. Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. For example, the lesson or unit could start with an activity, followed by teacher-led inquiry and end with direct instruction.

- ***Activity-based Learning***
  This approach is about learning by doing. It is particularly effective for teaching mathematical concepts and skills at primary and lower secondary levels, but is also effective at higher levels. Students engage in activities to explore and learn mathematical concepts and skills, individually or in groups. They could use manipulatives or other resources to construct meanings and understandings. From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results.

  For example, students use the virtual balance in AlgeTools™ to learn the concept of equations involving negative numbers, construct numerical and algebraic equations, and learn the process of solving linear equations by manipulating algebra discs on the virtual balance. During the activity, students communicate and share their understanding using concrete and pictorial representations. The role of the teacher is that of a facilitator who guides students through the concrete, pictorial and abstract levels of understanding by providing appropriate scaffolding and feedback.

- ***Teacher-directed Inquiry***
  This approach is about learning through guided inquiry. Instead of giving the answers, teachers lead students to explore, investigate and find answers on their own. Students learn to focus on specific questions and

ideas and are engaged in communicating, explaining and reflecting on their answers. They also learn to pose questions, process information and data and seek appropriate methods and solutions. This enhances the development of mathematical processes and 21st century competencies.

For example, students work together on a learning task in an ICT-enabled environment with guiding questions to investigate properties of geometric figures. They discuss and share their observations and findings, make conjectures regarding geometric properties and relationships, and justify them. The role of the teacher is that of a facilitator who asks questions to probe students' thinking, and guides students through the process of inquiry by directing them to specific questions and information to investigate, providing individual scaffolding, and challenging the abler students to extend and deepen their learning. Students are encouraged to communicate using appropriate mathematical language. The teacher also assesses students' understanding, identifies gaps and misconceptions, and reinforces key concepts through continuous interactive assessments.

- ***Direct Instruction***
  This approach is about explicit teaching. Teachers introduce, explain and demonstrate new concepts and skills. Direct instruction is most effective when students are told what they will be learning and what they are expected to be able to do. This helps them focus on the learning goals. Teachers draw connections, pose questions, emphasise key concepts, and role-model thinking. Holding students' attention is critical. Stimuli such as videos, graphic images, real-world contexts, and even humour, aid in maintaining a high level of attention.

  For example, in teaching problem solving, the teacher demonstrates how to use Polya's four-step problem-solving strategy[3] and models thinking aloud to make visible the thinking processes. The teacher probes students' conceptual understanding, corrects their misconceptions, and guides them in the process of solving problems. During lesson closure, the teacher reviews the key learning points of the lesson to consolidate the learning.

Phase 3 - *Mastery*

This is the final phase of learning where teachers help students consolidate and extend their learning. The mastery approaches include:

- ***Motivated Practice***
  Students need practice to achieve mastery. Practice can be motivating and fun. Practice must include repetition and variation to achieve proficiency and flexibility. Structuring practice in the form of games is one good strategy to make practice motivating and fun, while allowing for

[3] G. **Polya**, "**How to Solve It**", 2nd Ed., Princeton University Press, 1957.

repetition and variation. There should be a range of activities, from simple recall of facts to application of concepts.

- ***Reflective Review***
  It is important that students consolidate and deepen their learning through tasks that allow them to reflect on their learning. This is a good habit that needs to be cultivated from an early age and it supports the development of metacognition. Summarising their learning using concept maps, writing journals to reflect on their learning and making connections between mathematical ideas and between mathematics and other subjects should be encouraged. Sharing such reflections through blogs makes learning social.

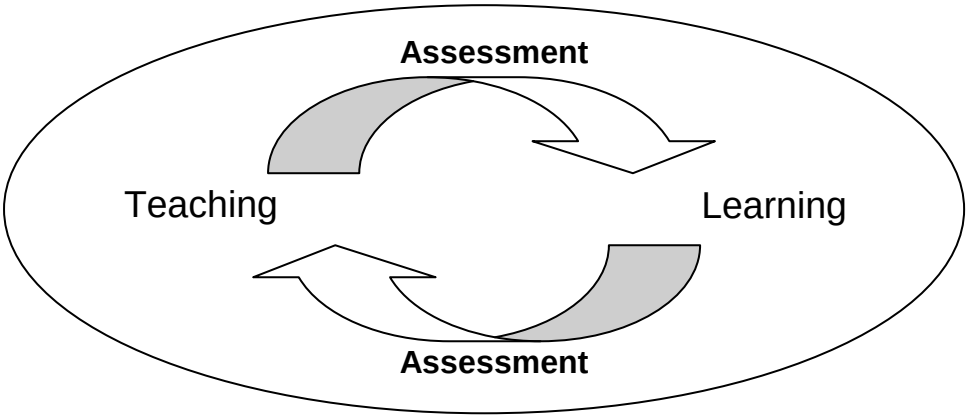
- ***Extended Learning***
  Students who are mathematically inclined should have opportunities to extend their learning. These can be in the form of more challenging tasks that stretch their thinking and deepen their understanding.

Assessment in the Classroom

Supporting Teaching and Learning in Mathematics

Role of assessment

Assessment is an integral part of the interactive process of teaching and learning, as illustrated in the diagram below. It is an ongoing process by which teachers gather information about students' learning to inform and support teaching. An important product of assessment is feedback. Feedback must be timely and rich. It must inform students where they are in their learning and what they need to do to improve their learning. It must also inform teachers what they need to do to address learning gaps and how to improve their instruction.



Range of assessment

Assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Teachers usually report the assessment result as a score or a grade.

- Formative and diagnostic assessments are used as assessment for learning to provide timely feedback to students on their learning, and to teachers on their teaching.

Assessment in the classroom should focus on helping students improve their learning. Therefore, they are primarily formative and diagnostic in purpose.

Though teachers are comfortable with the use of traditional pen-and-paper tests to find out how much students know and can do, there is value in exploring a wider variety of assessment strategies. These strategies allow teachers to gather information that is not easily available through traditional methods of assessment,

but are nevertheless valuable in supporting learning. Ultimately, the choice of assessment strategies must be guided by its purpose, that is, it must be fit-for-purpose.

Integrating assessment with instruction

It is important that teachers know what and when to assess student learning, and how to embed the assessment in the learning process. Assessment can be integrated into classroom discourse and activities using different assessment strategies. For example, teachers may watch students solve problems and get them to explain their strategies. Teachers may also engage students in assessing their own work and reflecting on their own learning and how to improve it. Both moment-by-moment assessment and planned assessment should be considered.

Effective questioning can scaffold learning and probe understanding. It creates teachable moments for teachers to correct a misconception, reinforce a point or expand on an idea. The questions can be open-ended to encourage students to consider alternative approaches. Sufficient wait-time is necessary so that students can formulate their thoughts, communicate and share their ideas, and hear the ideas of others. In the process, students learn to articulate their thinking and deepen their understanding, and develop confidence in talking about mathematics and using it. Teachers can assess students' thinking and understanding, and provide useful feedback to improve their learning.

Teachers can integrate performance assessments into the instructional process to provide additional learning experiences for students. This type of assessment requires students to apply their knowledge and skills in context, and the focus is on mathematical processes rather than on mathematics content. A rubric is useful to show teachers what to look for in students' work, but more importantly, it shows what is expected of students in terms of processes and quality of work. The rubric also provides a structured means of giving qualitative feedback. Teachers may allow students to assess their own performances so that they can reflect on their work and make improvements.

Assessment for learning calls for new ways of assessment in the classroom. It involves a change in teachers' roles and in the expectations of students. By integrating assessment and instruction, students will be more engaged in and will take greater ownership of their learning.

Chapter 4
O-Level Mathematics Syllabus

Aims of Syllabus
Syllabus Organisation
Content and Learning Experiences by Level

Aims of Syllabus

The O-Level Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics; and
- build confidence and foster interest in mathematics.

Syllabus Organisation

The syllabus is organised along 3 content strands with a listing of mathematical processes that cut across the 3 strands.

<table>
<tr><td colspan="3">3 Content Strands + 1 Process Strand</td></tr>
<tr><td>Number and Algebra</td><td>Geometry and Measurement</td><td>Statistics and Probability</td></tr>
<tr><td colspan="3">Mathematical Processes</td></tr>
</table>

Strand: Mathematical Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematical problem solving and beyond.

At the secondary level, students gradually move from inductive arguments to deductive arguments and justifications (e.g. to justify a mathematical statement using a short chain of logical reasoning or to disprove a statement by a counter example). They will continue to make connections among mathematical ideas, and between mathematics and the real world. They should be able to read and critique arguments that are supported by mathematics.

Greater attention will be given to applications and modelling at the secondary level, as students become more mature and aware of their immediate environment and phenomenon. Besides learning standard mathematical models, students should, under teacher guidance, develop an awareness and understanding of the mathematical modelling process. They work on real-world problems either individually or in groups. They would need to understand the real-world situation, make assumptions about the situation, devise a mathematical model to solve the problem, and interpret the solution in the context of the situation. The process of mathematical modelling widens and deepens students' understanding of mathematics, and helps them develop important 21st century skills, including collaboration, creativity, communication and critical thinking.

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to problem solving approaches such as the Polya's model and the steps in a modelling process in class. Teachers could "think aloud" to give attention to these processes and make them visible to students. Students should be given opportunities to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. Through practice, students will develop habits and strategies that will help them be better and more independent learners.

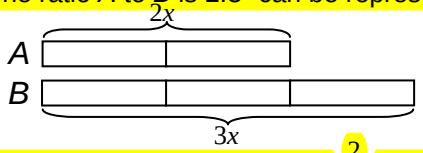
No.	Processes	Indicators
MATHEMATICAL PROCESSES		
MP1	**Reasoning, Communication and Connections**	• Use appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas • Reason inductively and deductively, including: ∗ Explaining or justifying/verifying a mathematical solution/statement ∗ Drawing logical conclusions ∗ Making inferences ∗ Writing mathematical arguments • Make connections within mathematics, between mathematics and other disciplines, and between mathematics and the real world
MP2	**Applications and Modelling**	• Apply mathematics concepts and skills to solve problems in a variety of contexts within or outside mathematics, including: ∗ Identifying the appropriate mathematical representations or standard models for a problem ∗ Using appropriate mathematical concepts, skills (including tools and algorithm) to solve a problem • Understand the nature of the mathematical modelling process, including: ∗ Formulating a real-world problem into a mathematical model by making suitable assumptions and simplification and identifying suitable mathematical representations ∗ Applying mathematics to solve the problem ∗ Interpreting the mathematical solution in the context of the problem ∗ Refining and improving the model
MP3	**Thinking Skills and Heuristics**	• Use thinking skills such as: ∗ Classifying ∗ Comparing ∗ Sequencing ∗ Generalising ∗ Induction ∗ Deduction ∗ Analysing (from whole to parts) ∗ Synthesizing (from parts to whole) • Use a problem-solving model such as Polya's model • Use heuristics such as: ∗ Drawing a diagram ∗ Tabulating ∗ Guess and check ∗ Working backwards ∗ Simplifying a problem ∗ Considering special cases

Content & Learning Experiences by Level

In this section, the content is listed by levels and learning experiences statements are included.

The learning experiences for the O-Level Mathematics syllabus should provide opportunities for students to:

- enhance conceptual understanding through use of various mathematical tools including ICT tools;
- apply concepts and skills learnt in real-world context;
- communicate their reasoning and connections through various mathematical tasks and activities; and
- build confidence and foster interest in mathematics.

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.1. primes and prime factorisation 1.2. finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation 1.3. negative numbers, integers, rational numbers, real numbers and their four operations 1.4. calculations with calculator 1.5. representation and ordering of numbers on the number line 1.6. use of <, >, ≤, ≥ 1.7. approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, and estimating the results of computation)	(a) Classify whole numbers based on their number of factors and explain why 0 and 1 are not primes. (b) Discuss examples of negative numbers in the real world. (c) Represent integers, rational numbers and real numbers on the number line as extension of whole numbers, fractions and decimals respectively. (d) Use algebra discs or the AlgeDisc™ application in AlgeTools™ to make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the 4 operations of integers. (e) Work in groups to estimate quantities (numbers and measures) in a variety of contexts, compare the estimates and share the estimation strategies. (f) Compare follow-through errors arising from intermediate values that are rounded to different degrees of accuracy. (g) Make estimates and check the reasonableness of answers obtained by calculators.
N2. Ratio and proportion	
2.1. ratios involving rational numbers 2.2. writing a ratio in its simplest form 2.3. problems involving ratio	(a) Discuss and explain how ratios are used in everyday life. (b) Use the concept of equivalent ratios to find the ratio a:b:c given the ratios a:b and b:c. (c) Make connections between ratios and fractions, use appropriate mathematical language to describe the relationship, and use algebra to solve problems, e.g. "The ratio *A* to *B* is 2:3" can be represented as:  The ratio 2:3 means "2 units to 3 units", "*A* is $\frac{2}{3}$ of *B*", or "*B* is $\frac{3}{2}$ of *A*". (d) Use the AlgeBar™ application in AlgeTools™ to formulate linear equations to solve problems. (Students can draw models to help them formulate equations.)

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N3. Percentage	
3.1. expressing one quantity as a percentage of another 3.2. comparing two quantities by percentage 3.3. percentages greater than 100% 3.4. increasing/decreasing a quantity by a given percentage (including concept of percentage point) 3.5. reverse percentages 3.6. problems involving percentages	(a) Collect examples of percentages from newspapers and magazines and discuss the meaning of percentage in each example. (b) Examine bills and receipts, etc. to find examples of the uses of percentages, e.g. discount, service charge, GST and other taxes and check the calculated values. (c) Make connections between percentages and fractions/decimals, e.g. "25% of a quantity is $\frac{1}{4}$ of the quantity", "20% of x is $0.2x$". (d) Discuss misconceptions, e.g. "If *A* is 5% more than *B*, then *B* is 5% less than *A*". (e) Use the AlgeBar™ application in AlgeTools™ to formulate linear equations to solve problems (Students can draw models to help them formulate equations).
N4. Rate and Speed	
4.1. concepts of average rate, speed, constant speed and average speed 4.2. conversion of units (e.g. km/h to m/s) 4.3. problems involving rate and speed	(a) Discuss examples of rates e.g. currency exchange rates, interest rates, tax rates, rate of rotation and speed. (b) Find out and compare the speeds of bicycles, cars, trains, aeroplanes and spaceships and their respective units to have a sense of their magnitude. (c) Explain the difference between average speed and constant speed and also explain why average speed is not the average of speeds.
N5. Algebraic expressions and formulae	
5.1. using letters to represent numbers 5.2. interpreting notations: • ab as $a \times b$ • $\frac{a}{b}$ as $a \div b$ or $a \times \frac{1}{b}$ • a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, . . .	(a) Use spreadsheets, e.g. Microsoft Excel, to * explore the concept of variables and evaluate algebraic expressions. * compare and examine the differences between pairs of expressions, e.g. $2n$ and $2+n$, n^2 and $2n$, $2n^2$ and $(2n)^2$. (b) Use algebra discs or the AlgeDisc™ application in AlgeTools™ to make sense of and interpret linear expressions with integral coefficients, e.g. $4x-3y$ and $-3(x-2)$.

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
• $3y$ as $y + y + y$ or $3 \times y$ • $3\,(x + y)$ as $3 \times (x + y)$ • $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$ 5.3. evaluation of algebraic expressions and formulae 5.4. translation of simple real-world situations into algebraic expressions 5.5. recognising and representing patterns/relationships by finding an algebraic expression for the nth term 5.6. addition and subtraction of linear expressions 5.7. simplification of linear expressions such as $-2(3x-5)+4x$ $\frac{2x}{3} - \frac{3(x-5)}{2}$ 5.8. use brackets and extract common factors	(c) Use the AlgeDisc™ application in AlgeTools™ to construct and simplify linear expressions with integral coefficients. (d) Work in groups to select and justify pairs of equivalent expressions. (e) Write algebraic expressions to express mathematical relationships, e.g. for the statement "There are 3 times as many boys as girls", if we let x be the number of girls, then the number of boys is $3x$. This can also be written as $b = 3g$, where b and g are the number of boys and girls respectively. (f) Explore number patterns and write algebraic expressions to represent the patterns.
N6. Functions and graphs	
6.1. Cartesian coordinates in two dimensions 6.2. graph of a set of ordered pairs as a representation of a relationship between two variables 6.3. linear functions $y = ax + b$ 6.4. graphs of linear functions 6.5. the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients)	(a) Play games, e.g. Battleship Game, that involve the use of 2D Cartesian coordinates to specify points. (b) Use a function machine to generate input and output values to illustrate the concept of function as "only one output for every input" and represent the function in verbal, tabular, graphical and algebraic forms. (c) Use a linear function to represent the relationship between two variables (such as distance and time when the speed is constant), show the relationship graphically and identify that the rate of change is the gradient of the graph. (d) Use a spreadsheet or graphing software to study how the graph of $y = ax + b$ changes when either a or b varies.

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N7. Equations and inequalities	
7.1. concepts of equation and inequality 7.2. solving linear equations in one variable 7.3. solving simple inequalities in the form $ax \le b$, $ax \ge b$, $ax < b$ and $ax > b$, where *a* and *b* are integers 7.4. solving simple fractional equations that can be reduced to linear equations such as $\frac{x}{3} + \frac{x-2}{4} = 3$ $\frac{3}{x-2} = 6$ 7.5. formulating a linear equation in one variable to solve problems	(a) Use the virtual balance in AlgeTools™ to explore the concepts of equation and to construct, simplify and solve linear equations with integral coefficients.. (b) Use the AlgeBar™ application in AlgeTools™ to formulate linear equations to solve problems (Students can draw models to help them formulate equations). (c) Formulate inequalities from real-world contexts.
N10. Problems in real-world contexts	
10.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc) 10.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 10.3. interpreting the solution in the context of the problem 10.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary One	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G1. Angles, triangles and polygons	
1.1. right, acute, obtuse and reflex angles 1.2. vertically opposite angles, angles on a straight line, angles at a point 1.3. angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles 1.4. properties of triangles, special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon), including symmetry properties 1.5. classifying special quadrilaterals on the basis of their properties 1.6. angle sum of interior and exterior angles of any convex polygon 1.7. properties of perpendicular bisectors of line segments and angle bisectors 1.8. construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractors, where appropriate	(a) Investigate the properties relating the sides and angles of a triangle, e.g. form triangles with different lengths to discover that the sum of two sides is greater than the third side, and that the longest side is opposite the largest angle. (b) Use GSP or other dynamic geometry software to explore a given type of quadrilateral (e.g. parallelogram) to discover its properties, e.g. diagonals of a parallelogram bisect each other. (c) Investigate the sum of the interior and exterior angles of polygons and obtain the formulae for them. (d) Recognise symmetric properties (rotational and line symmetry) in some special quadrilaterals and regular polygons. (e) Justify whether a mathematical statement is true or false. (f) Use GSP or other dynamic geometry software to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle.
G5. Mensuration	
5.1. area of parallelogram and trapezium 5.2. problems involving perimeter and area of plane figures 5.3. volume and surface area of prism and cylinder 5.4. conversion between cm^2 and m^2, and between cm^3 and m^3 5.5. problems involving volume and surface area of composite solids	(a) Make connections between the area of a parallelogram and that of a rectangle, and between the area of a trapezium and that of a parallelogram. (b) Identify the height corresponding to any given side of a triangle or quadrilateral that is taken as the base. (c) Visualise and sketch 3D shapes from different views. (d) Visualise and draw the nets of cubes, cuboids, prisms and cylinders for the calculation of surface area.

Content	Learning Experiences
Secondary One	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G8. Problems in real-world contexts	
8.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 8.2. interpreting the solution in the context of the problem 8.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.1. analysis and interpretation of: • tables • bar graphs • pictograms • line graphs • pie charts 1.2. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.3. explaining why a given statistical diagram leads to misinterpretation of data	(a) Construct tables, bar graphs, pictograms, line graphs and pie charts from given data. (b) Work collaboratively on a task to: • collect and classify data • present data using an appropriate statistical representation (including the use of software) • analyse data (c) Compare various statistical representations and justify why a particular representation is more suitable than others for a given situation. (d) Use data to make informed decisions, predictions and inferences.

Content	Learning Experiences
Secondary Two	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N2. Ratio and proportion	
2.4. map scales (distance and area) 2.5. direct and inverse proportion	(a) Interpret the various scales used on maps, floor plans and other scale drawings, and calculate the actual distance/length and area. (b) Work in groups to make a scale drawing of an existing or dream classroom/ bedroom and explain the choice of the scale used. (c) Discuss examples of direct and inverse proportion and explain the concepts using tables, equations and graphs.
N5. Algebraic expressions and formulae	
5.9. expansion of the product of algebraic expressions 5.10. changing the subject of a formula 5.11. finding the value of an unknown quantity in a given formula 5.12. use of: • $(a+b)^2 = a^2 + 2ab + b^2$ • $(a-b)^2 = a^2 - 2ab + b^2$ • $a^2 - b^2 = (a+b)(a-b)$ 5.13. factorisation of linear expressions of the form $ax + bx + kay + kby$ 5.14. factorisation of quadratic expressions $ax^2 + bx + c$ 5.15. multiplication and division of simple algebraic fractions such as $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$ 5.16. addition and subtraction of algebraic fractions with linear or quadratic denominator such as $\frac{1}{x-2} + \frac{2}{x-3}$ $\frac{1}{x^2-9} + \frac{2}{x-3}$ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$	(a) Use algebra manipulatives, e.g. algebra discs, to explain the process of expanding the product of two linear expressions of the form $px+q$, where p and q are integers, to obtain a quadratic expression of the form ax^2+bx+c. (b) Use the AlgeDisc™ application in AlgeTools™ to factorise a quadratic expression of the form ax^2+bx+c into two linear factors where a, b and c are integers. (c) Work in groups to identify and explain common errors in algebraic manipulations, e.g. $(x+y)^2 = x^2 + y^2$.

Content	Learning Experiences
Secondary Two	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N6. Functions and graphs	
6.6. quadratic functions $y = ax^2 + bx + c$ 6.7. graphs of quadratic functions and their properties: • positive or negative coefficient of x^2 • maximum and minimum points • symmetry	(a) Show graphically the relationship between the area of a square and the length of its side, and explain that the relationship is a function, but not a linear function. (b) Use a spreadsheet or graphing software to study how the graph of $y = ax^2 + bx + c$ changes when either a, b or c varies.
N7. Equations and inequalities	
7.6. graphs of linear equations in two variables ($ax + by = c$) 7.7. solving simultaneous linear equations in two variables by: • substitution and elimination methods • graphical method 7.8. solving quadratic equations in one variable by factorisation 7.9. formulating a pair of linear equations in two variables to solve problems	(a) Use Graphmatica, applets or other software to draw the graph of $ax + by = c$ (a straight line), check that the coordinates of a point on the straight line satisfy the equation, and explain why the solution of a pair of simultaneous linear equations is the point of intersection of two straight lines. (b) Draw the lines $x = a$ and $y = b$, and describe the lines and their gradients.
N10. Problems in real-world contexts	
10.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc) 10.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 10.3. interpreting the solution in the context of the problem 10.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Two	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G2. Congruence and similarity	
2.1. congruent figures 2.2. similar figures 2.3. properties of similar triangles and polygons: • corresponding angles are equal • corresponding sides are proportional 2.4. enlargement and reduction of a plane figure 2.5. scale drawings 2.6. solving simple problems involving congruence and similarity	(a) Examine whether two figures are congruent, by checking if one figure can be mapped onto the other figure under translation, rotation, and reflection. (b) Identify similar triangles/rectangles from cut-outs of triangles/rectangles and explain why they are similar. (c) Identify and suggest applications of congruency and similarity in real-world contexts, e.g. photocopying, tessellation patterns, etc.
G4. Pythagoras' theorem and trigonometry	
4.1. use of Pythagoras' theorem 4.2. determining whether a triangle is right-angled given the lengths of three sides 4.3. use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles	(a) Either (i) use strings of 12 units (e.g. 1 unit = 10 cm) to form a right-angled triangle with sides of whole unit lengths (e.g. 3 units, 4 units and 5 units) and find out if there is a relationship involving the three sides; or (ii) use cut-out pieces of two squares with sides 3 units and 4 units respectively to form a square of sides 5 units. (b) Discuss the use of trigonometric ratios in real life, e.g. finding the height of a tree/a building by measuring the angle of elevation with a clinometer.
G5. Mensuration	
5.6. volume and surface area of pyramid, cone and sphere	(a) Visualise and make connection between the volumes of pyramid and cone, and the volumes of pyramid/cone and the related prism/cylinder. (b) Make sense of the formulae for the volume and surface area of a sphere e.g. by relating to the formulae for the volume and curved surface area of the related cylinder.

Content	Learning Experiences
Secondary Two	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G8. Problems in real-world contexts	
8.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 8.2. interpreting the solution in the context of the problem 8.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.4. analysis and interpretation of: • dot diagrams • histograms • stem-and-leaf diagrams 1.5. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.6. explaining why a given statistical diagram leads to misinterpretation of data 1.7. mean, mode and median as measures of central tendency for a set of data 1.8. purposes and use of mean, mode and median 1.9. calculation of the mean for grouped data	(a) Construct dot diagrams, histograms (including equal and unequal class intervals) and stem-and-leaf diagrams from given data. (b) Predict, observe and explain how the different measures of central tendency are affected by extreme data values (or outliers). (c) Discuss the appropriate use of the measures of central tendency in different contexts.
S2. Probability	
2.1. probability as a measure of chance 2.2. probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)	(a) Discuss the concept of probability (or chance) using everyday events, including simple experiments such as tossing a coin. (b) Compare and discuss the experimental and theoretical values of probability using computer simulations.

Content	Learning Experiences
Secondary Three/Four	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.8. use of standard form $A\times10^n$, where n is an integer, and $1\le A<10$ 1.9. positive, negative, zero and fractional indices 1.10. laws of indices	(a) Discuss examples of very large and very small numbers, e.g. world population in billions (10^9) and size of atom in nanometres (10^{-9}), and express the numbers in standard form. (b) Describe and compare numbers written in index form, e.g. "Which is greater, 2^{10} or 10^2?", and explain how to multiply and divide such numbers using the laws of indices.
N6. Functions and graphs	
6.8. sketching the graphs of quadratic functions given in the form: • $y=(x-p)^2+q$ • $y=-(x-p)^2+q$ • $y=(x-a)(x-b)$ • $y=-(x-a)(x-b)$ 6.9. graphs of power functions $y=ax^n$, where $n=-2,-1,0,1,2,3$, and simple sums of not more than three of these 6.10. graphs of exponential functions $y=ka^x$, where a is a positive integer 6.11. estimation of the gradient of a curve by drawing a tangent	(a) Use Graphmatica or other graphing software to explore the characteristics of various functions. (b) Work in groups to match and justify sketches of graphs with their respective functions.
N7. Equations and inequalities	
7.10. solving quadratic equations in one variable by: • use of formula • completing the square for $y=x^2+px+q$ • graphical method 7.11. solving fractional equations that can be reduced to quadratic equations such as $\frac{6}{x+4}=x+3$	(a) Explain why there are no real solutions to a quadratic equation $ax^2+bx+c=0$ when b^2-4ac is negative. (b) Compare the methods of solving a linear inequality and the corresponding linear equation, and their solutions.

Content	Learning Experiences
Secondary Three/Four	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
$\frac{1}{x-2}+\frac{2}{x-3}=5$ 7.12. solving linear inequalities in one variable, and representing the solution on the number line 7.13. formulating a quadratic equation in one variable to solve problems	
N8. Set language and notation	
8.1. use of set language and the following notation: Union of *A* and *B* $A\cup B$ Intersection of *A* and *B* $A\cap B$ "… is an element of …" $\in$ "… is not an element of …" $\notin$ Complement of set *A* A' The empty set ϕ Universal set ξ *A* is a (proper) subset of *B* $A\subset B$ *A* is a not a (proper) subset of *B* $A\not\subset B$ 8.2. union and intersection of two sets 8.3. Venn diagrams	(a) Use set language and notation to specify and describe sets of numbers, shapes and objects, to determine whether an object belongs to a set or not, and to classify objects into distinct sets. (b) Use Venn diagrams to represent relationships between two sets, and between a set and its elements.
N9. Matrices	
9.1. display of information in the form of a matrix of any order 9.2. interpreting the data in a given matrix 9.3. product of a scalar quantity and a matrix 9.4. problems involving addition, subtraction and multiplication of matrices	(a) Justify if two matrices can be multiplied by checking the orders of the matrices. (b) Discuss some applications of matrix multiplication, e.g. decoding messages and transformation matrices for movie making.

Content	Learning Experiences
Secondary Three/Four	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N10. Problems in real-world contexts	
10.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple and compound interest, taxation, instalments, utilities bills, money exchange, etc) 10.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 10.3. interpreting the solution in the context of the problem 10.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Three/Four	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G2. Congruence and similarity	
2.7. determining whether two triangles are: • congruent • similar 2.8. ratio of areas of similar plane figures 2.9. ratio of volumes of similar solids	(a) Construct triangles with given conditions, e.g. "3 sides", "3 angles", "2 sides, 1 angle", and "2 angles, 1 side", and examine what conditions are necessary for congruency/similarity.
G3. Properties of circles	
3.1. symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length • the line joining an external point to the centre of the circle bisects the angle between the tangents 3.2. angle properties of circles: • angle in a semicircle is a right angle • angle between tangent and radius of a circle is a right angle • angle at the centre is twice the angle at the circumference • angles in the same segment are equal • angles in opposite segments are supplementary	(a) Use paper folding to visualise symmetric properties of circles, e.g. "the perpendicular bisector of a chord passes through the centre". (b) Use GSP or other dynamic geometry software to explore the properties of circles, and use geometrical terms correctly for effective communication.
G4. Pythagoras' theorem and trigonometry	
4.4. extending sine and cosine to obtuse angles 4.5. use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle 4.6. use of sine rule and cosine rule for any triangle 4.7. problems in two and three dimensions including those involving angles of elevation and depression and bearings	(a) Visualise height, north direction, right-angled triangle, etc. from 2D drawings of 3D situations. (b) Use the sine and cosine rules to articulate the relationships between the sides and angles of a triangle, e.g. the lengths of the sides are proportional to sine of the corresponding angles, Pythagoras theorem is a special case of the cosine rule, etc.

Content	Learning Experiences
Secondary Three/Four	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G5. Mensuration	
5.7. arc length, sector area and area of a segment of a circle 5.8. use of radian measure of angle (including conversion between radians and degrees)	(a) Find the arc length and sector area by considering them as fractions of the circumference and area of circle respectively. (b) Visualise the size of an angle of 1 radian, and estimate the size of angles in radians.
G6. Coordinate geometry	
6.1. finding the gradient of a straight line given the coordinates of two points on it 6.2. finding the length of a line segment given the coordinates of its end points 6.3. interpreting and finding the equation of a straight line graph in the form $y = mx + c$ 6.4. geometric problems involving the use of coordinates	(a) Extend their intuitive understanding of gradient as ratio of vertical change to horizontal change to a formal treatment using the coordinates of two points on the line. (b) Use GSP or other dynamic geometry software to explore and describe the gradients of straight lines, including the gradient of a vertical line as undefined, and to investigate how the signs of $y_2 - y_1$ and $x_2 - x_1$ affect the sign of the gradient of a straight line.
G7. Vectors in two dimensions	
7.1. use of notations: $\begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{AB}$, $\mathbf{a}$, $\left\lvert\overrightarrow{AB}\right\rvert$ and $\lvert\mathbf{a}\rvert$ 7.2. representing a vector as a directed line segment 7.3. translation by a vector 7.4. position vectors 7.5. magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$ 7.6. use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors 7.7. multiplication of a vector by a scalar 7.8. geometric problems involving the use of vectors	(a) Represent graphically the sum and difference of two vectors, and a multiple of a vector. (b) Give examples to illustrate the resultant of two vectors.

Content	Learning Experiences
Secondary Three/Four	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G8. Problems in real-world contexts	
8.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 8.2. interpreting the solution in the context of the problem 8.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.10. quartiles and percentiles 1.11. range, interquartile range and standard deviation as measures of spread for a set of data 1.12. analysis and interpretation of: • cumulative frequency diagrams • box-and-whisker plots 1.13. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.14. calculation of the standard deviation for a set of data (grouped and ungrouped) 1.15. using the mean and standard deviation to compare two sets of data	(a) Compare the means and standard deviations of two sets of data. (b) Discuss examples of inappropriate representations of data from newspapers and other sources, e.g. whether certain representations are misleading.
S2. Probability	
2.3. probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) 2.4. addition and multiplication of probabilities (mutually exclusive events and independent events)	(a) Discuss and differentiate between mutually exclusive and non-mutually exclusive events, and between independent and dependent events.

Chapter 5
N(A)-Level Mathematics Syllabus

Aims of Syllabus
Syllabus Organisation
Content by Strand
Content and Learning Experiences by Level

Aims of Syllabus

The N(A)-Level Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem-solving;
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics; and
- build confidence and foster interest in mathematics.

Syllabus Organisation

The syllabus is organised along 3 content strands with a listing of mathematical processes that cut across the 3 strands.

<table>
<tr><td colspan="3">3 Content Strands + 1 Process Strand</td></tr>
<tr><td>Number and Algebra</td><td>Geometry and Measurement</td><td>Statistics and Probability</td></tr>
<tr><td colspan="3">Mathematical Processes</td></tr>
</table>

Strand: Mathematical Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematical problem solving and beyond.

At the secondary level, students gradually move from inductive arguments to deductive arguments and justifications (e.g. to justify a mathematical statement using a short chain of logical reasoning or to disprove a statement by a counter example). They will continue to make connections among mathematical ideas, and between mathematics and the real world. They should be able to read and critique arguments that are supported by mathematics.

Greater attention will be given to applications and modelling at the secondary level, as students become more mature and aware of their immediate environment and phenomenon. Besides learning standard mathematical models, students should, under teacher guidance, develop an awareness and understanding of the mathematical modelling process. They work on real-world problems either individually or in groups. They would need to understand the real-world situation, make assumptions about the situation, devise a mathematical model to solve the problem, and interpret the solution in the context of the situation. The process of mathematical modelling widens and deepens students' understanding of mathematics, and helps them develop important 21st century skills, including collaboration, creativity, communication and critical thinking.

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to problem solving approaches such as the Polya's model and the steps in a modelling process in class. Teachers could "think aloud" to give attention to these processes and make them visible to students. Students should be given opportunities to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. Through practice, students will develop habits and strategies that will help them be better and more independent learners.

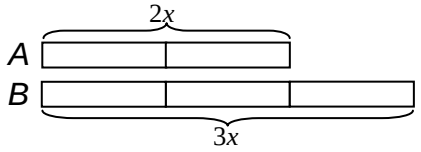
No.	Processes	Indicators
MATHEMATICAL PROCESSES		
MP1	**Reasoning, Communication and Connections**	• Use appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas • Reason inductively and deductively, including: ∗ Explaining or justifying/verifying a mathematical solution/statement ∗ Drawing logical conclusions ∗ Making inferences ∗ Writing mathematical arguments • Make connections within mathematics, between mathematics and other disciplines, and between mathematics and the real world
MP2	**Applications and Modelling**	• Apply mathematics concepts and skills to solve problems in a variety of contexts within or outside mathematics, including: ∗ Identifying the appropriate mathematical representations or standard models for a problem ∗ Using appropriate mathematical concepts, skills (including tools and algorithm) to solve a problem • Understand the nature of the mathematical modelling process, including: ∗ Formulating a real world problem into a mathematical model by making suitable assumptions and simplification and identifying suitable mathematical representations ∗ Applying mathematics to solve the problem ∗ Interpreting the mathematical solution in the context of the problem ∗ Refining and improving the model
MP3	**Thinking Skills and Heuristics**	• Use thinking skills such as: ∗ Classifying ∗ Comparing ∗ Sequencing ∗ Generalising ∗ Induction ∗ Deduction ∗ Analysing (from whole to parts) ∗ Synthesizing (from parts to whole) • Use a problem-solving model such as Polya's model • Use heuristics such as: ∗ Drawing a diagram ∗ Tabulating ∗ Guess and check ∗ Working backwards ∗ Simplifying a problem ∗ Considering special cases

Content & Learning Experiences by Level

In this section, the content is listed by levels and learning experiences statements are included.

The learning experiences for the N(A)-Level Mathematics syllabus should provide opportunities for students to:

- enhance conceptual understanding through use of various mathematical tools including ICT tools;
- apply concepts and skills learnt in real-world context;
- communicate their reasoning and connections through various mathematical tasks and activities; and
- build confidence and foster interest in mathematics.

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.1. primes and prime factorisation 1.2. finding highest common factor (HCF) and lowest common multiple (LCM), squares, cubes, square roots and cube roots by prime factorisation 1.3. negative numbers, integers, rational numbers, real numbers and their four operations 1.4. calculations with calculator 1.5. representation and ordering of numbers on the number line 1.6. use of <, >, ≤, ≥ 1.7. approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, and estimating the results of computation)	(a) Classify whole numbers based on their number of factors and explain why 0 and 1 are not primes. (b) Discuss examples of negative numbers in the real world. (c) Represent integers, rational numbers, and real numbers on the number line as extension of whole numbers, fractions and decimals respectively. (d) Use algebra discs or the AlgeDisc™ application in AlgeTools™ to make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the 4 operations of integers. (e) Work in groups to estimate quantities (numbers and measures) in a variety of contexts, compare the estimates and share the estimation strategies. (f) Compare follow-through errors arising from intermediate values that are rounded to different degrees of accuracy. (g) Make estimates and check the reasonableness of answers obtained from calculators.
N2. Ratio and proportion	
2.1. comparison between two or more quantities by ratio 2.2. relationship between ratio and fraction 2.3. dividing a quantity in a given ratio 2.4. ratios involving rational numbers 2.5. equivalent ratios 2.6. writing a ratio in its simplest form 2.7. problems involving ratio	(a) Discuss and explain how ratios are used in everyday life. (b) Use the concept of equivalent ratios to find the ratio a:b:c given the ratios a:b and b:c. (c) Make connections between ratios and fractions, use appropriate mathematical language to describe the relationship, and use algebra to solve problems, e.g. "The ratio *A* to *B* is 2:3" can be represented as:  "The ratio of 2:3 means "2 units to 3 units", "*A* is $\frac{2}{3}$ of *B*", or "*B* is $\frac{3}{2}$ of A".

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N3. Percentage	
3.1. expressing percentage as a fraction or decimal 3.2. expressing one quantity as a percentage of another 3.3. comparing two quantities by percentage 3.4. percentages greater than 100% 3.5. increasing/decreasing a quantity by a given percentage (including concept of percentage point) 3.6. finding percentage increase/decrease 3.7. reverse percentages 3.8. problems involving percentages	(a) Collect examples of percentages from newspapers and magazines and discuss the meaning of percentage in each example. (b) Examine bills and receipts, etc. to find examples of the uses of percentages, e.g. discount, service charge, GST and other taxes and check the calculated values. (c) Make connections between percentages and fractions/decimals, e.g. "25% of a quantity is $\frac{1}{4}$ of the quantity", "20% of x is $0.2x$". (d) Discuss misconceptions, e.g. "If *A* is 5% more than *B*, then *B* is 5% less than *A*".
N4. Rate and Speed	
4.1. relationships between distance, time and speed 4.2. writing speed in different units (e.g. km/h, m/min, m/s and cm/s) 4.3. concepts of average rate, speed, constant speed and average speed 4.4. conversion of units (e.g. km/h to m/s) 4.5. calculation of speed, distance or time given the other two quantities 4.6. problems involving rate and speed	(a) Discuss examples of rates, e.g. currency exchange rates, interest rates, tax rates, rate of rotation and speed. (b) Find out and compare the speeds of bicycles, cars, trains, aeroplanes and spaceships and their respective units to have a sense of their magnitude. (c) Explain the difference between average speed and constant speed and also explain why average speed is not the average of speeds.

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N5. Algebraic expressions and formulae	
5.1. using letters to represent numbers 5.2. interpreting notations: • ab as $a \times b$ • $\frac{a}{b}$ as $a \div b$ or a x $\frac{1}{b}$ • a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, . . . • $3y$ as $y + y + y$ or $3 \times y$ • $3\,(x + y)$ as $3 \times (x + y)$ • $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$ 5.3. evaluation of algebraic expressions and formulae 5.4. translation of simple real-world situations into algebraic expressions 5.5. recognising and representing patterns/relationships by finding an algebraic expression for the nth term 5.6. addition and subtraction of linear expressions 5.7. simplification of linear expressions with integral coefficients such as $2(x-3y)$ $4x-2(3x-5)$ $3(x-y)-(2y+x)-y$	(a) Use spreadsheets, e.g. Microsoft Excel, to * explore the concept of variables and evaluate algebraic expressions. * compare and examine the differences between pairs of expressions, e.g. $2n$ and $2+n$, n^2 and $2n$, $2n^2$ and $(2n)^2$. (b) Use algebra discs or the AlgeDisc™ application in AlgeTools™ to make sense of and interpret linear expressions with integral coefficients, e.g. $4x-3y$ and $-3(x-2)$. (c) Use the AlgeDisc™ application in AlgeTools™ to construct and simplify linear expressions with integral coefficients. (d) Work in groups to select and justify pairs of equivalent expressions. (e) Write algebraic expressions to express mathematical relationships, e.g. for the statement "There are 3 times as many boys as girls", if we let x be the number of girls, then the number of boys is $3x$. This can also be written as $b = 3g$, where b and g are the number of boys and girls respectively. (f) Explore number patterns and write algebraic expressions to represent the patterns.
N7. Equations and inequalities	
7.1. concepts of equation 7.2. solving linear equations with integral coefficients in one variable 7.3. formulating a linear equation in one variable to solve problems	(a) Use the virtual balance in AlgeTools™ to explore the concepts of equation, and to construct, simplify and solve linear equations with integral coefficients. (b) Use the AlgeBar™ application (for whole numbers) in AlgeTools™ to formulate linear equations to solve problems (Students can draw models to help them formulate the equations.)

Content	Learning Experiences
Secondary One	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N8. Problems in real-world contexts	
8.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc) 8.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 8.3. interpreting the solution in the context of the problem 8.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary One	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G1. Angles, triangles and polygons	
1.1. right, acute, obtuse and reflex angles 1.2. vertically opposite angles, angles on a straight line, angles at a point 1.3. angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles 1.4. properties of triangles	(a) Investigate the properties relating the sides and angles of a triangle, e.g. form triangles with different lengths to discover that the sum of two sides is greater than the third side, and that the longest side is opposite the largest angle.

Content	Learning Experiences
Secondary One	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G5. Mensuration	
5.1. area of parallelogram and trapezium 5.2. problems involving perimeter and area of plane figures 5.3. volume and surface area of prism and cylinder 5.4. conversion between cm^2 and m^2, and between cm^3 and m^3 5.5. problems involving volume and surface area of composite solids	(a) Make connections between the area of a parallelogram and that of a rectangle, and between the area of a trapezium and that of a parallelogram, e.g. using paper folding/cutting. (b) Identify the height corresponding to any given side of a triangle or quadrilateral that is taken as the base. (c) Visualise and sketch 3D shapes from different views. (d) Visualise and draw the nets of cubes, cuboids, prisms and cylinders for the calculation of surface area.
G7. Problems in real-world contexts	
7.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 7.2. interpreting the solution in the context of the problem 7.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.1. analysis and interpretation of: • tables • bar graphs • pictograms • line graphs • pie charts 1.2. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.3. explaining why a given statistical diagram leads to misinterpretation of data	(a) Construct tables, bar graphs, pictograms, line graphs and pie charts from given data. (b) Work collaboratively on a task to: • collect and classify data • present data using an appropriate statistical representation (including the use of software) • analyse data (c) Compare various statistical representations and justify why a particular representation is more suitable than others for a given situation. (d) Use data to make informed decisions, predictions and inferences.

Content	Learning Experiences
Secondary Two	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N2. Ratio and proportion	
2.8. map scales (distance and area) 2.9. direct and inverse proportion	(a) Interpret the various scales used on maps, floor plans and other scale drawings, and calculate the actual distance/length and area. (b) Work in groups to make a scale drawing of an existing or dream classroom/ bedroom and explain the choice of the scale used. (c) Discuss examples of direct and inverse proportion and explain the concepts using tables, equations and graphs.
N5. Algebraic expressions and formulae	
5.8. simplification of linear expressions with fractional coefficients such as $\frac{2x}{3}-\frac{3(x-5)}{2}$ 5.9. expansion of the product of two linear expressions 5.10. use brackets and extract common factors 5.11. use of: • $(a+b)^2=a^2+2ab+b^2$ • $(a-b)^2=a^2-2ab+b^2$ • $a^2-b^2=(a+b)(a-b)$ 5.12. factorisation of quadratic expressions ax^2+bx+c 5.13. multiplication and division of simple algebraic fractions such as $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4}\div\frac{9a^2}{10}$	(a) Use algebra manipulatives, e.g. algebra discs to explain the process of expanding the product of two linear expressions of the form $px+q$, where p and q are integers, to obtain a quadratic expression of the form ax^2+bx+c. (b) Use the AlgeDisc™ application in AlgeTools™, to factorise a quadratic expression of the form ax^2+bx+c into two linear factors where a, b and c are integers. (c) Work in groups to identify and explain ~~some~~ common errors in algebraic manipulations, such as $(x+y)^2=x^2+y^2$.

Content	Learning Experiences
Secondary Two	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N6. Functions and graphs	
6.1. Cartesian coordinates in two dimensions 6.2. graph of a set of ordered pairs as a representation of a relationship between two variables 6.3. linear functions $y = ax + b$ 6.4. graphs of linear functions 6.5. the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients)	(a) Play games, e.g. Battleship Game, that involve the use of 2D Cartesian coordinates to specify points. (b) Use a function machine to generate input and output values to illustrate the concept of function as "only one output for every input" and represent the function in verbal, tabular, graphical and algebraic forms. (c) Use a linear function to represent the relationship between two variables (such as distance and time when the speed is constant), show the relationship graphically and identify that the rate of change is the gradient of the graph. (d) Use a spreadsheet or graphing software to study how the graph of $y = ax + b$ changes when either *a* or *b* varies.
N7. Equations and inequalities	
7.4. solving linear equations in one variable (including fractional coefficients) 7.5. concept and properties of inequality 7.6. solving simple inequalities in the form $ax \le b$, $ax \ge b$, $ax < b$ and $ax > b$, where a and b are integers. 7.7. solving simple fractional equations that can be reduced to linear equations such as $\frac{x}{3} + \frac{x-2}{4} = 3$ $\frac{3}{x-2} = 6$ 7.8. graphs of linear equations in two variables ($ax + by = c$) 7.9. solving simultaneous linear equations in two variables by: • substitution and elimination methods • graphical method 7.10. formulating a linear equation in one variable or a pair of linear equations in two variables to solve problems	(a) Formulate inequalities from real-world contexts. (b) Use Graphmatica, applets or other software to draw the graph of $ax + by = c$ (a straight line), check that the coordinates of a point on the straight line satisfy the equation, and explain why the solution of a pair of simultaneous linear equations is the point of intersection of two straight lines. (c) Draw the lines $x = a$ and $y = b$, and describe the lines and their gradients. (d) Use the AlgeBar™ application in AlgeTools™ to formulate linear equations to solve problems (Students can draw models to help them formulate equations.)

Content	Learning Experiences
Secondary Two	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N8. Problems in real-world contexts	
8.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple interest, taxation, instalments, utilities bills, money exchange, etc) 8.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 8.3. interpreting the solution in the context of the problem 8.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (c) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Two	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G1. Angles, triangles and polygons	
1.5. properties of special quadrilaterals and regular polygons (pentagon, hexagon, octagon and decagon) including symmetry properties 1.6. classifying special quadrilaterals on the basis of their properties 1.7. angle sum of interior and exterior angles of any convex polygon 1.8. properties of perpendicular bisectors of line segments and angle bisectors 1.9. construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractors, where appropriate	(a) Use GSP or other dynamic geometry software to explore a given type of quadrilateral (e.g. parallelogram) to discover its properties, e.g. diagonals of a parallelogram bisect each other. (b) Investigate the sum of the interior and exterior angles of polygons and obtain the formulae for them. (c) Recognise symmetric properties (rotational and line symmetry) in some special quadrilaterals and regular polygons. (d) Justify whether a mathematical statement is true or false. (e) Use GSP or other dynamic geometry software to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle.

Content	Learning Experiences
Secondary Two	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G2. Congruence and similarity	
2.1. congruent figures 2.2. similar figures 2.3. properties of similar triangles and polygons: • corresponding angles are equal • corresponding sides are proportional	(a) Examine the pictures of two congruent figures and check if one figure can be mapped onto the other under translation, rotation and reflection. (b) Identify similar triangles/rectangles from cut-outs of triangles/rectangles and explain why they are similar.
G4. Pythagoras' theorem and trigonometry	
4.1. use of Pythagoras' theorem 4.2. determining whether a triangle is right-angled given the lengths of three sides	(a) Either (i) use a string of length 12 units (e.g. 1 unit = 10 cm) to form a right-angled triangle with sides of whole-unit lengths (e.g. 3 units, 4 units and 5 units) and find out if there is a relationship between the three sides; or (ii) use cut-out pieces of two squares with sides of 3 units and 4 units respectively to form a square of sides 5 units.
G5. Mensuration	
5.6. volume and surface area of pyramid, cone and sphere	(a) Visualise and make connections between the volumes of pyramid and cone, and the volumes of pyramid/cone and the related prism/cylinder. (b) Make sense of the formulae for the volume and surface area of a sphere, e.g. by relating to the formulae for the volume and curved surface area of the related cylinder.
G7. Problems in real-world contexts	
7.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 7.2. interpreting the solution in the context of the problem 7.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Two	
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.4. analysis and interpretation of: • dot diagrams • histograms • stem-and-leaf diagrams 1.5. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.6. explaining why a given statistical diagram leads to misinterpretation of data 1.7. mean, mode and median as measures of central tendency for a set of data 1.8. purposes and uses of mean, mode and median 1.9. calculation of the mean for grouped data	(a) Construct dot diagrams, histograms (including equal and unequal class intervals) and stem-and-leaf diagrams from given data. (b) Predict, observe and explain how the different measures of central tendency are affected by changing data values. (c) Discuss the appropriate use of the measures of central tendency in different contexts.
S2. Probability	
2.1. probability as a measure of chance 2.2. probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)	(a) Discuss the concept of probability (or chance) using everyday events, including simple experiments, e.g. tossing a coin, and use language such as “certain”, “likely” and “unlikely”. (b) Compare and discuss the experimental and theoretical values of probability using computer simulations.

Content	Learning Experiences
Secondary Three/Four	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.8. use of standard form $A\times10^n$, where n is an integer, and $1\le A<10$ 1.9. positive, negative, zero and fractional indices 1.10. laws of indices	(a) Discuss examples of very large and very small numbers, e.g. world population in billions (10^9) and size of atom in nanometres (10^{-9}), and express the numbers in standard form. (b) Describe and compare numbers written in index form, e.g. "Which is greater, 2^{10} or 10^2?", and explain how to multiply and divide such numbers using the laws of indices.
N5. Algebraic expressions and formulae	
5.14. factorisation of linear expressions of the form $ax + bx + kay + kby$ 5.15. expansion of the product of algebraic expressions 5.16. changing the subject of a formula 5.17. finding the value of an unknown quantity in a given formula 5.18. addition and subtraction of algebraic fractions with linear or quadratic denominator such as $\frac{1}{x-2}+\frac{2}{x-3}$ $\frac{1}{x^2-9}+\frac{2}{x-3}$ $\frac{1}{x-3}+\frac{2}{(x-3)^2}$	(a) Work in groups to identify and explain common errors in algebraic fractions.
N6. Functions and graphs	
6.6. quadratic functions $y=ax^2+bx+c$ 6.7. graphs of quadratic functions and their properties: • positive or negative coefficient of x^2 • maximum and minimum points • symmetry 6.8. graphs of power functions $y=ax^n$, where n = –2, –1, 0, 1, 2, 3, and simple sums of not more than three of these	(a) Show graphically the relationship between the area of a square and the length of its side, and explain that the relationship is a function, but not a linear function. (b) Use a spreadsheet or graphing software to study how the graph of $y=ax^2+bx+c$ changes when either *a*, *b* or *c* varies. (c) Use Graphmatica or other graphing software to explore the characteristics of various functions. (d) Work in groups to match and justify sketches of graphs with their respective functions.

Content	Learning Experiences
Secondary Three/Four	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
6.9. graphs of exponential functions $y = ka^x$, where a is a positive integer 6.10. estimation of the gradient of a curve by drawing a tangent	
N7. Equations and inequalities	
7.11. solving quadratic equations in one variable by: • factorisation • use of formula • completing the square for $y = x^2 + px + q$ • graphical method 7.12. solving fractional equations that can be reduced to quadratic equations such as $\frac{6}{x+4} = x+3$ $\frac{1}{x-2} + \frac{2}{x-3} = 5$ 7.13. formulating a quadratic equation in one variable to solve problems	(a) Explain why there are no real solutions to a quadratic equation $y = ax^2 + bx + c$ when $b^2 - 4ac$ is negative.
N8. Problems in real-world contexts	
8.1. solving problems based on real-world contexts: • in everyday life (including travel plans, transport schedules, sports and games, recipes, etc) • involving personal and household finance (including simple and compound interest, taxation, instalments, utilities bills, money exchange, etc) 8.2. interpreting and analysing data from tables and graphs, including distance-time and speed-time graphs 8.3. interpreting the solution in the context of the problem 8.4. identifying assumptions made and the limitations of the solution	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs, tables and formulae/equations). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Three/Four	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G2. Congruence and similarity	
2.4. enlargement and reduction of a plane figure 2.5. scale drawings 2.6. solving simple problems involving congruence and similarity	(a) Identify and suggest applications of congruency and similarity in real-world contexts, e.g. photocopying, tessellation patterns, etc.
G3. Properties of circles	
3.1. symmetry properties of circles: • equal chords are equidistant from the centre • the perpendicular bisector of a chord passes through the centre • tangents from an external point are equal in length • the line joining an external point to the centre of the circle bisects the angle between the tangents 3.2. angle properties of circles: • angle in a semicircle is a right angle • angle between tangent and radius of a circle is a right angle • angle at the centre is twice the angle at the circumference • angles in the same segment are equal • angles in opposite segments are supplementary	(a) Use paper folding to visualise symmetric properties of circles, e.g. the perpendicular bisector of a chord passes through the centre. (b) Use GSP or other dynamic geometry software to explore the properties of circles, and use geometrical terms correctly for effective communication.
G4. Pythagoras' theorem and trigonometry	
4.3. use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles 4.4. extending sine and cosine to obtuse angles 4.5. use of the formula $\frac{1}{2}ab\sin C$ for the area of a triangle	(a) Discuss the use of trigonometric ratios in real life, e.g. finding the height of a tree/building by measuring the angle of elevation with a clinometer. (b) Visualise height, north direction and right-angled triangle, etc. from 2D drawings of 3D situations. (c) Use the sine and cosine rules to articulate the relationships between the sides and angles of a triangle e.g. the lengths of the sides are proportional to sine of the corresponding angles, Pythagoras theorem is a special case of the cosine

Content	Learning Experiences
Secondary Three/Four	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
4.6. use of sine rule and cosine rule for any triangle 4.7. problems in two and three dimensions including those involving angles of elevation and depression and bearings	rule, etc.
G5. Mensuration	
5.7. arc length as fraction of the circumference and sector area as fraction of the area of a circle 5.8. area of a segment 5.9. use of radian measure of angle (including conversion between radians and degrees) 5.10. problems involving the arc length, sector area of a circle and area of a segment	(a) Find the arc length and sector area by considering them as fractions of the circumference and area of circle respectively. (b) Visualise the size of an angle of 1 radian and estimate the size of angles in radians.
G6. Coordinate geometry	
6.1. finding the gradient of a straight line given the coordinates of two points on it 6.2. finding the length of a line segment given the coordinates of its end points 6.3. interpreting and finding the equation of a straight line 6.4. geometric problems involving the use of coordinates	(a) Extend their intuitive understanding of gradient as ratio of vertical change to horizontal change to a formal treatment using the coordinates of two points on the line. (b) Use GSP or other dynamic geometry software to explore and describe the gradients of straight lines, including the gradient of a vertical line as undefined, and to investigate how the signs of $y_2 - y_1$ and $x_2 - x_1$ affect the sign of the gradient of a straight line.
G7. Problems in real-world contexts	
7.1. solving problems in real-world contexts (including floor plans, surveying, navigation, etc) using geometry 7.2. interpreting the solution in the context of the problem 7.3. identifying the assumptions made and the limitations of the solution	(a) Work on tasks that incorporate some or all elements of the mathematical modelling process.

Content	Learning Experiences
Secondary Three/Four	
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.10. quartiles and percentiles 1.11. range, interquartile range and standard deviation as measures of spread for a set of data 1.12. analysis and interpretation of: • cumulative frequency diagrams • box-and-whisker plots 1.13. calculation of the standard deviation for a set of data (grouped and ungrouped) 1.14. using the mean and standard deviation to compare two sets of data	(a) Compare the means and standard deviations of two sets of data. (b) Discuss examples of inappropriate representations of data from newspapers and other sources, e.g. whether certain representations are misleading.
S2. Probability	
2.3. probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) 2.4. addition and multiplication of probabilities (mutually exclusive events and independent events)	(a) Discuss and differentiate between mutually exclusive and non-mutually exclusive events, and between independent and dependent events.