Overcoming counterexamples in secondary school geometry

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Outline of the workshop

• Example 1

• The main points of the workshop

• Example 2

• Summary
Let’s prove it!

A task from textbooks

As shown in the diagram on the right, in isosceles triangle ABC where AB = AC, line BD is drawn from point B perpendicular to side AC, and line CE is drawn from point C perpendicular to side AB. Prove that BD = CE.

Using a congruence condition for right triangles: two right triangles are congruent if the hypotenuses and a pair of corresponding angles are equal.
As shown in the diagram on the right, in isosceles triangle ABC where $AB = AC$, line $BD$ is drawn from point $B$ perpendicular to side $AC$, and line $CE$ is drawn from point $C$ perpendicular to side $AB$. Prove that $BD = CE$.

By proof, we can establish that the statement is true in general.

However,

✓ Is it really true?
✓ Let’s investigate by drawing various shapes of isosceles triangles ABC.
Modification of the statement - True when $\angle A \leq 90^\circ$

Proof of the generalised statement by using the initial proof

Generalisation of the statement
Side $AB \rightarrow$ Line $AB$
Authentic Mathematical Activity

• “[I]nformal, quasi-empirical, mathematics ... grow[s] ... through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations” (Lakatos, 1976, p. 5).

• “[W]e need to explore authentic, exciting and meaningful ways of incorporating experimentation and proof in mathematics education, in order to provide students with a deeper, more holistic insight into the nature of our subject” (De Villiers, 2010, p. 220).

Example 1

By drawing various diagrams
Let’s try another example!

A task from textbooks

As shown in the right diagram, there are four points A, B, C, and D on circle O. Point P is the intersection point of lines AC and BD. Prove \( \triangle PAB \sim \triangle PDC \).

Using a similarity condition for right triangles: two triangles are similar if two pairs of corresponding angles are equal.
As shown in the right diagram, there are four points A, B, C, and D on circle O. Point P is the intersection point of lines AC and BD. Prove $\triangle PAB \sim \triangle PDC$.

By proof, we can establish that the statement is true in general.

However,

✓ Is your proof valid for all cases?
✓ Let’s investigate by moving point A on various places on circle O.
Modification of the proof (the reason for $\angle BAP = \angle BDC$)

The inscribed angle theorem

→ The inscribed quadrilateral theorem

Summary

Critical Thinking

- True?
- Improve if not true.
  (Example 1)
- Valid?
- Improve if not valid.
  (Example 2)

By drawing various diagrams