Overcoming counterexamples in secondary school geometry

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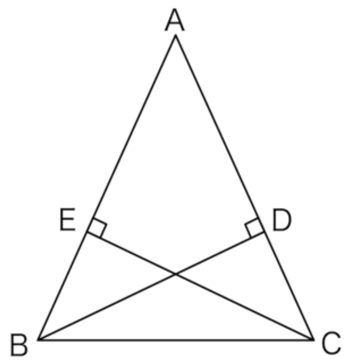
Outline of the workshop

- Example 1
- The main points of the workshop
- Example 2
- Summary

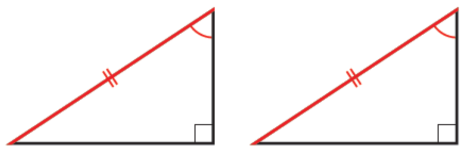
Let’s prove it!

A task from textbooks

As shown in the diagram on the right, in isosceles triangle ABC where AB = AC, line BD is drawn from point B perpendicular to side AC, and line CE is drawn from point C perpendicular to side AB. Prove that BD = CE.



Using a congruence condition for right triangles: two right triangles are congruent if the hypotenuses and a pair of corresponding angles are equal.



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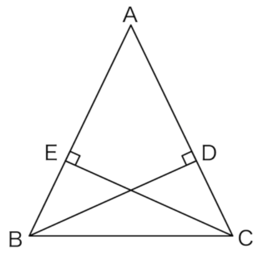
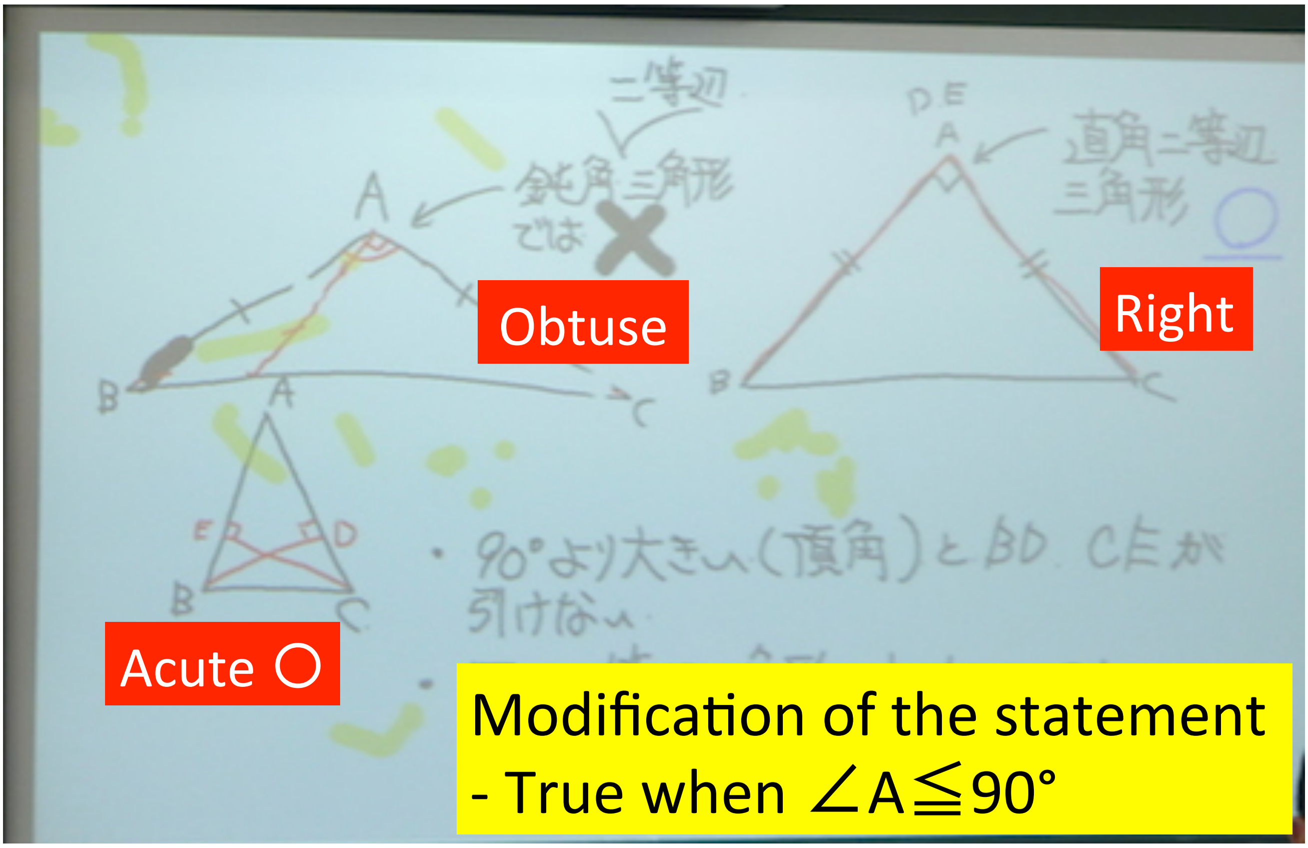


By proof, we can establish that the statement is true in general.

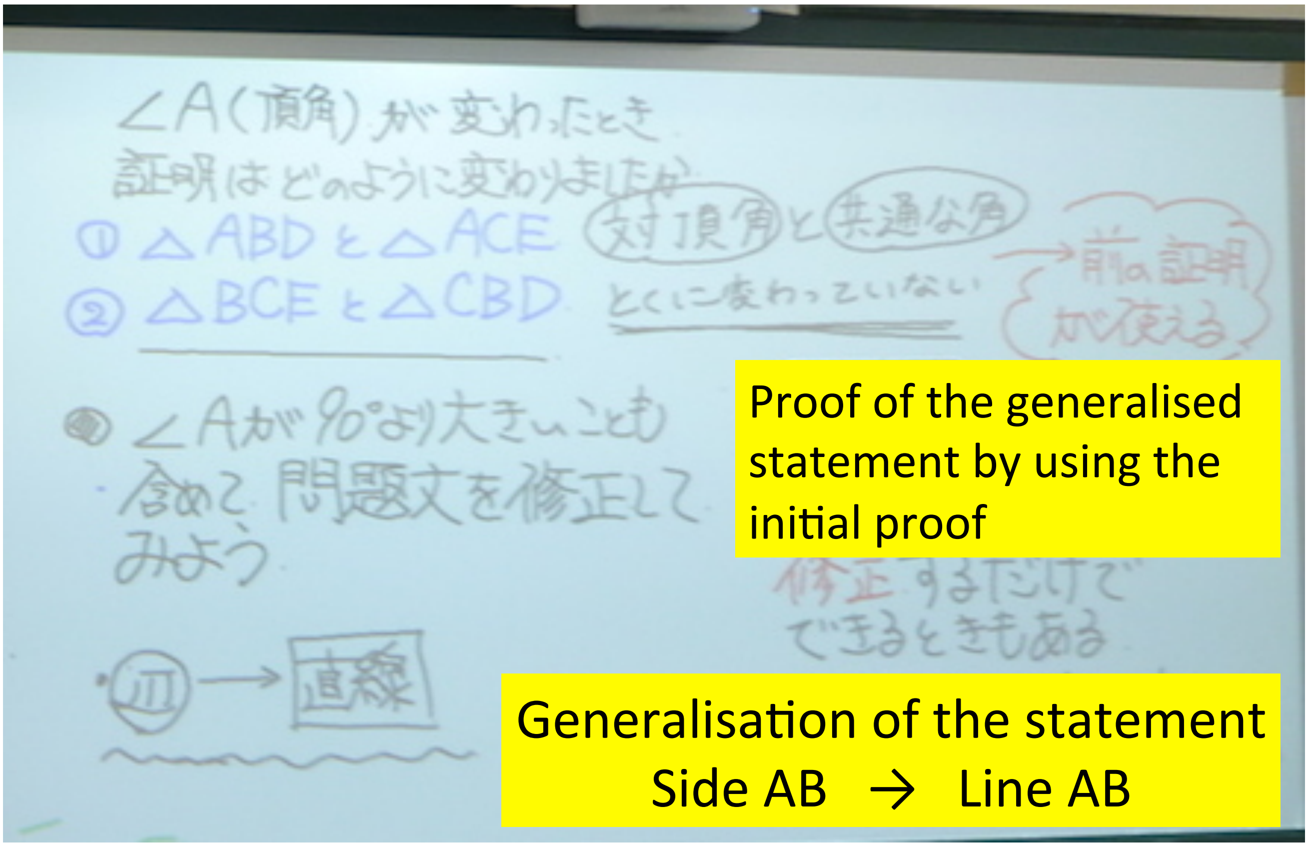
However,

- ✓ Is it really true?
- ✓ Let's investigate by <u>drawing various shapes</u> of isosceles triangles ABC.

Classroom Lesson



Classroom Lesson

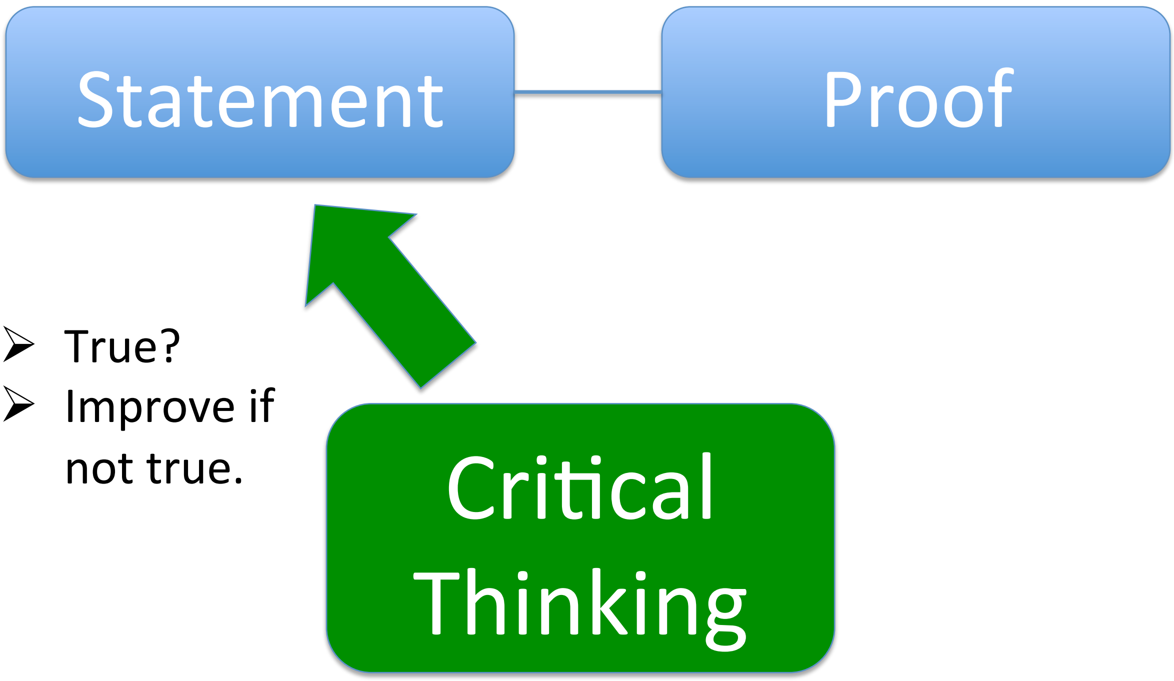


Authentic Mathematical Activity

- "[I]nformal, quasi-empirical, mathematics … grow[s] … through the incessant improvement of guesses by speculation and criticism, by the logic of proofs and refutations" (Lakatos, 1976, p. 5).

- "[W]e need to explore authentic, exciting and meaningful ways of incorporating experimentation and proof in mathematics education, in order to provide students with a deeper, more holistic insight into the nature of our subject" (De Villiers, 2010, p. 220).

Example 1



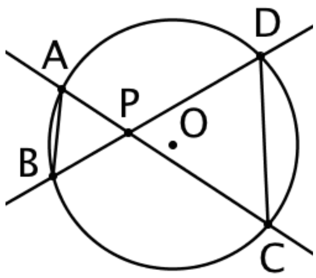
- True?
- Improve if not true.

By drawing various diagrams

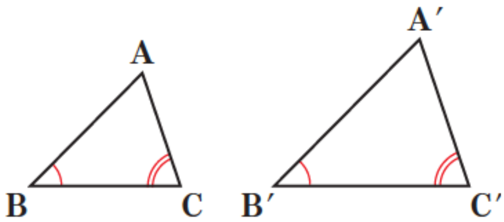
Let's try another example!

A task from textbooks

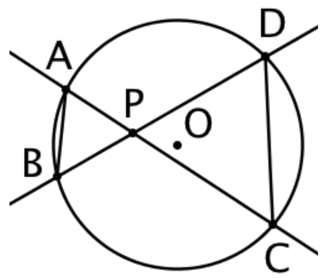
As shown in the right diagram, there are four points A, B, C, and D on circle O. Point P is the intersection point of lines AC and BD. Prove △PAB ∽ △PDC.



Using a similarity condition for right triangles: two triangles are similar if two pairs of corresponding angles are equal.



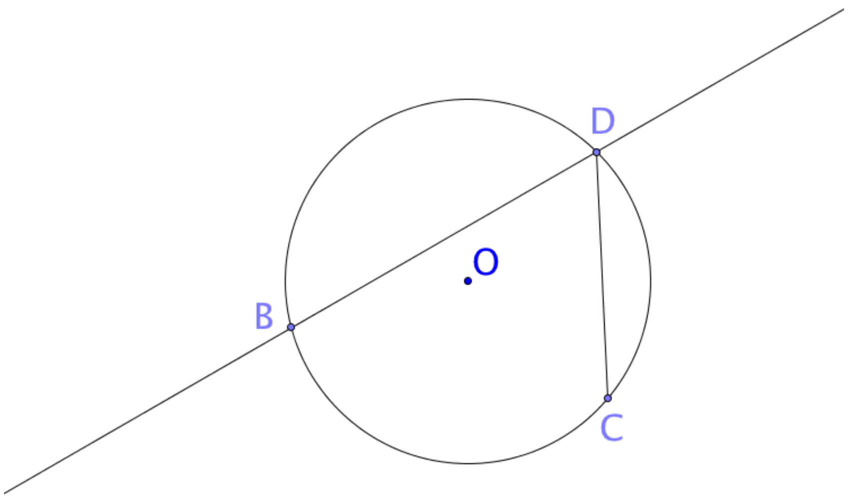
As shown in the right diagram, there are four points A, B, C, and D on circle O. Point P is the intersection point of lines AC and BD. Prove △PAB ∽ ΔPDC.

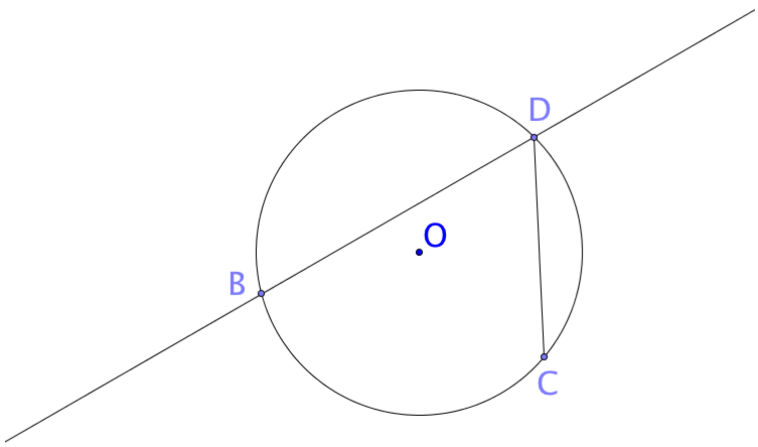


By proof, we can establish that the statement is true in general.

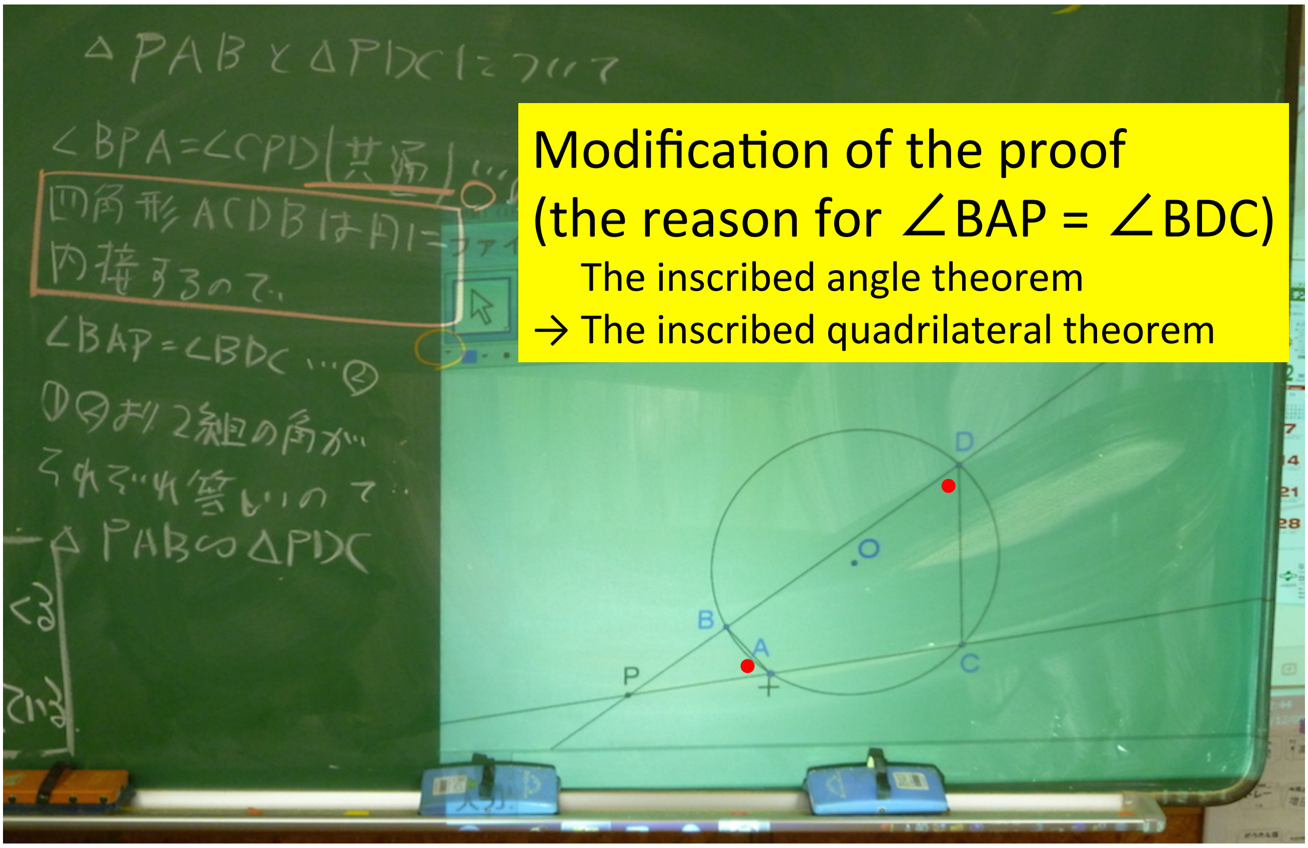
However,

- ✓ Is your proof valid for all cases?
- ✓ Let's investigate by <u>moving point A on various places</u> on circle O.





Classroom Lesson



Summary

Statement — Proof

Critical Thinking

- True?
- Improve if not true.

(Example 1)

- Valid?
- Improve if not valid.

(Example 2)

By drawing various diagrams