ICME-2012

On Geometry for Development of Critical Thinking

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Enhancing Critical Thinking with Proving in Geometry

Proving in Geometry	Critical Thinking
Establishing the universality of properties	Establishing the truth of statements logically
Clarifying implicit theorems in a proof	Revealing implicit evidences in an explanation
Organizing figural properties	Organizing knowledge
Discovering new properties based on proofs	Discovering new knowledge based on explanations
Overcoming counterexamples	Making use of proofs and refutations

Strongly Recommended to Impose Proving in Curriculum for Future Generations

Importance of Proof & Proving

The teaching and learning of proof is a key component of mathematics and thus of mathematics curricula (Hanna & de Villiers, 2008; 2012).

TIMSS 2011 results

http://timssandpirls.bc.edu/timss2011/

Country	Average Scale Score		
Korea, Rep. of	613 (2.9)	▲	
[2] Singapore	611 (3.8)	▲	
Chinese Taipei	609 (3.2)	▲	
Hong Kong SAR	586 (3.8)	▲	
Japan	570 (2.6)	▲	
[2] Russian Federation	539 (3.6)	▲	
[3] Israel	516 (4.1)	▲	
Finland	514 (2.5)	▲	
[2] United States	509 (2.6)	▲	
‡ England	507 (5.5)		
Hungary	505 (3.5)		
Australia	505 (5.1)		
Slovenia	505 (2.2)	▲	
[1] Lithuania	502 (2.5)		
TIMSS Scale Centerpoint	500		
Italy	498 (2.4)		

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Teaching and Learning a Proof in Japan

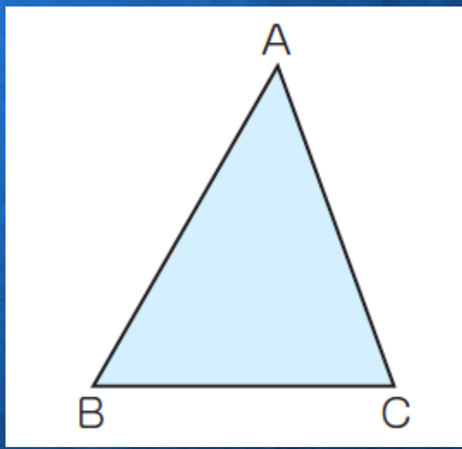
Grade	4	5	6	7	8	9
Proof	Informal				Formal	
Activity	Learning a informal proof •Thinking logically, deductively •Expressing precisely				Learning a formal proof in paragraph modes •What is it? •Why is it needed? •How to construct it?	
Ex.	◆Explain the sum of inner angel of triangle inductively and deductively				◆Prove the sum of inner angel of triangle ◆Prove the properties of triangles ◆Prove the properties of quadrilaterals	

Gap in the Geometry Curriculum

Grade	Contents	Proving
7	Plane Geometry: Symmetry, Basic constructions, Circle and sector	Informal
	Space Geometry: Solids and spatial figures, Surface area and volume of solids	
8	How to explore figures: Properties of parallel lines and angles, Properties of congruent figures, Conditions of congruent triangles	
	Proof & Proving :What is it? How to construct?	
	Figural properties and proof: Triangles, Quadrilaterals	Formal
9	Figures and similarity	
	Inscribed angle and central angle	
	Pythagoras' theorem	

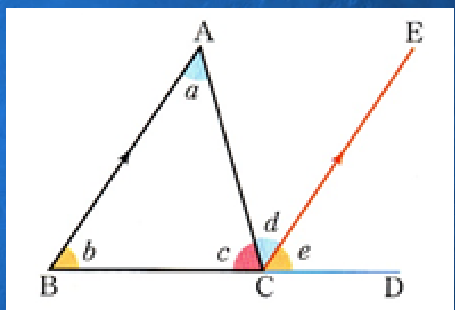
How do you prove it to establish the universality?

- The sum of the three interior angles of a triangle is 180° -



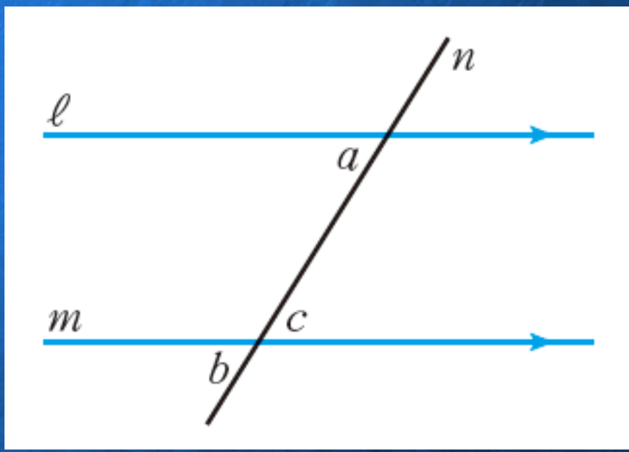
Okamoto etc. (2012). "Gateway to the future math2", Keirinkan: Osaka.

What kinds of theorems used?



$$\angle a = \angle d$$
$$\angle b = \angle e$$
$$\angle a + \angle b + \angle c = \angle d + \angle e + \angle c$$
$$\angle d + \angle e + \angle c = 180^\circ$$
$$\angle a + \angle b + \angle c = 180^\circ$$

Why is Properties of parallel lines true?



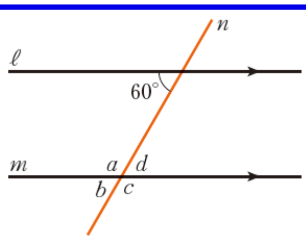
Prove ∠a = ∠C

Okamoto etc. (2012). "Gateway to the future math2", Keirinkan: Osaka.

Proofs in Textbook

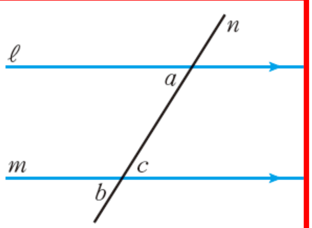
Generic Explanation

Line n is drawn across two parallel lines ℓ and m, as shown in the figure on the right. What can you say about the measure of angles ∠a, ∠b, ∠c, and ∠d?



Proof

When $\ell /\!/ m$ in the figure on the right, corresponding angles ∠a and ∠b are equal and vertical angles ∠b and ∠c are equal. This means that alternate interior angles ∠a and ∠c are also equal.
In other words,

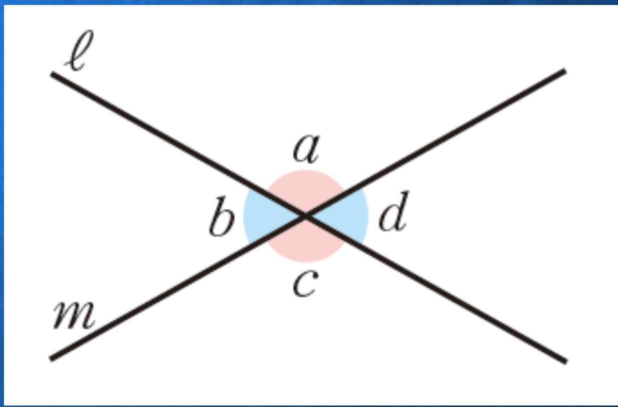


Properties of parallel lines (corresponding angles)

Vertical angles are equal

Okamoto etc. (2012). "Gateway to the future math2", Keirinkan: Osaka

Why are vertical angles equal?



Vertical angles are equal

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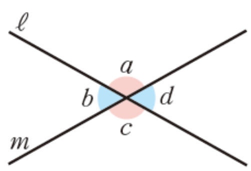
Proofs in Textbook

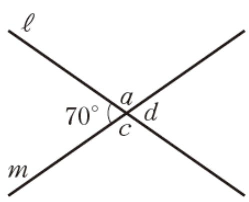
When two lines intersect as in the figure on the right, four angles are formed around the point of intersection.
Angles opposite each other, such as $\angle a$ and $\angle c$, are called **vertical angles.**



$\angle b$ and $\angle d$ are also vertical angles.

Generic Explanation

When $\angle b = 70^\circ$, $\angle a$ and $\angle c$ are both $180^\circ - 70^\circ$, so we know that $\angle a = \angle c$.



Proof

This relationship can also be expressed

$$\angle a = 180^\circ - \angle b,\ \angle c = 180^\circ - \angle b$$

So this holds true no matter what the measure of $\angle b$.

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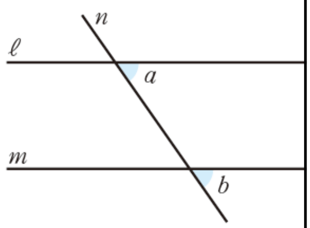
Why is it true?

Properties of parallel lines (corresponding angles)

Extend What should we do?

Use a set square to draw a line parallel to line ℓ.

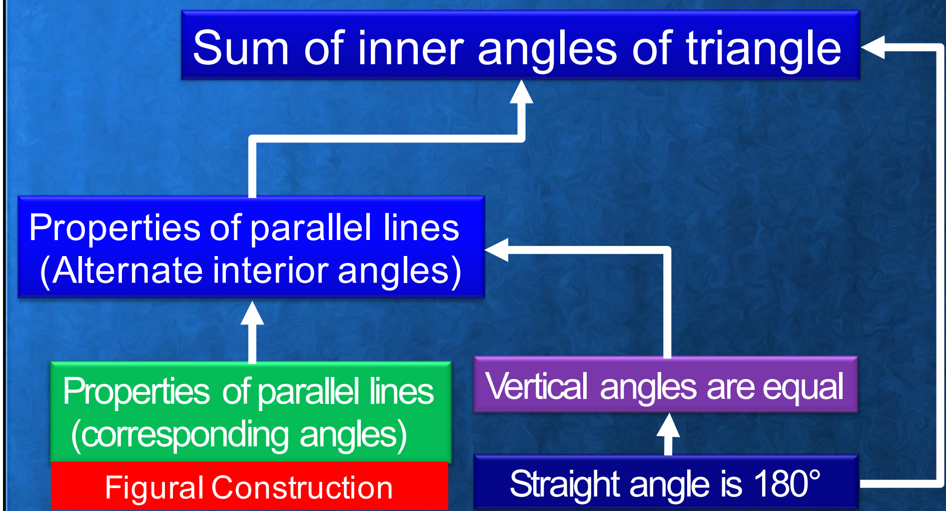
When we use the method in ✤ to draw parallel lines, we are using the fact that if corresponding angles $\angle a$ and $\angle b$ in the figure on the right are equal, $\ell /\!/ m$. In other words,



If $\angle a = \angle b$, then $\ell /\!/ m$.

Figural Construction

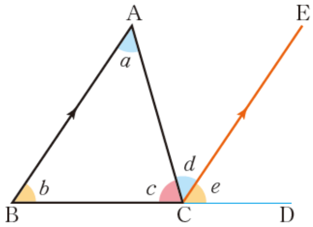
Organizing Figural Properties



What can we find from this proof?

make sure the three angles of a triangle add up to 180°.

As you can see in the figure on the right, D is on a line formed by extending side BC of △ABC. Line CE is drawn parallel to side BA and through point C.



In this case,

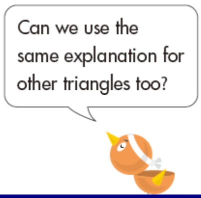
The alternate interior angles of parallel lines are equal, so $\angle a = \angle d$. ……①

The corresponding angles of parallel lines are equal, so $\angle b = \angle e$. ……②

Knowing ① and ② allows us to find the sum of the three angles of △ABC:

$$\angle a + \angle b + \angle c = \angle d + \angle e + \angle c$$
$$= \angle BCD$$

The three points B, C, and D are on the same line, so ∠BCD=180°. This means that the sum of the three angles of a triangle is 180°.



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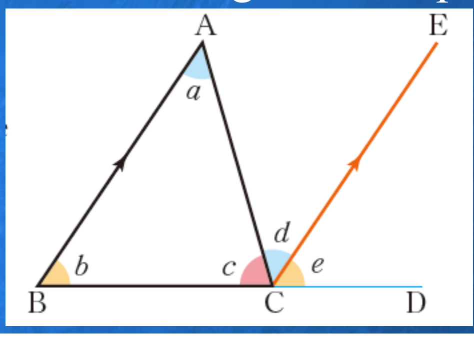
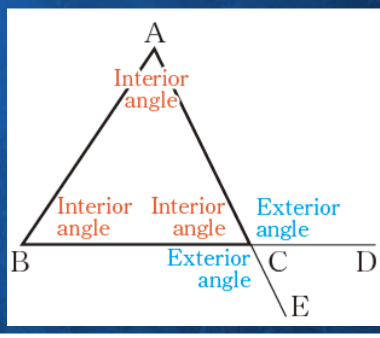
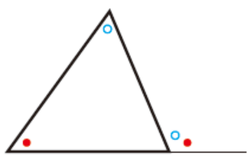
Discovering New Properties Based on Proofs





Properties of interior and exterior angles of triangles

1. The sum of the three interior angles of a triangle is 180°.
2. The measure of an exterior angle of a triangle is equal to the sum of the two non-adjacent interior angles.



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