Contributed by TOH Tin Lam
Statistics and Probability:
Exploring probability with law of large numbers and sample space K3SP2-1

Standards	Activities
Experiment with the help of coins and dice to explore the distribution of ratio and understand the law of large numbers	Experiment 1: Using Coins • Use a normal coin. Toss the coin 10 times. Record down the number of Heads (H) and the number of Tails (T) in the ten throws. • Compare with your friends who also carry out this experiment. • Compare the numbers of H and T you obtained. • Repeat the same experiment 20 times and record down the number of H and T. • Repeat the same experiment 50 times and record down the number of H and T. • What do you observe about the number of H and T you obtain for 50 throws? • What do you expect to be the distribution of H and T if you are required to throw 1000 times? What about 10 000 times? Explain. • The above may be carried out either using actual coins or using the software WINSTAT to simulate the entire processes. *Suggestion:* Carry out using actual coins for using up to 20 throws and for more throws, use the WINSTAT simulation software. Experiment 2: Dice • Use a usual die. Toss it 24 times. Record down the number of 1, 2, 3, 4, 5 and 6 that appear for the total of 24 throws. • Compare with your friends who carry out the same experiment. What do you know about the recordings by you and your friends? • Repeat the same experiment by throwing the die 60 times. Record down the number of 1, 2, 3, 4, 5 and 6 that appear for the 60 throws. What do you observe about your recordings (if any) that you observe? • Suppose you are asked to repeat this experiment 600 times. What would you expect the readings to me? What about if you were to repeat 60000 throws? • Can you explain why that is so? • Note that you can use WINSTAT to carry out the above experiment. We suggest

<table>
<tr><td></td><td>that you can use the simulation software for more than 30 throws.</td></tr>
<tr><td>Use the idea of equally likely for inference to get the value of probability</td><td>Students to re-examine the definition of probability as $\frac{\text{Total number of favorable outcomes}}{\text{Total number of possible outcomes}}$.
Consider the following scenario:
Consider throwing two fair coins (each with H and T). There are three possible outcomes: (1) 2 H; (2) 1H1T; (3) 2T. So using the above definition for probability, the probability of getting 1H1T = $\frac{1}{3}$.
Critique the above argument.
Draw a table of representation as follows. Based on the table, what is the probability of getting 1H1T?<table><tr><td>1^{st} \ 2^{nd}</td><td>H</td><td>T</td></tr><tr><td>H</td><td></td><td></td></tr><tr><td>T</td><td></td><td></td></tr></table>
Critique what is incorrect with the above argument proposed.</td></tr>
<tr><td>Analyze the situation with tables to represent the sample space and using it for predicting with probability.</td><td>Consider the Monty Hall Problem
There are 10 doors. A prize is found behind only one of the doors. You choose one door. However, after this, the deejay will open up eight doors behind which there are no prizes. You are given one choice: to keep to your original choice or given a chance to change your decision (this would mean the door that is still not open). Would you want to use the second chance to change the other choice? Use a diagram to justify your choice.
There are 3 doors. A prize is found behind only one of the doors. You choose one door. However, after this, the deejay will open up one door behind which there is no prize. You are given one choice: to keep to your original choice or given a chance to change your decision (this would mean the door that is still not open). Would you want to use the second chance to change the other choice? Use a diagram to justify your choice.
How has the sample space change in the process? How has your probability of winning changed?</td></tr>
</table>