

Lesson Plan for Mathematics (4th Grade)

Teacher: Hiroshi TANAKA (Elementary School, University of Tsukuba)

Theme	How to Construct Lessons that Get Students to Enjoy Observing Phenomena, and Discovering and Verifying Regularities in Them
-------	---

1. Subject: The secrets of the subtraction relay (using three-digit numbers)

2. About the research theme

In studying calculation, repetitive drills are often used to achieve the demanded level of skills. While there is no doubt that the ability to do basic calculations is important, focusing only on calculations may result in building up a mistaken image in children that "Mathematics abilities are the same as calculation skills."

While doing calculation exercises is essential, the author wants to improve the students' "number sense" and also wants students to appreciate discovering regularities through inductive thinking.

In the case of using 3, 5, and 7:

$$\begin{array}{r} 753 \\ -357 \\ \hline 396 \end{array}$$

In this lesson, we use Kaprekar's constant, a mysterious regularity in subtraction, to make students relive the process of discovering a kind of interesting regularity in the world of numbers that mathematicians of past appreciated at the same age.

Then, we repeat the same calculation using the numbers 3, 9, and 6 from the result. Repeating it several times always gives 495 at the end.

First, students pick three cards from a set of cards with numbers from 1 to 9. Out of the numbers that can be created by arranging these cards, the students subtract the smallest such number from the greatest such number. For example, in the case of 3, 5, and 7, students would subtract 357 from 753 as shown above. The result is 396.

Next, repeat the same subtraction using 3, 9, and 6. Using cards, students continue to do this kind of subtraction repetitively. While repeating the subtraction, they should notice that they all get the same number, 495. In fact, no matter what numbers they used in the beginning, with three digits the result is always 495. This is called Kaprekar's constant.

Look at the results of the subtractions. These numbers share some additional kinds of interesting regularity. First, students notice that tens digit is always 9. They should also be able to explain the reason for this. Moreover, they will probably notice that the hundreds digit and the ones digit always make 9.

Students who find such discoveries interesting would normally want to expand their discoveries by trying the same thing with two- or four-digit numbers. In fact, for two digits, the final result will be 9, and for four digits, it will be 6174.

The author aims to use this lesson so that students can experience discovering attractions of the world of numbers, although they think calculation is boring.

Due to the special status of this lesson as a one-time lesson, this subject has been chosen as a means of developing students' "ways of thinking for inductive discoveries." In this lesson, utilizing the method for subtracting three-digit numbers, we aim to get students to enjoy intellectual discoveries in the world of mathematics.

3. Development

(1)Objective

After repeating the "three-digit subtraction relay," students look back on the results of their calculations, and discover regularities in them by comparing the results for different numbers.

(2)Development

Contents	Teacher's attitude to support students' activities
<p>Trying the "subtraction relay": Understanding the rule and doing the calculations. Example: Using 3, 5, 7</p> $\begin{array}{r} 753 \\ -357 \\ \hline 396 \end{array} \quad \begin{array}{r} 963 \\ -369 \\ \hline 594 \end{array} \quad \begin{array}{r} 954 \\ -459 \\ \hline 495 \end{array}$ <p>T: Who can do the most calculations? C: Huh? C: I get the same result every time.</p> <p>Trying it starting with different numbers: T: Maybe we picked the inappropriate numbers the first time. Let's try it with other numbers. C: I still get the same result. C: That's strange... but interesting.</p> <p>Looking through the calculations done by students to find regularities: C: It's always the same. T: What is it that's always the same? C: I always get the same results. C: The middle number is always 9. C: The hundreds digit and the ones digit always make 9.</p> <p>Looking back on the calculations on the blackboard and summarizing the discoveries of the lesson.</p>	<p>The teacher and a student should do the calculations at the blackboard together to make sure that all students understand how to do it.</p> <p>Students should start with the example shown on the blackboard. When they get 495, they realize that the same calculation is repeated.</p> $\begin{array}{r} 954 \\ -459 \\ \hline 495 \end{array} \quad \begin{array}{r} 954 \\ -459 \\ \hline 495 \end{array}$ <p>This confuses students. Suggest them to try again with different numbers. Some students may not be good at subtracting three-digit numbers. Depending on their level, including some brush-up may be necessary.</p> <p>A larger amount of data is needed for students to be able to discover the regularities by themselves. Make considerations so that many calculations remain on the blackboard.</p> <p>To make it easier to look through the results, calculations should be written on cards.</p> <p>Suggest considering the cases for two and four digits before finishing the class.</p>