# The education of all young people in the 21st Century requires a strong definition of numeracy

**Max Stephens**
*The University of Melbourne, Australia*

*Second International Education Cooperation Symposium, Tokyo 23 January 2005*

# Outline of presentation

- Defining "numeracy". What does this term mean for school education and for life beyond school education?
- Strong and weak definitions of numeracy ("old thinking" and "new thinking"). What is the reason for this change?
- Some practical illustrations of what it means to be numerate in the 21st century.

# *Part 1*

## Defining Numeracy

# Two key themes

- A major policy objective of all governments is to ensure that all students attain sound foundations in literacy and numeracy
- In our increasingly globalized economies, we cannot have one definition (**実り多い**) of numeracy for rich countries and another definition of numeracy (**粗末な**) for those in poor and developing countries

# Numeracy – a new term

- *Numeracy* is a recent concept that has different meanings for different people. The term “numeracy” first occurred in England in the Crowther Report (1959)
- It was re-defined in the Cockroft Report (1982) as “the skills and dispositions needed by ordinary people in work and daily life(**万民が仕事や生活で必要とする技能と能力**)”

# Mathematical literacy

- In the USA, some mathematics educators and politicians(**行政官**) prefer terms like “quantitative literacy(**量的識字力**)” or “mathematical literacy(**数学的識字力**)”
- The focus is the same: to be able to use the mathematics one has learned: in life, at home, in paid work, in further study, and in community and civic life

# OECD – PISA (2000)

- PISA assessment focuses on young people's ability to apply their knowledge and skills to real-life problems and situations
  - Are students able to analyse, reason and communicate their ideas effectively?
  - Do they have the capacity and strategies to continue learning throughout their lives?

# OECD – PISA (2000)

- The OECD considers that
  - mathematics, science and technology are sufficiently pervasive in modern life that
    - personal fulfilment, employment and full participation in society increasingly require an adult population, which is not only able to read and write, but also *mathematically, scientifically and technologically literate*
- Mathematical literacy = numeracy

# PISA defines *mathematical literacy*

- as mathematical knowledge and skills for life …
- as an individual's capacity
  - to identify and understand the role that mathematics plays in the world
  - to make well-founded judgements and
  - to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen

# Numeracy – an Australian definition

*Numeracy=everyone’s business (1997)*, which was funded by the Australian Government, provides a definition which underpins current policy:

To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life

**数量図形的思考力を備えているとは、自宅での生活、そして職業、コミュニティや市民としての生活する際の、一般的な課題に遭遇した際に数学を、効果的に利用することである。**

(Numeracy=everyone’s business, p. 10)

# What is numeracy about?

- The United Kingdom focus of numeracy has been largely on number and measurement.  But now embraces space, and data as well
- In Australia, *Numeracy=everyone's business (1977)*, for example, acknowledges that numeracy must encompass **all the key concepts and skills of mathematics (数学のすべての鍵概念･技能)**.
- Research shows that many students are unable to take their mathematics beyond the mathematics classroom
  - numeracy must pervade (**浸透する**) all areas of the curriculum

# Numeracy and success at school

- Students who cannot make connections between the mathematics they have been taught and the mathematical knowledge they need for their other school subjects
  - are the most likely to ‘drop out’ of high school
  - are also most ‘at risk’ of being unemployed after leaving school

# Three different emphases

- *Early years of school* - the focus of ‘numeracy’ is on building understanding of key ideas of number and measurement
- *Middle years of school* - the focus of ‘numeracy’ expands to include making links between mathematics and other school subjects and its uses beyond school

# Three different emphases

- *Senior high school* - how well do young people use and apply their mathematical knowledge and skills to a range of contexts they are likely to meet in their other school subjects, in further study and training, and in community life
- This view of numeracy is used in *OECD's* PISA study (results for 2003 just released)

# *Part 2*

## Strong and weak definitions of numeracy or mathematical literacy

# Public discourse

- The word “Numeracy” is an artefact (**人工的所産**) of public debate (**政治的論争**) to draw attention to those aspects of mathematics that are considered important for all students regardless of their intended specialisation
- In Japanese, the closest equivalents may be “suryo zuke tekina shikoryoku” **(数量図形的思考力)** or Basics ‘Kiso Kihon’ in the words of the National Course of Study

# Strong and weak definitions

- The word, ‘numeracy’ belongs to the sphere of basic education and to the compulsory years of schooling
- There are strong (**実り多い**) definitions of numeracy, and weak/narrow (**粗末な**) definitions
- In countries, where the compulsory years of schooling are limited to the elementary school, there tends to be a limited or weak (**粗末な**) definition of numeracy
- There students require basic primary education and basic numeracy (number, measurement and money) to perform basic tasks in one’s job and at home (**= numeracy: weak definition**) (**粗末な**)

# Do we want two definitions?

- As a society's expectations of general or basic education increase, definitions of numeracy become stronger (**実り多い定義が求められる**)
- Two definitions of numeracy divide rich and poor countries: strong definition (**実り多い**) for developed countries and a weak/narrow definition (**粗末な**) for developing countries
- Two definitions perpetuate (**永続させる**) inequalities, especially economic and educational imbalances

# Changing economies - Japan

- In Japan between 1990 and 2000, the percentage of labour force engaged in
  - agriculture and related industries fell by 28%
  - manufacturing and mining fell by 12%
  - knowledge and service related industries increased by 15%
- knowledge and service related industries now constitute 65% of the total labour force (Labour situation in Japan, 2003)

# Changing economies - Thailand

- In Thailand between 1990 and 2002, the percentage of labour force engaged in
  - agriculture fell from 64% to 36%
  - manufacturing & mining rose from 10% to 16%
  - knowledge and service related industries increased from 26% to 48%
- knowledge and service related industries now constitute nearly half of Thailand's labour force

(Asian Development Bank: Key Indicators of Developing Asian and Pacific Countries, 2003)

# Changing economies

- In many countries, manufacturing and agriculture are becoming less labour intensive and more capital intensive
- Capital intensive work requires fewer people and higher mathematical literacy
- The growing knowledge and service related industries also require higher mathematical literacy

# Changing work patterns

- A working life in the 21st century will not be spent in one job, or with one employer, or with one company, like Japan in the 1980s (Horio, 1997)
- Today’s generation of young workers will need to be retrained in the skills the need and may change their work several times in a working life (Morishima, 2000)

# An ‘old’ world view

- Agricultural and industrial societies could allow many students to drop out of school mathematics.
- There were always plenty of low skilled jobs for these students to undertake
- In these societies, it was sufficient to select those who could work in technology, engineering and other careers which required a strong mathematical preparation

# An ‘old’ world view

- The education system was designed to identify those who were likely to succeed in these “elite” careers
- Schools adopted a pyramid approach to mathematics, selecting and rejecting, and expecting that many students would fail along the way (**人々を序列化し少数の成功者と多数の失敗者をその過程で選別する**) (see Uzawa, 2000)

# An ‘old’ world view

- Mathematics was used to select students at the end of primary school for a limited number of places in secondary schools
- In these societies, people also entered a stable labour market (**安定した労働市場**) where jobs were well defined and where a person’s entire working life might well be spent in one job or with one company
- As a result, people could be trained in the skills required to do their work over the course of a working life

# Rapid social change

- Information and service driven societies require quite different knowledge bases.  Their labour base is structured quite differently.  There are fewer and fewer unskilled jobs.
- Manufacturing and agriculture are both becoming capital(**資本**) intensive and less labour intensive.
- Capital intensive work requires people with high ability to process numerical, statistical, and graphical information (**= numeracy: strong definition**) (**実り多い**).

# Rapid social change

- Today, the labour market is not so stable
- Jobs can disappear or can be re-defined in terms of the necessary knowledge and skills required to carry them out
- There is a rapid growth in part-time work, “on-call work”, self-employed work and in small businesses that respond to market forces
- The school system needs to prepare young people for a very different world of work from that which their parents knew

# Mathematical literacy

- PISA 2003 used a strong framework for mathematical literacy in terms of:
  - mathematical content:
    - quantity
    - space and shape
    - change and relationships, and
    - uncertainty

# Mathematical literacy

- PISA 2003 used a strong framework for mathematical literacy in terms of:
  - situations or contexts where mathematics is used
    - personal
    - educational/occupational
    - public
    - scientific

# Mathematical literacy

- PISA 2003 used a strong framework for mathematical literacy in terms of:
  - mathematical competencies
    - thinking and reasoning
    - argumentation
    - communication
    - modelling
    - representation
    - problem posing and solving
    - using symbolic, formal and technical language and operations

# *Part 3*

Some illustrations of mathematical literacy needed for the 21st Century

# From the middle school

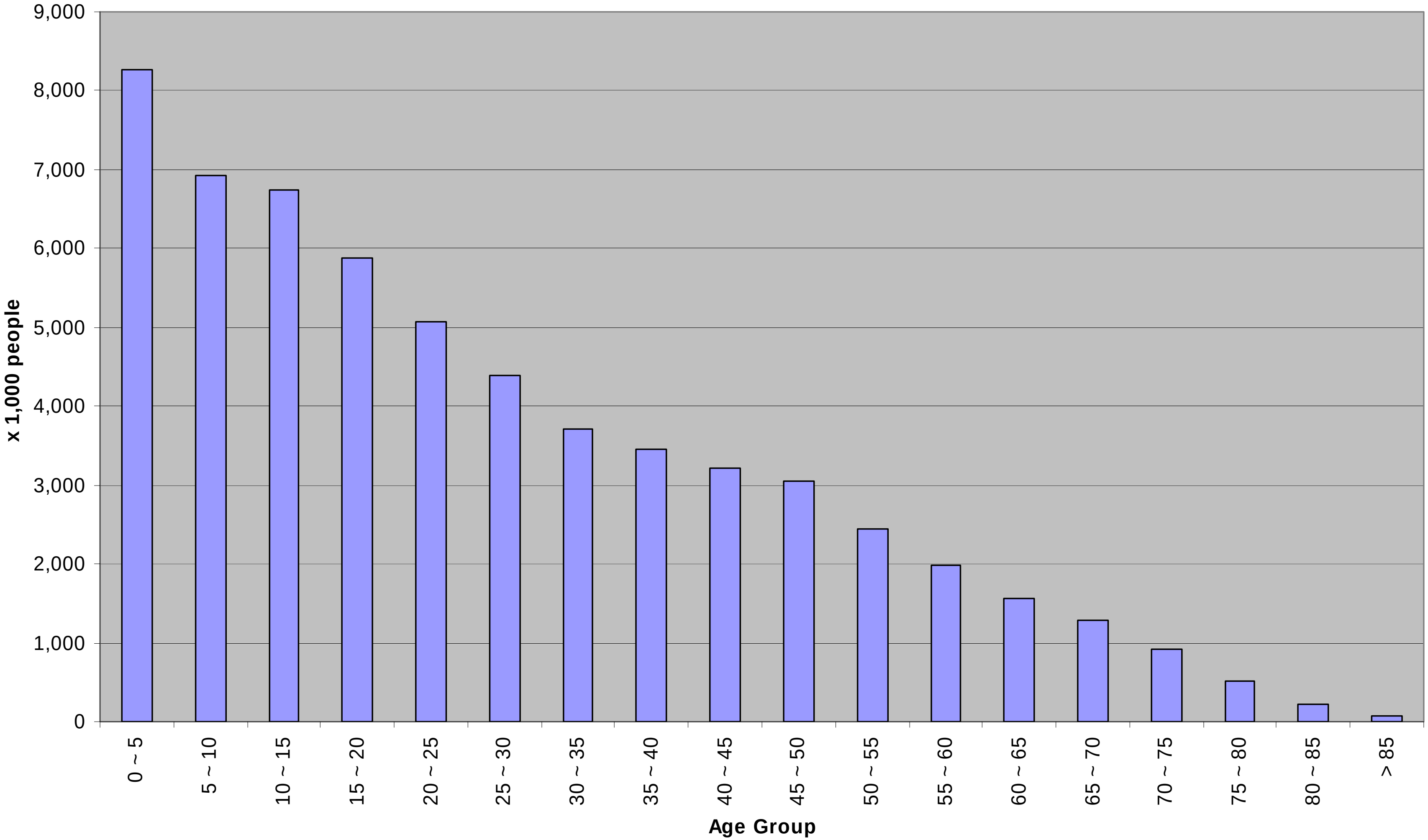
Investigating Japan’s population

3-4 lessons

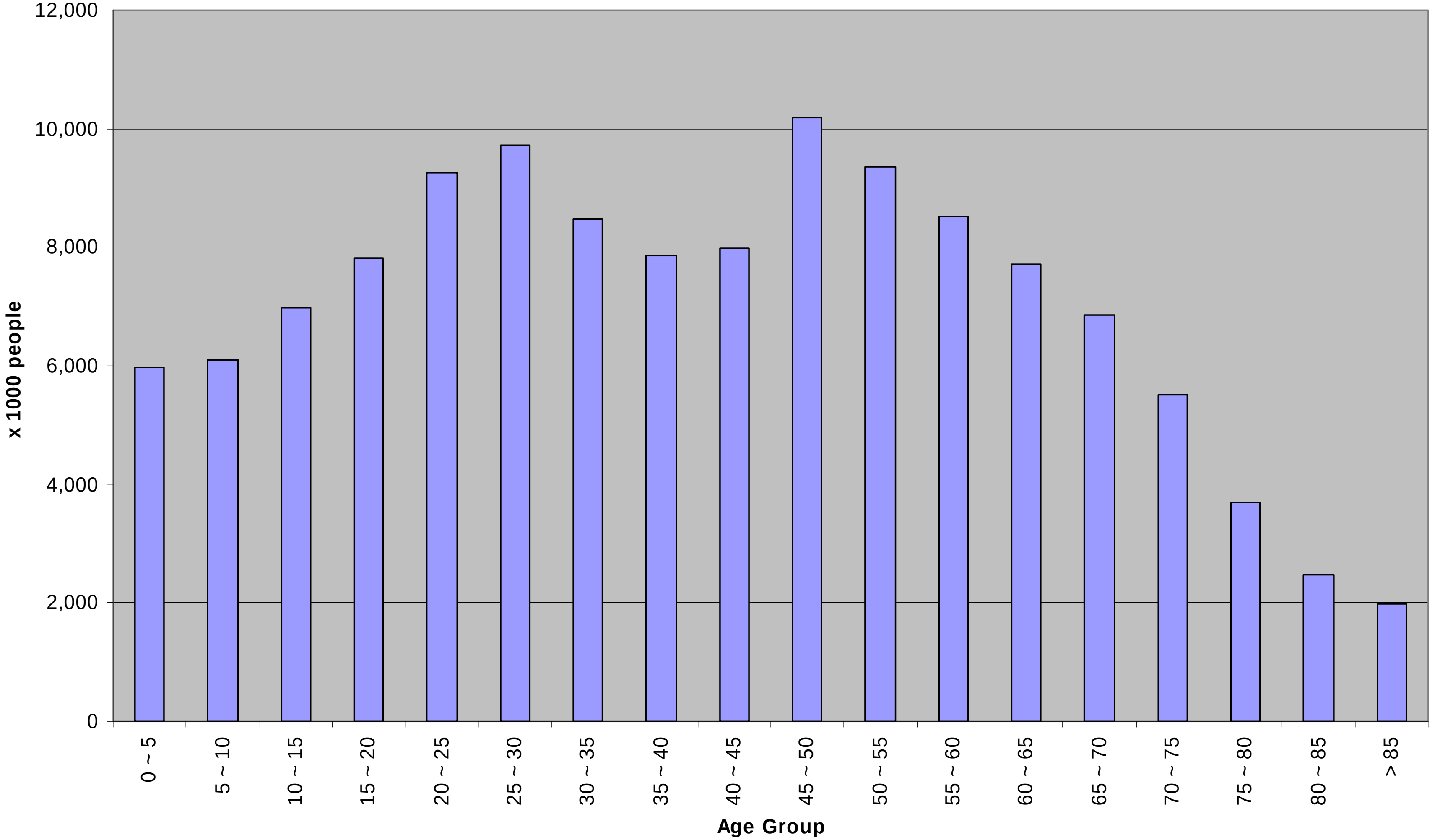
# Population of Japan

- From the newspapers and from their social studies class, students know that Japan’s population is “ageing”
- To investigate what this means the students looked at Japan’s population in 1925 and 1998
- They used EXCEL to make a graph from the numerical data

# Population of Japan in 1925



# Population of Japan in 1998



# Japan’s population

- Students noticed very different shape of the two graphs
- Median age in both cases is quite different
- They found population data for Australia (1921 & 1998) and USA (1925 & 1990)
- They used technology to create graphs for Australia and USA

# Japan’s population

- Size of population is different, but students compared the *shapes* of the graphs from the 1920s and 1990s
- Japan, Australia and the USA all have a similar “population profile” in the 1920s
- Students explained what was similar from the graphs from the 1920s

# Japan’s population

- In the 1990s the profile of Japan’s population is very different from Australia and the USA
- Students identified key differences
- What problems are likely to arise for Japan in the future?
- What predictions can be made?

# Japan’s population

- Technology helps students to present complex information in a way that they can see the problem and make some analysis
- At the conclusion of their study
  - Students provided some feedback to the teacher at the end of this topic
  - Their teacher gave students some further questions to consider

# From the middle/senior high school

Looking at the environment

2 lessons

# Reducing Bluegill population in Lake Biwa

- Bluegill fish have become a problem in Lake Biwa
- Shiga prefecture has a plan to catch a certain number each year to reduce the population
- Students in Years 8 and 9 at Tennoji Junior/senior high school investigated this topic (ICME-10: Yanagimoto, 2004)

# Reducing Bluegill population in Lake Biwa

- Students used graphics calculators to do the mathematical modelling of this theme:
  - According to the fishery section of Shiga prefecture in 1999, it is estimated that there are between 1500 *t* and 3000 *t* of bluegills in Lake Biwa
  - They plan to catch 300 *t* each year to halve the population of blue gills in 10 years
  - The budget for this is ¥ 55 000 000

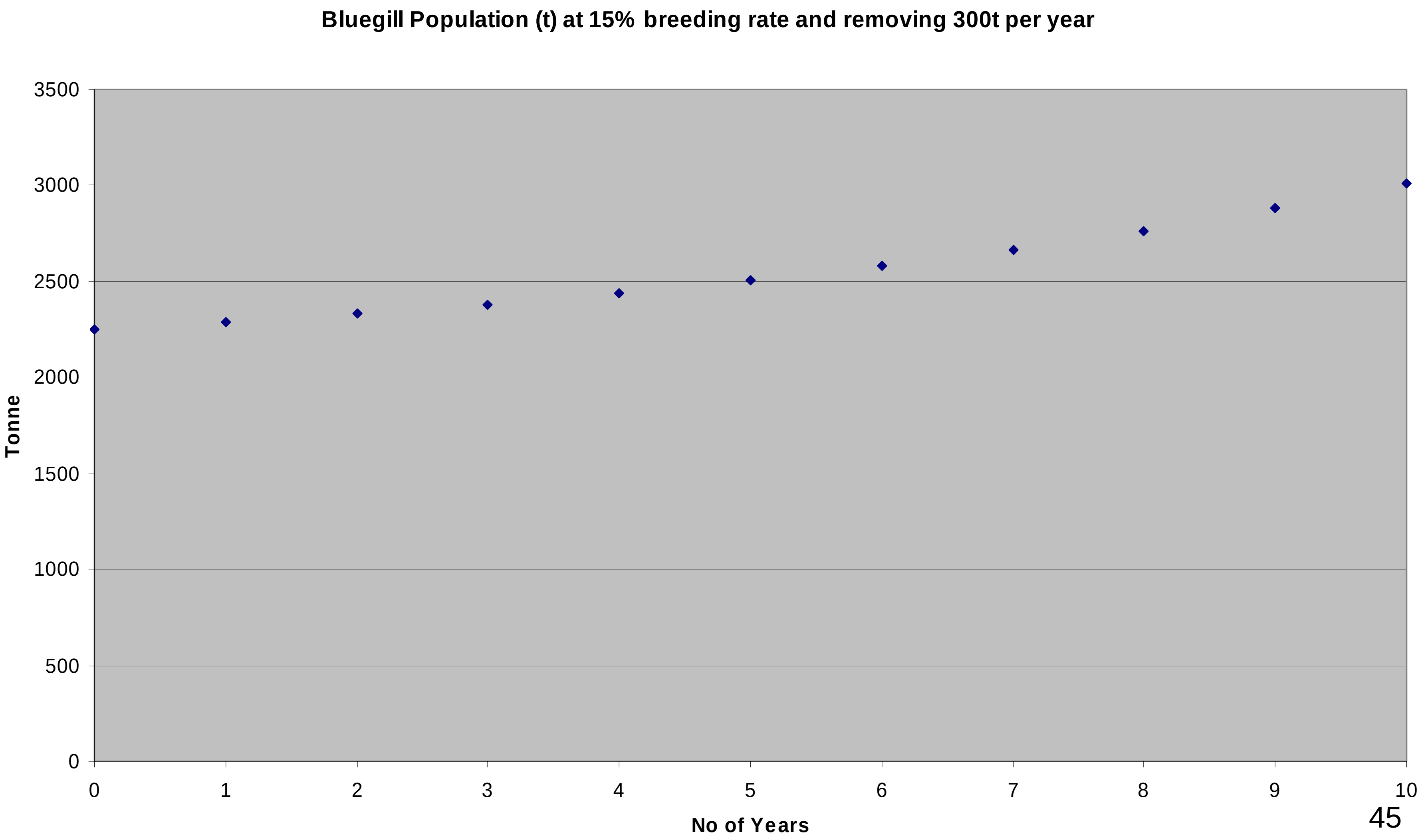
# Reducing Bluegill population in Lake Biwa

- Assumptions used by the students in their model:
  - (1) the present population of bluegills in Lake Biwa is between 1500 *t* and 3000 *t*
  - (2) the breeding rate of bluegills is constant
  - (3) the capture of bluegills is 300 *t* every year
  - (4) other conditions are not considered

# Reducing Bluegill population in Lake Biwa

- Students used Casio graphic calculator fx-9700GE and its table and graph menu
- The students initially
  - Used 15% as the breeding rate of bluegills,
  - Input the recurrence formula
    - $A_{n+1} = A_n + 0.15\, A_n - 300$
    - estimating the present amount of bluegills at 2250 *t* ($A_0$) as the average of 1500 *t* and 3000 *t*
    - Let n be from 0 year to 10 years

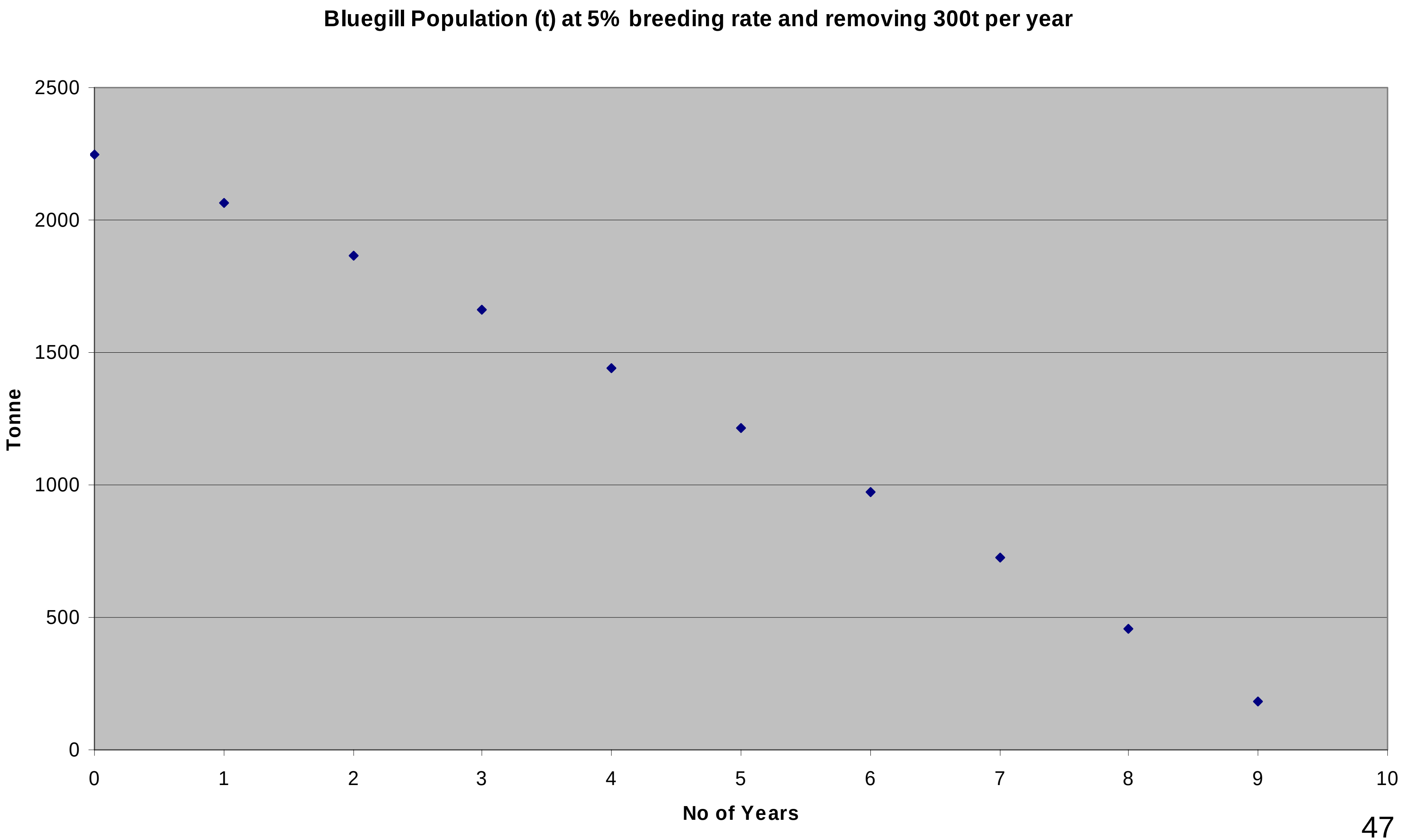
# Reducing Bluegill Population in Lake Biwa



# Reducing Bluegill population in Lake Biwa

- The graph shows that with 15% breeding rate the population of bluegills increases
- Next, the students took 5% as the breeding rate of bluegills and used the same recurrence formula
- In this case, the population of bluegills decreases immediately and becomes zero after nine years

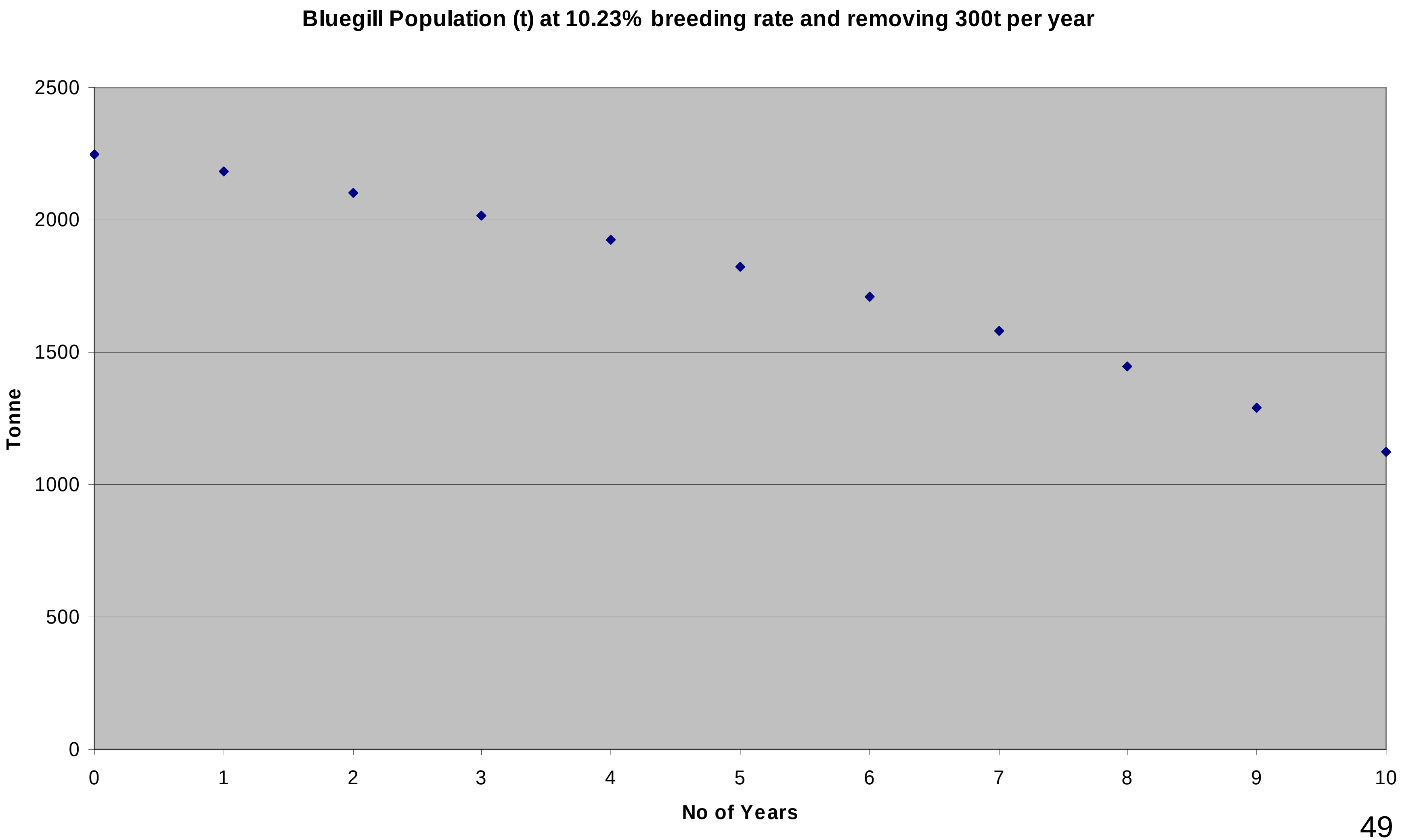
# Reducing Bluegill Population in Lake Biwa



# Reducing Bluegill population in Lake Biwa

- Students then tried various breeding rates
- They showed that if the breeding rate is 10.23% the number of bluegills becomes half in 10 years
- Students then did some extensions.
- They used 1500 *t* and 3000 *t* separately as the actual population and combined these numbers with different breeding rates

# Reducing Bluegill Population in Lake Biwa



# Reducing Bluegill population in Lake Biwa

- Students could quickly create different graphical representations using fx-9700GE
- Students concluded that
  - mathematics is useful (essential?) in investigating environmental problems
  - validity of the model depends on the accuracy of the breeding rate and population estimate
  - more detailed modelling may be needed to solve the problem

# Conclusions

- In these examples of mathematical literacy
  - Focus is on using mathematics to investigate a practical problem
  - Using mathematics to create an image or representation of reality
  - Access to technology is important
  - The most difficult mathematics is not always required

# Conclusions

- In these examples some considerations become very important:
  - What is the quality and reliability of the data?
  - Does the real situation need to be simplified?
  - What results are given?
  - What predictions can be made?
  - What limitations or restrictions need to be placed on these predictions?

# Summary

- In Japanese, there may be no exact equivalent for the term “Numeracy”
- “Mathematical literacy” as used by PISA may be more useful expression
- In the past, numeracy focussed on calculation and simple measurement
- This weak definition is not sufficient for young people in the 21st century

# For the 21st century

- Representing, interpreting and analysing quantitative information is central to mathematical literacy (numeracy)
- Computer and hand-held technology makes *representation* easier and quicker
- In an information society, representing and analysing information is a key element of mathematical literacy