SEAMEO School Network Program provided by University of Tsukuba

Mathematics Education to Develop Students Agency: The Case of Fractions

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why? II

Lesson 1: What is fraction?

Lectured by: Masami Isoda, Prof/PhD., University of Tsukuba, Japan

With support of: Raimundo Olfos, Prof/PhD., and Soledad Estrella, Prof/PhD., Pontificia Universidad Catolica de Valparaiso, Chile

With participation of: Ms. Laura Lopez Zarate, Colombia

Ms. Mei Nakada, Japan

Mr. Diego Solis, Costa Rica

筑波大学 University of Tsukuba

Review

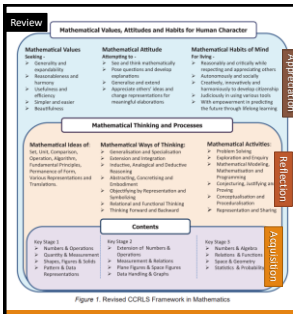


Figure 1. Revised CCRLS Framework in Mathematics

On OECD Education for 2030 (2018), Student Agency means students who "Learned how to learn".

Developing students who learn mathematics by and for themselves.

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why?

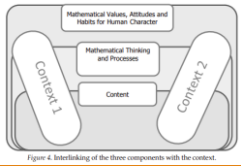


Figure 4. Interlinking of the three components with the context.

Chapter 1: What is fraction?

The professor asked the teachers:

There is 2m tape. Where is the position of ⅔ m? Show it by ↓

0m 1m 2m

Isoda, M. (1993)

Possible Answers:

Som: 0m 1m 2m ↑

Any: 0m 1m 2m ↑

What do you want to do next? Isoda & Katagiri (2012)

I would like to ask why?

Yes, we would like to discuss!

Mathematical Values
Seeking -
- Generality and expendability
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

Mathematical Attitude
Attempting to -
- See and think mathematically
- Pose questions and develop explanations
- Generalise and extend
- Appreciate others' ideas and change representations for meaningful elaborations

Page 7.a

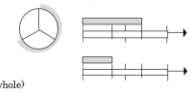
Professor: You are good teachers, aren't you? Because you already have the custom to ask why to others and discuss. It means that you usually engage in a similar activity by yourself. You already have the mind set for learning how to learn by yourself!

Discussion 1: Why do you think so?

Som: It is larger than 1m because 2/3 means:

Ano: No, the whole is divided by 3, then shaded the 2 third of 3 parts

Som: Yeah, I should draw like that.



Any: Wow, 2/3 is less than 1. It is like this:

Som & Ano: ..continue..(talking about part-whole)

Mathematical Attitude
Attempting to -
- See and think mathematically
- Pose questions and develop explanations
- Generalise and extend
- Appreciate others' ideas and change representations for meaningful elaborations

Page 7.b

Any: ...continue... (talking about part-whole)

Prof: I could not understand well all of your explanations and diagrams. What is the original question?

Any: Where is the 2/3m?

Prof: Yes, then, the denomination 'meter' is missing in all of your explanations. We should write the meter on the number line. Can you explain 2/3m, again?

Any: It is 2/3 of 1m

Som: It is 2/3 of 2m

0m 1m

Prof: Now we can discuss what differences are there between these two ideas.

The denomination of quantity is important. When we explain something with the situation, we usually omit some words which are already known in the situation. In this case, the unit of meter 'm' itself is written in the task; however the meter was missing from their explanation and diagrams. In their discussion, if the denomination of quantity is missing, we are not sure which part you are explaining.

People who participated in the discussion; however, not sure which answer is appropriate.

Mathematical Ideas of:
Set, Unit, Comparison, Operation, Algorithm, Fundamental Principles, Permanence of Form, Various Representations and Translations.

Mathematical Ways of Thinking:
- Generalisation and Specialisation
- Extension and Integration
- Inductive, Analogical and Deductive Reasoning
- Abstracting, Concretising and Embodiment
- Objectifying by Representation and Symbolizing
- Relational and Functional Thinking
- Thinking Forward and Backward

Page 8.a

Then, Professor asked.

What is Fraction?

Discussion 2: What is Fraction?

Ano: It is a part of the whole.

Prof: It is not clear. Could you explain it more exactly?

Ano: When the whole is divided into parts, fraction is the number of the parts in the whole.

Prof: Still are missing some important terms. Can anyone support?

Oth: When the whole is divided into 'm equally parts', 'fraction n/m' is 'n pieces of the parts in the whole'.

Any: The whole for 2/3m is 1m.

Som: The whole for 2/3m is 2m.

What is the whole, here?

Mathematical Ideas of:
Set, Unit, Comparison, Operation, Algorithm, Fundamental Principles, Permanence of Form, Various Representations and Translations.

Page 8.b

Discussion 3: Which one should be the whole?

Any: 1m

Som: 2m

Ano: 1m or 2m, which one shall we chose for the whole, in this task?

Any: The whole for 2/3m is 1m

Som: The whole for 2/3m is 2m.

Oth: How can we discuss?

In mathematics, generally applicable idea is strong. For checking it, we have to think 'For Example, ..."

Mathematical Values
Seeking -
- Generality and expandability
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

Mathematical Attitude
Attempting to -
- See and think mathematically
- Pose questions and develop explanations
- Generalise and extend
- Appreciate others' ideas and change representations for meaningful elaborations

Page 9.a

Discussion 4: For Example, if............

Prof: For example, if I change the task from 2/3m to 1/2m what will happen?

Any: The 1/2m is 0.5m.

Som: The 1/2 of 2m is 1m.

Ano: No, what you are saying is that 1/2m is 1m. However, 1/2m is 0.5m, isn't it?

Pro: If I change the original question '2/3m' to '1/1m' what will happen?

Oth: If Som's idea, 1/1m is the 1/1 of 2m, thus 2m. 1/1m =2m. It is strange.

Prof: Yes, we can generalize Any's idea and not generalize Som's idea. When we say 2/3m, the whole is denominated by the 2/3 'meters': The unit of meters is 1m in any time.

What did you learn from this class? Let's write your own resume based on what you learned.

Page 9.b

Questions for professional development 1

Q1. Why did the author choose this story as for the introductory chapter?

Q2. Professor asked several questions in the class. Which question is most important in this class? Why do you think so?

Q3. Do you think the definition of fraction in the class is appropriate? Why do you think so?

Q4. Explain the class using the term of appreciation.

Q5. If you conduct this task in your class or your teacher training program, what is your objective?

Q6. In your class, do you ask your students or teachers 'what do you want to do next?'. If you do ask, when do you ask it? If you do not ask, why?

Page 10.a

Q7. Please explain the professor's questioning and values from the viewpoint of the following.

Three Major Objectives for Education as for Future Preparation

1. Human Character Formation ∋ {Developing Mindset, Attitude, Value} *appreciation*
2. Learning How to Learn ∋ {Knowing how to develop and reconstruct} *reflection*
3. Knowledge and Skills ∋ {Understanding and Proficiency} *acquisition*

Dizon, D., Ahmad, J., Isoda, M. (2017)

Q8. What is the explanation in mathematics at elementary school level? Using the discussion in class, please explain what it is from the following three perspectives.

Explanation of:
1. Meaning, such as the base for the reasoning using different representation;
2. Significance or Objective, why I would like to think such a way;
3. Procedure, how I did.

Isoda, M.(2008, 2009)

Q9. In the argumentation, for progressive dialectic, the way of discussion below is known as meaningful: 'If your saying is true, what will happen?'
In the class, Professor has used this dialectic method. Where did he use it?

Page 10.b

References

Isoda, M. (1993). The logic for Argumentation in Mathematics Classroom. (written in Japanese:礒田正美(1993).算数授業における説得の論理を探る.北海道教育大学教科教育学研究図書編集委員会編.教科と子どもとことば：言語で探る教科教育.東京：東京書籍) English translation: Isoda, M. (2008). Getting Others' Perspectives through the Hermeneutic Effort: Paper presented at TSG 26, ICME11 on *http://tsg.icme11.org/document/get/803*

Isoda, M. (2008). Teaching Mathematics through Argumentation for Junior High School. *(written in Japanese:* 礒田正美(2008). 思考・判断・表現による『学び直し』を求める数学の授業改善：新学習指導要領が求める対話:アーギュメンテーションによる学び方学習.東京：明治図書出版) A theory for dialectic communication to develop mathematics knowledge and thinking was in English: Isoda, M. (2007). Developing Mathematical Thinking in Classroom, APEC Project report on *http://www.criced.tsukuba.ac.jp/math/apec/apec2007/progress_report/specialists_session/Masami_Isoda.pdf*

Isoda, M. (2009). Teaching Mathematics through Argumentation for Elementary School *(written in Japanese:* 礒田正美(2009). 思考・判断・表現による「学び直し」を求める算数の授業改善：新学習指導要領が求める言語活動：アーギュメンテーションの実現. 東京：明治図書出版) A theory for dialectic communication to develop mathematics knowledge and thinking was in English: Isoda, M. (2007). Developing Mathematical Thinking in Classroom, APEC Project Report on *http://www.criced.tsukuba.ac.jp/math/apec/apec2007/progress_report/specialists_session/Masami_Isoda.pdf*

Isoda, M. & Katagiri,S. (2012). Mathematical Thinking: How to Develop it in the classroom. NJ: World Scientific. The part of 'What do you want to do next' can be downloaded: *http://www.worldscientific.com/doi/suppl/10.1142/8163/suppl_file/8163_chap01.pdf*

Dizon, D., Ahmad, J., Isoda, M. (2017). SEAMEO Basic Education Standards (SEA-BES): Common Core Regional Learning Standards (CCRLS) in Mathematics and Science. Penang, Malaysia: SEAMEO RECSAM *https://www.researchgate.net/publication/335465253_SEAMEO_Basic_Education_Standards_SEA-BES_Common_Core_Regional_Learning_Standards_CCRLS_in_Mathematics_and_Science*

The textbook used in this course must be cited as shown below:

Masami Isoda (2013). Fraction for Teachers: Knowing What before Planning How to Teach. Tokyo: CRICED, University of Tsukuba.