SEAMEO School Network Program provided by CRICED-University of Tsukuba

Mathematics Education to Develop Students Agency: The Case of Fractions

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why? II

Lesson 3: Addition with different denominator

Lectured by: Masami Isoda, Prof/PhD., University of Tsukuba, Japan
With support of: Maitree Inprasitha, Assoc. Prof/Phd., Narumon Changsri, Assis. Prof./PhD,, and Nisakorn Boonsena, PhD., Auijit Pattanajak, Assoc. Prof., Khon Kaen University, Thailand
With participation of: Ms. Laura Lopez Zarate, Colombia
Mr. Diego Solis, Costa Rica

Chapter 3: Addition of Fraction with Different Denominators

The professor asked the teachers:

In addition of the fractions, we teach the case of the same denominator at first and then we introduce the case of different denominator.

In the case of $\frac{1}{2}+\frac{1}{3}$

We have two bottles with $\frac{1}{2}$L and $\frac{1}{3}$L of milk. How much L in total? Can you imagine the student's answer? Isoda (1996)

Possible Answers:

Som: Yes, we have children who calculate as follows: $\frac{1}{2}+\frac{1}{3}=\frac{1+1}{2+3}=\frac{2}{5}$

Any: Yes, we have. However we have to teach: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

Questions for teachers.

1) If you teach the following as a procedure of multiplication (at 6th grade), $\frac{1}{2}\times\frac{3}{5}=\frac{1\times 3}{2\times 5}$ why you do not teach the following as a procedure of addition (at 4th grade)? $\frac{1}{2}+\frac{3}{5}=\frac{1+3}{2+5}$

2) How can you proceduralize the meaning?

3) What meaning is necessary to be proceduralized?

Page 20.a

How can you plan their argument?

I would like to ask why?

I would like to ask to draw diagrams to explain the meaning.

Professor: Yes, we would like to listen their ideas. Can you imagine their diagrams for explanation? How do you teach to find the common denominator?

In Japan, before students learn the addition with different denominators, they learn the addition with the same denominator.

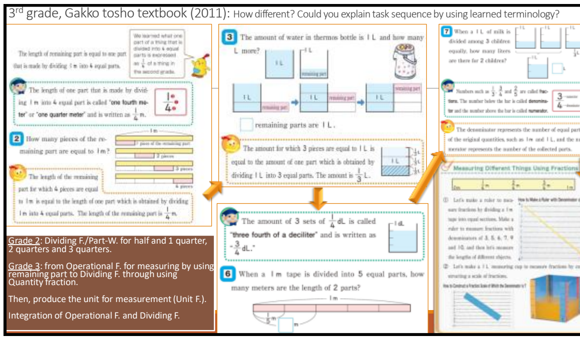
- Exercise

Exercise1: Let's plan the process of dialectic discussion between two different ideas through the drawing diagrams which support each of answers.

Exercise 2: When you plan, what are the expected knowledge for children before the class and what knowledge do they have, actually, for those two ideas? What shall they learn before this class for enabling them such a discussion? What shall we teach before you plan your class?

Page 20.b

3rd grade, Gakko tosho textbook (2011): How different? Could you explain task sequence by using learned terminology?



Grade 2: Dividing F./Part-W. for half and 1 quarter, 2 quarters and 3 quarters.

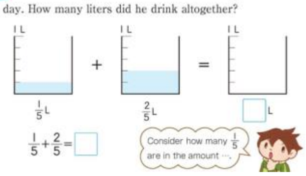
Grade 3: from Operational F. for measuring by using remaining part to Dividing F. through using Quantity fraction.

Then, produce the unit for measurement (Unit F.).

Integration of Operational F. and Dividing F.

Addition of the same denominator at 3rd grade: Could you explain the task by using learned terminology?

1 Akira drank $\frac{1}{5}$ L of milk yesterday and $\frac{2}{5}$ L of milk today. How many liters did he drink altogether?



A Sample of Arguments:

Teacher: Good, now we have different answers. What shall we do?

C1: How did they get their answers?

C2: Yes.

Teacher: Then, we would like to ask: how do you get it?

C3: $\frac{1}{2}+\frac{1}{3}=\frac{1+1}{2+3}=\frac{2}{5}$. I added numerators and denominators each.

C4: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$. I tried to find the common denominator.

Teacher: Do they explain in the same way?

C: (No, answer).

Teacher: Do you have any questions for them?

C1: I have a question for C3, why you added the numerators?

C3: As I explained, $\frac{1}{2}+\frac{1}{3}=\frac{1+1}{2+3}=\frac{2}{5}$. I added numerators and denominators.

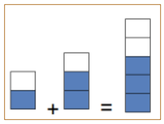
Page 21.a

C4: The total, $\frac{2}{5}$ is smaller than $\frac{1}{2}$. If $\frac{1}{3}$L of milk is added to $\frac{1}{2}$L of milk is less than $\frac{1}{2}$ L. It is very much strange, isn't it?

C3: Yes, it's strange...then I cannot add denominators?

C5: No, it is not strange. I will explain the bottles like this:

In the case of fractions, it is true.



C3: Aha, yes, it is!

Teacher: Then, you say in the case of fractions, it is possible to say that the total from $\frac{1}{2}$L of milk and $\frac{1}{3}$L of milk is less than $\frac{1}{2}$L.

C: No! Yes!

Teacher: Yes or No? It looks C5 is less supported. Why?

C4: The milk bottle of C5, the size is not the same.

C5: Yes, the shape of the milk bottles are not usually the same.

When you teach Dividing F., do you teach the whole? Should it be the same?

When you teach shading the diagram for learning Dividing F., what is the object of counting?

When you teach drawing the diagram for F., do you ask the whole to be the same?

By using Dividing Fraction without quantity, we should note we can not compare and add! Dividing Fraction can be compared and added **only** in the case the whole is kept.

Page 21.b

C4: ....

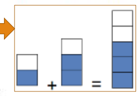
Teacher: Why did you use the common denominator, C4?

C4: Because we added when the denominators are the same.

We learned the addition of fractions in the case when the denominators are the same.

C2: Yes, we know the addition of the same denominator. In that case, we added the numerator. This task is not the same.

Teacher: Did you draw a diagram, like C5?



C4: No, I did not. I think that it is the rule.

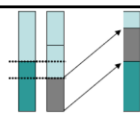
C5: Oh, the rule? I could explain my idea by the diagram.

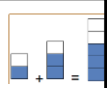
Please count in my diagram.

The way of C3 will be the right rule, not C4.

Page 22.a

C6: I draw a similar diagram, not the same as C5:





It shows that the value is larger than the parts.

Teacher: Now, we have two drawings. Which diagram is reasonable for establishing the way of calculation?

C3: I do not think C6 is explaining C4: $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.

It may explain the part of $\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, however it does not explain $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}$.

Why you have to change the fraction on $\frac{1}{6}$?

It is Unit F. on Operational F.

Teacher: It means that why C4 has to change the denominator, right?

C3: Is that a rule, also? If it is the rule, my own is better because we can explain the idea by counting like C5.

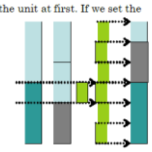
Let's produce the procedure based on the meaning which is represented by diagrams!

Which diagrams are appropriate to explain the meaning of addition?

Why the whole should be the same?

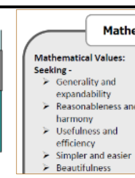
Page 22.b

C6: If we have to explain it by counting, we have to share the unit at first. If we set the unit for counting from the difference between $\frac{1}{2}$ and $\frac{1}{3}$:



I could count six times of the unit.

Thus, the unit for counting is $\frac{1}{6}$.



C5: Aha, I got it. However, why do you use the unit for counting by the difference? Is it occasionally? Can you do that every time? Or, you already knew the difference in fractions with different denominator?

C6: Not sure, however, if you consider every time, do you think your idea also works every time?

Teacher: Yes, in mathematics, we develop generally applicable ideas. It must be important point for discussion.

C5: Yes.

In mathematics, general idea is strong!

Page 23.a

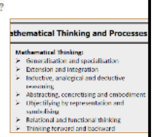
C4: Wow, did you add the denominators in the case that the denominators are the same?

C5: What?

C4: For example, how do you think in the case of $\frac{1}{2}+\frac{1}{2}$.

We already learned it is $\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2}=1$, right?

But if your idea is true, $\frac{1}{2}+\frac{1}{2}=\frac{1+1}{2+2}=\frac{2}{4}=\frac{1}{2}$. It is strange.



C3,C5: Wow, yes, it is strange. Is it the reason, if the denominators are the same, we add the numerators?

C4: Yes, now I understand why I did it. That is the objective why I changed the denominators into common denominators! However, how can we produce the common denominator for the addition of fractions in the case of different denominators. I am still not sure of the ways of calculation.

Teacher: Aha, for considering generally, we checked each proposed way by C3 and C4 on the different denominators by the known case of the same denominator. Then, the way of C4 works and the way of C3 does not. However, the diagram by C5 still looks fine. Why did it produce the inappropriate answer?

In mathematics, general idea is strong!

For generalization, we should develop students who say "For example"!

Counter example is used for dialectic.

Page 23.b

C6: The size of the bottle should be the same. In this case, if we draw the diagram of a 1L milk bottle, it is fine.

Teacher: Yes, we need to write the size of quantity on the diagram. In this case, we should draw 1L as for the whole bottle size in every diagram. We should use the same size bottle for explaining the addition and subtraction of fractions. In the diagram of C5, we counted different size of fractions. $\frac{1}{2}$L is counted one and $\frac{1}{3}$L is counted one. We cannot count one, two because the size is not the same. The diagram by C6 used $\frac{1}{6}$ L for the same counting unit.

C4: Wow, this is the reason why I used the common denominator. Still not sure of the ways.

Teacher: Then, from now, we would like to find the shorter and simple way of calculation by considering the common denominator.

Page 24.a

Questions for professional development 3

Q13. In the argument, you may see the same discussion which you read in Chapter 1 and it will be explained by the several terminologies as mentioned in Chapter 2. Let's explain the argument by the terminologies. And explain your appreciation about the terminologies for understanding the difficulty of fraction and what is the necessary content for teaching.

Q14. In the argument, what kind of explanation did you find? Please read each explanation from the viewpoint of Chapter 1, Q8. Why do we need the diagram? What C4 wished to say? For developing children who learn mathematics by/for themselves, what type of argument you would like to establish in your classroom.

Q15. If you are not used to draw a diagram for explaining fractions by yourself, the argument might be difficult for you to understand. The difficulty that you recognized is based on the unknown. It means that it is the chance for learning. At first, let's discuss about which part do you feel a difficulty and then, talk about what you shall learn.

Q16. When you feel a difficulty, your children also feel the same. Please ask your children who knows addition of the same denominator and equivalent fraction, but not yet learned the case of different denominators to read this discussion. And ask them, how do they read? It is also a good chance to learn from children for knowing what is the task for their learning and what is necessary for your preparations.

Q17. The argument itself was implemented in a class (see Isoda, 1996) and not unusual in Japan. When we compared the classes, major difference is the teaching before the class. What kinds of content shall we teach before this class?

Page 24.b

References

Isoda, M. (1996). Problem Solving Approach with Dialectic. *(written in Japanese:多様な考えを生み練り合う問題解決授業：意味とやり方のずれによる葛藤と納得の授業づくり. 東京：明治図書出版)* in English: Isoda, M. (2007). Developing Mathematical Thinking in Classroom, APEC Project report on *http://www.criced.tsukuba.ac.jp/math/apec/apec2007/progress_report/specialists_session/Masami_Isoda.pdf*

Masami Isoda et al. (2010). Mathematics Education Theories for Lesson Study: Problem Solving Approach and the Curriculum through Extension and Integration, Journal of Japan Society of Mathematical Education, 92(11). 1-158.

Shin Hitotsumatsu et al. (2005). Study with your friends: Mathematics for Elementary School (12 vols). Tokyo: Gakkotosyo.

Masami Isoda et al. (2010). Study with your friends: Mathematics for Elementary School (12 vols). Tokyo: Gakkotosyo.

Masami Isoda, Tenoch Esaú Cedillo Ávalos, et al. (2012). Matemáticas para la educación normal (11vols). Estado de México : Pearson Educación de México.

Tenoch Cedillo, Masami Isoda, et al. (2013) Matemáticas para la educación normal : guía para el aprendizaje y enseñanza de la aritmética, Estado de México : Pearson Educación de México.

The textbook used in this course must be cited as shown below:

Masami Isoda (2013). Fraction for Teachers: Knowing What before Planning How to Teach. Tokyo: CRICED, University of Tsukuba.