SEAMEO School Network Program provided by CRICED-University of Tsukuba

Mathematics Education to Develop Students Agency: The Case of Fractions

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why? II

Lesson 4: Fraction as Number

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Chapter 4: When does Fraction become Number?

Number system is the set with mathematical structures such as the relationship of equivalence, greater/less and operations (addition and multiplication). Mathematically, fraction n/m is an element of rational numbers, if n and m are integers and m≠ 0. If n or m are irrational or complex, it becomes a part of a larger number set.

In school mathematics, when we extend numbers we usually discuss existence/necessity/significance, equivalence/larger/smaller, and four arithmetic operations. If fraction is completely learned, we can recognize the fraction as a part of number set. It is a representation of rational numbers which are bridged with decimals. In other words, fraction is not the number until it has been fully completed.

On this content, every teacher should know that the dividing fraction is not the number. It represents the part whole relationship actions on the object such as half of a Pizza. Quantity fraction can be arranged on the number line for quantity as well as other numbers such as decimal numbers. Dividing fraction usually begins from the whole and it is not easy to represent on the number line as long as what is the whole which we discussed on Chapter 1.

Equivalent Fraction

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$... is the equivalent fraction of $\frac{1}{2}$. At the lower grades the dividing fraction, which is represented by paper folding activity, is learned for knowing the half and the quarter, two quarters, and three quarters. It is a necessary activity for knowing the procedure to get the half and the quarter. However, the paper folding activity for dividing fraction is usually missing $\frac{2}{6}$ because it cannot be represented by the folding procedure into half.

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• Equivalent Fraction

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As shown on the Figure 3a in Chapter 2, we can divide any string into any number of equally parts. On this basis, we can draw the diagram of number lines which shows the equivalence (Figure 5).

Please note, the number line includes more than 1 on Figure 5 like a ray. If it is represented by a segment between 0 and 1, it is just a representation of dividing fraction which show a part-whole relationship. As discussed in Chapter 2, operational fraction is majored by a unit fraction (remaining part) and the number of unit fraction is a numerator and thus, it can be extended more than 1. Quantity fraction functions to show what is 1 unit-quantity.

• Fraction is Expression:

When fraction can be seen as an expression, it supports to see fraction as a part of number.

• Fraction as Quotient

Fraction is related with the dividing activity which includes both meanings of quotative and dividing fraction on situations (see Chapter 2). In both situations, fraction represents the value of division such as $1\div 3=\frac{1}{3}$. We call it **fraction as quotient**. From this definition, fraction connected with a number as well as the whole number (See figure 6a and 6b) because answer of division should be a number on the perspective of the permanence of form.

1 Follow the instructions below by using this number line.

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3 Fractions, Decimal Numbers and Whole Numbers

The Quotients of Divisions and Fractions

1 When we divide 2 L of milk among ☐ students equally, how many liters will each student receive?

2÷☐

① Enter the numbers from 1 to 5 in the ☐ and calculate the answers.

2÷☐ 2÷☐ 2÷☐ 2÷☐ 2÷☐

② Divide the above equations into 3 groups based on the answers.

ⓐ Answers that are whole numbers. (☐)

ⓑ Answers that are expressed exactly as decimal numbers. (☐)

ⓒ Answers that are not expressed exactly as decimal numbers. (☐)

2÷3 is 0.666..., so this cannot be expressed exactly as a decimal number because there is no end.

③ When 2 L is divided equally among 3 students, how many liters does each student receive?

ⓐ Color in the portion for one student.

ⓑ How many liters is one portion?

Let's see how to express the quotient of a division problem when it cannot be expressed exactly as a decimal number.

Figure 6a. Gakkō Tosho Grade 5. (vol.2. p28, 2005: vol.1. p128, 2011)

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1 L 1 L 1 L

The amount for one student: $\frac{1}{3}$ L $\frac{1}{3}$ L — 1 L — The amount for one student

The amount for one student when 1 L is divided into 3 equal parts → ☐ L

The amount for one student when 2 L is divided to 3 equal parts → ☐ L

2÷3=☐

2 How many meters is the length of each section when a 3m string is divided into 4 equal parts?

① Write an equation.

② What is the length of one section?

1÷4 (m) 2÷4 (m) 3÷4 (m)

3÷4=☐

The quotient of a division problem in which a whole number is divided by another whole number can be expressed as a fraction. ●÷■=●/■

The quotient can be expressed precisely as a fraction.

Express the quotients in these problems as fractions.

① 1÷6 ② 5÷8 ③ 4÷3 ④ 9÷7

Figure 6b. Gakkō Tosho Grade 5. (vol.2. p30, 2005: vol.1. p130, 2011)

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If children can see the fraction as quotient, now fraction as a part of number which can be seen as the expression of division such as $\frac{1}{3}=1\div 3$. If fraction is given by the expression of division, the meaning of equivalence fraction is also understandable because it is not the mysterious property only appeared on the fraction. In any arithmetic operations, it has similar properties. When the equivalence of fraction is recognized on the bases of the fraction as quotient, all four operations complete the properties of equivalence on their expressions.

On mathematics, it is the discussion of equivalent class.

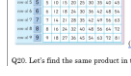
• Questions for professional development 4

Q18. Let's find the equivalent properties of addition and subtraction operations: Gakkotosho Grade 1.

(Gakkotosho Grade 1, pp82-83, pp92-93, 2005: pp96-97, pp106-107, 2011)

Q19. In Chapter 2, Q11, there are two meanings of dividing activity. Which activity can you see on the figure 6a and 6b and explain why.

Multiplication Table



(Gakkotosho Grade vol.2, p.92, 2

Q20. Let's find the same product in the multiplication table.

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The textbook used in this course must be cited as shown below:

Masami Isoda (2013). Fraction for Teachers: Knowing What before Planning How to Teach. Tokyo: CRICED, University of Tsukuba.