SEAMEO School Network Program provided by CRICED-University of Tsukuba

Mathematics Education to Develop Students Agency: The Case of Fractions

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why? II

Lesson 5: Fraction in relation to ratio and proportion

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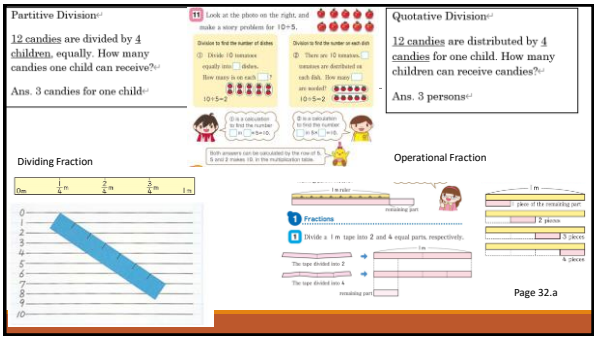
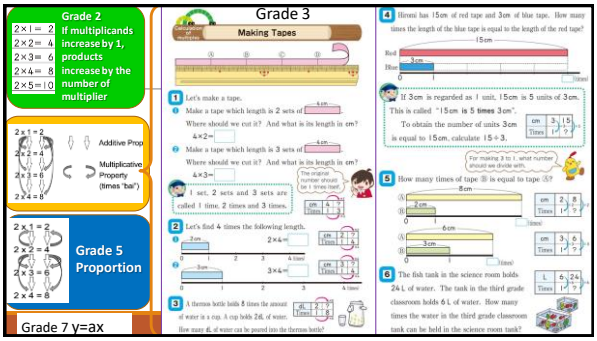
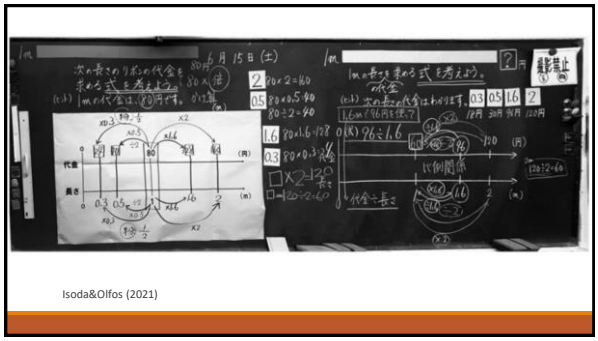


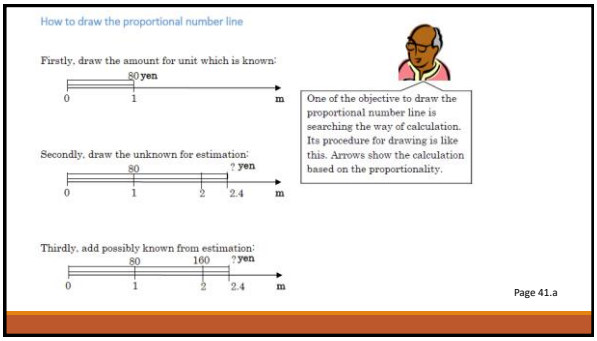
Terminology

	Multiplication	Division	Quantities	Ratio
Partitive Division: 12 candies are divided by 4 children, equally. How many candies one child can receive? Ans. 3 candies for one child	(how many children) x (how many for each child)	Partitive Division Situations (how many for each child or dish)	Division of Different Quantities	Ratio (Rate) Speed (km) : (hour) Population Density (person): (km^2)
Quotative Division: 12 candies are distributed by 4 candies for one child. How many children can receive candies? Ans. 3 persons	(how many dishes) x (how many for each dish)	Quotative Division Situations (how many children or dishes	Division of Same Quantities	Ratio (Boys) : (Girls) (Boys) : (Human)

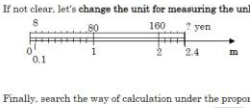




Isoda&Olfos (2021)

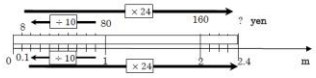


If not clear, let's **change the unit for measuring the unknown**:



The price of 2.4m is not sure. If I **change the unit** 'm' to 'cm,' it is 80 yen for 100cm. How much in 240cm. Aha, 8 yen for 10cm· it means 0.1m!

Finally, search the way of calculation under the proportionality:



$80 \times 2.4 = ?$ The length corresponds the price. **The re measuring process is represented by the arrows.** The arrow sequence represents $80 \div 10 \times 24$. Aha!!

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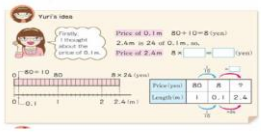
Exercise

Let's get the price for 2.4m by the proportional number line.

For knowing how and what, please explain the way to use the proportional number line by yourself.



(Gakko Tosho Grade 5, Vol.1.pp30.2005/pp32.2011)

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Questions for professional development 5

The following tasks show the various context of ratio. Let's answer the following questions.

Q22. 1) Solve the following task. Provide the answer using percentage.

2) Explain the feature of the task using the following terms: Partitive division, Different Quantities.

3 Some students checked the number of passengers on airlines one day. Which plane is more crowded?

Number of Passengers and Seats

	Small plane	Large plane
Number of passengers	117	442
Number of seats	130	520

A number that is expressed by the quantity being compared when the basic quantity is made 1, like a shooting record or crowding, is called "a ratio."

Ratio = Quantity being compared ÷ Basic quantity

(Gakko Tosho, Grade 5, vol.2, pp57-58, 2005: pp85-86, 2011)

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Q23. 1) Solve the following tasks and explain the base quantity for each task.

2) Explain the role of tape and number line diagram.

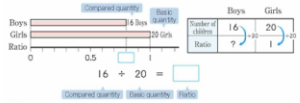
3) Explain the situation using the quotative division and the same quantity.

4) $16 \div 20 = \frac{16}{20} = \frac{4}{5}$. Can we explain the task by the part whole relationship?

The Ratio of 2 Quantities

We can express the proportion between 2 quantities even if one of them is not a part of the other.

4 There are 16 boys and 20 girls in Keiko's class. Let's find the ratio of the number of boys to the number of girls.



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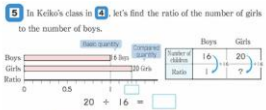
5 In Keiko's class in **4**, let's find the ratio of the number of girls to the number of boys.



The ratio will change if we change the basic quantity. In some cases, the ratio will become larger than 1.

(Gakko Tosho, Grade 5, vol.2, pp59, 2005: pp87, 2011)

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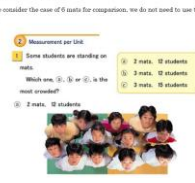
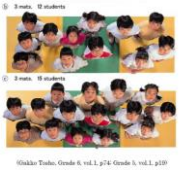
Q24

1) Solve the task using ratio.

2) If we consider the case of 6 mats for comparison, we do not need to use the idea of ratio.





(Gakko Tosho, Grade 6, vol.1, p74-Grade 5, vol.1, p59)

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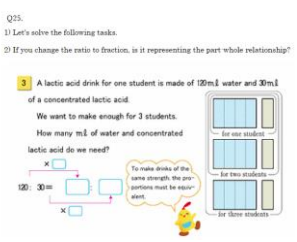
Q25.
1) Let's solve the following tasks.
2) If you change the ratio to fraction, is it representing the part whole relationship?

3 A lactic acid drink for one student is made of 120mℓ water and 30mℓ of a concentrated lactic acid.
We want to make enough for 3 students.
How many mℓ of water and concentrated lactic acid do we need?

120 : 30 = □ : □

To make drinks of the same strength, the proportions must be equivalent.

for one student
for two students
for three students

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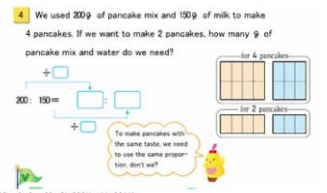
4 We used 200g of pancake mix and 150g of milk to make 4 pancakes. If we want to make 2 pancakes, how many g of pancake mix and water do we need?

200 : 150 = □ : □

for 4 pancakes
for 2 pancakes

To make pancakes with the same taste, we need to use the same proportion, don't we?

(Grade 6, vol2,p35,2005: p11, 2011)

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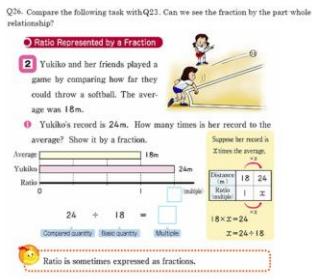
Q26. Compare the following task withQ23. Can we see the fraction by the part whole relationship?

Ratio Represented by a Fraction

2 Yukiko and her friends played a game by comparing how far they could throw a softball. The average was 18m.

❶ Yukiko's record is 24m. How many times is her record to the average? Show it by a fraction.

Average 18m
Yukiko 24m
Ratio 0 1 □(multiple)

Suppose her record is x times the average.

Distance (m)	18	24
Ratio (multiple)	1	x

×x ×x

24 ÷ 18 = □
Compared quantity Basic quantity Multiple

18×x=24
x=24÷18

Ratio is sometimes expressed as fractions.

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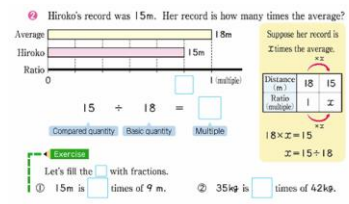
❷ Hiroko's record was 15m. Her record is how many times the average?

Average 18m
Hiroko 15m
Ratio 0 1 □(multiple)

Suppose her record is x times the average.

Distance (m)	18	15
Ratio (multiple)	1	x

×x ×x

15 ÷ 18 = □
Compared quantity Basic quantity Multiple

18×x=15
x=15÷18

Exercise
Let's fill the □ with fractions.
① 15m is □ times of 9 m. ② 35kg is □ times of 42kg.

(Gakko Tosyo,Grade6, vol2,p27,2005: Grade 6, vol.1,p53,2011)

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Masami Isoda, Raimundo Olfos. Edited. (2021). *Teaching Multiplication with Lesson Study*. Cham: Springer.

The textbook used in this course must be cited as shown below:

Masami Isoda (2013). Fraction for Teachers: Knowing What before Planning How to Teach. Tokyo: CRICED, University of Tsukuba.