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Fraction for Teachers

Knowing What before Planning How to Teach

Masami Isoda, PhD/Prof. Director of CRICED, University of Tsukuba, Japan

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Preface

Education is the work to prepare for the future. Developing children who learn mathematics by and for themselves is one of the major issues on mathematics education reforms in the world (See such as Isoda & Katagiri, 2012). After the comparative study of mathematics classroom such as TIMSS video study in 90s, Japanese lesson study is the world-shared methodology as for the tools for professional development because the study indirectly demonstrated the quality of Japanese mathematics teaching and it is established by the lesson study. However, people often misunderstand the lesson study as for the talking about the class rather than studying subject matter. They enjoy the classroom observation likely listening to the music or watching the theatre. However, through listening to the music, and even if we enjoy talking about actors, we cannot prepare the good player ourselves. In Japanese lesson study, most efforts are done for the preparation of the class. The misunderstanding originated due to the limitation of the content guidebook to refer in English. On this reason, I have developed several resources which show the theory for the purpose to improve mathematics education with researches in the world.

For the workshop of SMASE-INSET project under Japan International Cooperation Agency (JICA), Japan and Federal Ministry of Education (FME), Nigeria, this booklet includes the essential theory for enabling teachers to plan the class for developing children who learn mathematics by and for themselves. It focused on the innovation of elementary school mathematics based on the content which is well written in the textbooks in each country and known by teachers. The workshop done in Nigeria was based on the author's experience in Central and South America, South East Asia and Pacific as well as in Japan.

May 7, 2013

Masami Isoda, PhD

CRICED, University of Tsukuba, Japan

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Pictures of the English Edition of Japanese-Mathematics Textbook are extracted from **'Study with Your Friends MATHEMATICS for Elementary School** (Gakko-Tosho; 2005)'. When user extracts the pictures from the booklet, he/she needs the permission from Gakko-Tosho: Katsuaki Serizawa (e-mail: katsuaki.serizawa@gakuto.co.jp), GAKKO TOSHO CO., LTD. 3-10-36 Higashi-jujo Kita, Tokyo, 114-0001, Japan. **https://support.gakuto.co.jp/mathematics-textbook/**

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Akinori Ito (JICA, PNG project)

About the Author

Affiliation and Address:

CRICED/Graduate School of Education, University of Tsukuba, 305-8572, Japan.

Website:

http://math-info.criced.tsukuba.ac.jp/

Title:

PhD (Education), Waseda University;

Honorary PhD (Mathematics Education), Khon Kaen University, Thailand;

Associate Professor, University of Tsukuba (1998-).

Education:

Master Program of Education, University of Tsukuba (1984).

Academic Membership:

Advisory Board Member, History and Pedagogy of Mathematics (2002-);

Project Overseer (Director), APEC Lesson Study Project in Mathematics (2006-);

Council Member, World Association of Lesson Study (2011-);

Chief Editor, Journal of Japan Society of Mathematical Education (2010-).

Awarded by:

Minister of Education (MEXT), Japan, as for the Best Educational Software in the Year 2005;

President of the Japan Publishers Association as for the Most Beautiful Book of the Year 2010 in the area of natural science;

Honorary PhD (Mathematics Education) by Khon Kaen University, Thailand as for his contribution to Khon Kaen University and Thailand, 2011.

Book for Acquisitions Program of Chilean Authors, Selected by National Committee of Culture and Art, Government of Chile. 2012.

President of the University of Tsukuba, as for the Best Faculty Member of the Year 2012. 2013.

Further CV and Publications:

Japanese Full

http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/profile.html

English Part
http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/prof_e.html

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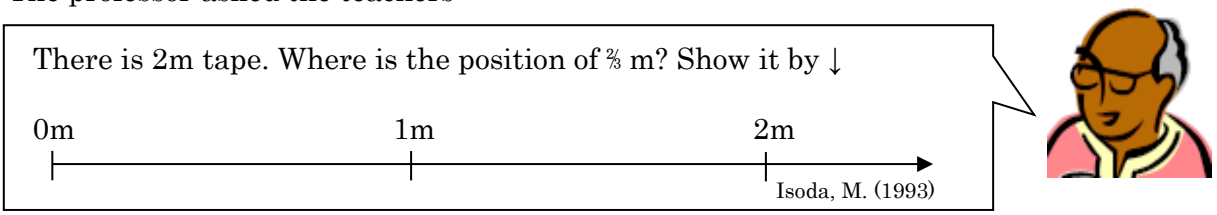
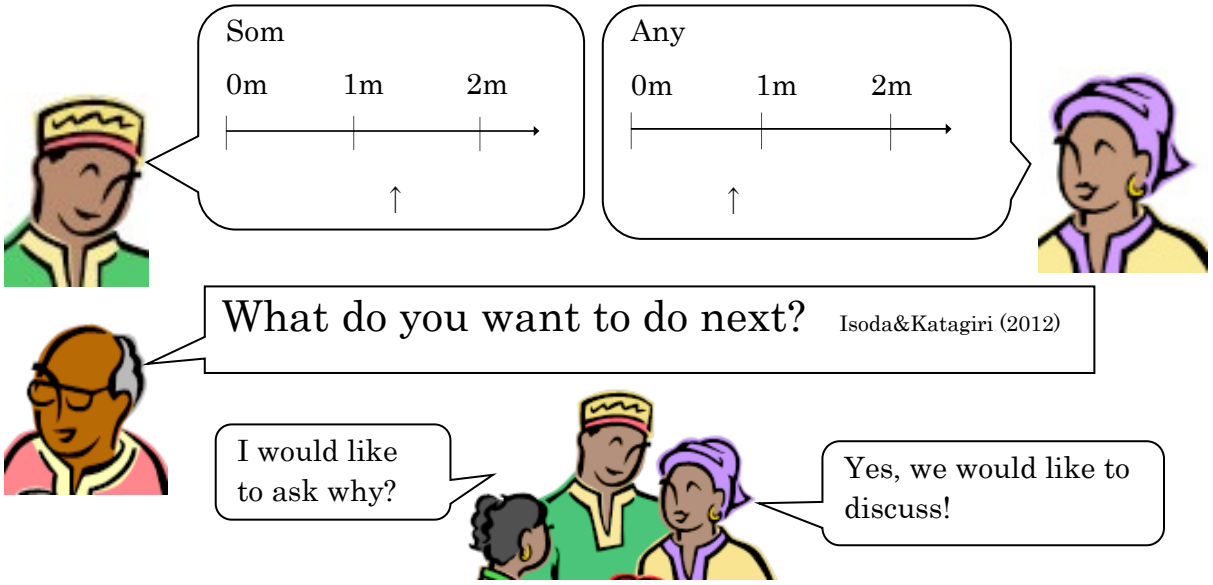
Chapter 1: What is fraction?

The professor asked the teachers:



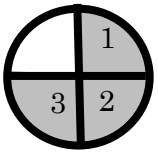
Possible Answers:



Professor: You are good teachers, aren't you? Because you already have the custom to ask why to others and discuss. It means that you usually engage in a similar activity by yourself. You already have the mind set for learning how to learn by yourself!

Discussion 1: Why do you think so?

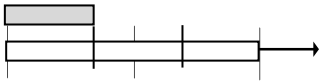
Som: It is larger than 1m because 2/3 means:



Ano: No, the whole is divided by 3, then shaded the 2 third of 3 parts

Som: Yeah, I should draw like that.

Any: Wow, 2/3 is less than 1. It is like this:



Som & Ano: ..continue..(talking about part-whole)

http://math-info.criced.tsukuba.ac.jp/museum/dbook_site/

Any: ...continue... (talking about part-whole)

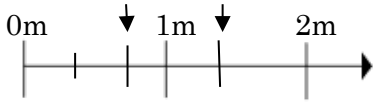
Prof: I could not understand well all of your explanations and diagrams. What is the original question?

Any: Where is the 2/3m?

Prof: Yes, then, the denomination 'meter' is missing in all of your explanations. We should write the meter on the number line. Can you explain 2/3m, again?

Any: It is 2/3 of 1m

Som: It is 2/3 of 2m

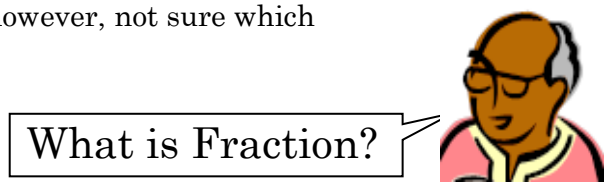


Prof: Now we can discuss what differences are there between these two ideas.

The denomination of quantity is important. When we explain something with the situation, we usually omit some words which are already known in the situation. In this case, the unit of meter 'm' itself is written in the task; however the meter was missing from their explanation and diagrams. In their discussion, if the denomination of quantity is missing, we are not sure which part you are explaining.

People who participated in the discussion; however, not sure which answer is appropriate.

Then, Professor asked.



Discussion 2: What is Fraction?

Ano: It is a part of the whole.

Prof: It is not clear. Could you explain it more exactly?

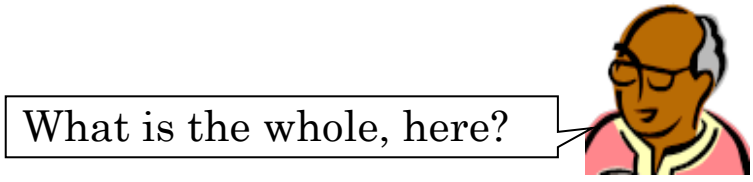
Ano: When the whole is divided into parts, fraction is the number of the parts in the whole.

Prof: Still are missing some important terms. Can anyone support?

Oth: When the whole is divided into 'm equally parts', 'fraction n/m' is 'n pieces of the parts in the whole'.

Any: The whole for 2/3m is 1m.

Som: The whole for 2/3m is 2m.



Discussion 3: Which one should be the whole?

Any: 1m

Som: 2m

Ano: 1m or 2m, which one shall we chose for the whole, in this task?

Any: The whole for 2/3m is 1m

Som: The whole for 2/3m is 2m.

Oth: How can we discuss?

In mathematics, generally applicable idea is strong. For checking it, we have to think 'For Example, ..."

Discussion 4: For Example, if.....

Prof: For example, if I change the task from 2/3m to 1/2m what will happen?

Any: The 1/2m is 0.5m.

Som: The 1/2 of 2m is 1m.

Ano: No, what you are saying is that 1/2m is 1m. However, 1/2m is 0.5m, isn't it?

Pro: If I change the original question '2/3m' to '1/1m' what will happen?

Oth: If Som's idea, 1/1m is the 1/1 of 2m, thus 2m. 1/1m =2m. It is strange.

Prof: Yes, we can generalize Any's idea and not generalize Som's idea. When we say 2/3m, the whole is denominated by the 2/3 'meters': The unit of meters is 1m in any time.

What did you learn from this class? Let's write your own resume based on what you learned.

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Questions for professional development 1

Q1. Why did the author choose this story as for the introductory chapter?

Q2. Professor asked several questions in the class. Which question is most important in this class? Why do you think so?

Q3. Do you think the definition of fraction in the class is appropriate? Why do you think so?

Q4. Explain the class using the term of appreciation.

Q5. If you conduct this task in your class or your teacher training program, what is your objective?

Q6. In your class, do you ask your students or teachers 'what do you want to do next?'. If you do ask, when do you ask it? If you do not ask, why?

Q7. Please explain the professor's questioning and values from the viewpoint of the following.

Three Major Objectives for Education as for Future Preparation

1. Human Character Formation ∋ {Developing Mindset, Attitude, Value} *appreciation*
2. Learning How to Learn ∋ {Knowing how to develop and reconstruct} *reflection*
3. Knowledge and Skills ∋ {Understanding and Proficiency} *acquisition*

Dizon, D., Ahmad, J., Isoda, M. (2017)

Q8. What is the explanation in mathematics at elementary school level? Using the discussion in class, please explain what it is from the following three perspectives.

Explanation of:

1. Meaning, such as the base for the reasoning using different representation;
2. Significance or Objective, why I would like to think such a way;
3. Procedure, how I did.

Isoda, M.(2008, 2009)

Q9. In the argumentation, for progressive dialectic, the way of discussion below is known as meaningful: 'If your saying is true, what will happen?'

In the class, Professor has used this dialectic method. Where did he use it?

http://math-info.criced.tsukuba.ac.jp/museum/dbook_site/

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