Fraction for Teachers

Knowing What before Planning How to Teach



Masami Isoda, PhD/Prof. Director of CRICED, University of Tsukuba, Japan

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Preface



Education is the work to prepare for the future. Developing children who learn mathematics by and for themselves is one of the major issues on mathematics education reforms in the world (See such as Isoda & Katagiri, 2012). After the comparative study of mathematics classroom such as TIMSS video study in 90s, Japanese lesson study is the world-shared methodology as for the tools for professional development because the study indirectly demonstrated the quality of Japanese mathematics teaching and it is established by the lesson study. However, people often misunderstand the lesson study as for the talking about the class rather than studying subject matter. They enjoy the classroom observation likely listening to the music or watching the theatre. However, through listening to the music, and even if we enjoy talking about actors, we cannot prepare the good player ourselves. In Japanese lesson study, most efforts are done for the preparation of the class. The misunderstanding originated due to the limitation of the content guidebook to refer in English. On this reason, I have developed several resources which show the theory for the purpose to improve mathematics education with researches in the world.

For the workshop of SMASE-INSET project under Japan International Cooperation Agency (JICA), Japan and Federal Ministry of Education (FME), Nigeria, this booklet includes the essential theory for enabling teachers to plan the class for developing children who learn mathematics by and for themselves. It focused on the innovation of elementary school mathematics based on the content which is well written in the textbooks in each country and known by teachers. The workshop done in Nigeria was based on the author's experience in Central and South America, South East Asia and Pacific as well as in Japan.

May 7, 2013

Masami Isoda, PhD

CRICED, University of Tsukuba, Japan

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About the Author

Affiliation and Address:

CRICED/Graduate School of Education, University of Tsukuba, 305-8572, Japan.

Website:

http://math-info.criced.tsukuba.ac.jp/

Title:

PhD (Education), Waseda University;

Honorary PhD (Mathematics Education), Khon Kaen University, Thailand;

Associate Professor, University of Tsukuba (1998-).

Education:

Master Program of Education, University of Tsukuba (1984).

Academic Membership:

Advisory Board Member, History and Pedagogy of Mathematics (2002-);

Project Overseer (Director), APEC Lesson Study Project in Mathematics (2006-);

Council Member, World Association of Lesson Study (2011-);

Chief Editor, Journal of Japan Society of Mathematical Education (2010-).

Awarded by:

Minister of Education (MEXT), Japan, as for the Best Educational Software in the Year 2005;

President of the Japan Publishers Association as for the Most Beautiful Book of the Year 2010 in the area of natural science;

Honorary PhD (Mathematics Education) by Khon Kaen University, Thailand as for his contribution to Khon Kaen University and Thailand, 2011.

Book for Acquisitions Program of Chilean Authors, Selected by National Committee of Culture and Art, Government of Chile. 2012.

President of the University of Tsukuba, as for the Best Faculty Member of the Year 2012. 2013.

Further CV and Publications:

Japanese Full

http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/profile.html

English Part http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/prof_e.html

Table of Content

Preface	2
About the Author	4
Table of Content	5
Chapter 1: What is fraction?	7
Questions for professional development 1	10
Major Reference and Further readings 1	11
Chapter 2: Dividing Fraction and Quantity Fraction	12
Questions for professional development 2	15
Dividing Fraction	16
Exercise:	17
Operational (measuring) Fraction	17
Euclidian Algorithm	18
Exercise	18
Quantity Fraction	18
Quantity Fraction Unit Fraction for measuring unit	18 19
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2	18 19 19
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators	18 19 19 20
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise	18 19 19 20 20
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3	 18 19 19 20 20 24
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3 Major Reference and Further readings 3	 18 19 19 20 20 24 25
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3 Major Reference and Further readings 3 Chapter 4: When does Fraction become Number?	 18 19 19 20 20 24 25 26
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3 Major Reference and Further readings 3 Chapter 4: When does Fraction become Number? Equivalent Fraction	 18 19 20 20 24 25 26 26
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3 Major Reference and Further readings 3 Chapter 4: When does Fraction become Number? Equivalent Fraction Fraction is Expression	 18 19 20 20 24 25 26 26 27
Quantity Fraction Unit Fraction for measuring unit Major Reference and Further Readings 2 Chapter 3: Addition of Fraction with Different Denominators Exercise Questions for professional development 3 Major Reference and Further readings 3 Chapter 4: When does Fraction become Number? Equivalent Fraction Fraction is Expression Fraction as Quotient	 18 19 20 20 24 25 26 26 27 27

Major Reference and Further readings 4	
Chapter 5. Fraction in relation to ratio and proportion	
Questions for professional development 5	
Fraction as Ratio	
Exercise	
Rule of Three	
Tape and Number-Line diagram: Proportional number line	
Exercise	
How to draw the proportional number line	
Major Reference and Further readings 5	
Chapter 6. Multiplication and Division of Fraction (1)	
Number line	43
Multiplication	
How to find the expression from the situated problem	
Exercise	
Fraction × Whole Number	
Exercise	
Fraction \times Fraction	
Area Diagram	
Exercise	
Major Reference and Further Readings 6	
Chapter 7. Multiplication and Division of Fraction (2)	51
Rules of Multiplication and Division	51
Exercise:	51
Exercise: Fraction divided by Fractions	53
Major Reference and Further Readings 7	54
Further Readings	55

Chapter 3: Addition of Fraction with Different Denominators

The professor asked the teachers:

In addition of the fractions, we teach the case of the same denominator at first and then we introduce the case of different denominator.

In the case of $\frac{1}{2} + \frac{1}{3}$

We have two bottles with $\frac{1}{2}$ L and $\frac{1}{3}$ L of milk. How much L in total? Can vou imagine the student's answer? Isoda (1996) Possible Answers:



Professor: Yes, we would like to listen their ideas. Can you imagine their diagrams for explanation? How do you teach to find the common denominator?

Exercise

- Exercise1: Let's plan the process of dialectic discussion between two different ideas through the drawing diagrams which support each of answers.
- Exercise 2: When you plan, what are the expected knowledge for children before the class and what knowledge do they have, actually, for those two ideas? What shall they learn before this class for enabling them such a discussion? What shall we teach before you plan your class?

A Sample of Arguments:

Teacher: Good, now we have different answers. What shall we do?

C1: How did they get their answers?

C2: Yes.

Teacher: Then, we would like to ask: how do you get it?

C3: $\frac{1}{2} + \frac{1}{3} = \frac{1+1}{2+3} = \frac{2}{5}$, I added numerators and denominators each.

C4: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$, I tried to find the common denominator.

Teacher: Do they explain in the same way?

C: (No, answer).

Teacher: Do you have any questions for them?

C1: I have a question for C3, why you added the numerators?

C3: As I explained, $\frac{1}{2} + \frac{1}{3} = \frac{1+1}{2+3} = \frac{2}{5}$, I added numerators and denominators.

C4: The total, $\frac{2}{5}$ is smaller than $\frac{1}{2}$. If $\frac{1}{3}$ L of milk is added to $\frac{1}{2}$ L of milk is less than $\frac{1}{2}$ L. It is very much strange, isn't it?

C3: Yes, it's strange...then I cannot add denominators?

C5: No, it is not strange. I will explain the bottles like this:

In the case of fractions, it is true.



C3: Aha, yes, it is!

Teacher: Then, you say in the case of fractions, it is possible to say that the total from $\frac{1}{2}$ L of milk and $\frac{1}{3}$ L of milk is less than $\frac{1}{2}$ L.

C: No! Yes!

Teacher: Yes or No? It looks C5 is less supported. Why?

C4: The milk bottle of C5, the size is not the same.

C5: Yes, the shape of the milk bottles are not usually the same.

C4:

Teacher: Why did you use the common denominator, C4?

C4: Because we added when the denominators are the same.

We learned the addition of fractions in the case when the denominators are the same.

C2: Yes, we know the addition of the same denominator. In that case, we added the numerator. This task is not the same.

Teacher: Did you draw a diagram, like C5?

- C4: No, I did not. I think that it is the rule.
- C5: Oh, the rule? I could explain my idea by the diagram.

Please count in my diagram.

The way of C3 will be the right rule, not C4.

C6: I draw a similar diagram, not the same as C5:



It shows that the value is larger than the parts.

- Teacher: Now, we have two drawings. Which diagram is reasonable for establishing the way of calculation?
- C3: I do not think C6 is explaining C4: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$.

It may explain the part of $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, however it does not explain $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$.

Why you have to change the fraction on $\frac{1}{6}$?

Teacher: It means that why C4 has to change the denominator, right?

C3: Is that a rule, also? If it is the rule, my own is better because we can explain the idea by counting like C5.

C6: If we have to explain it by counting, we have to share the unit at first. If we set the unit for counting from the difference between $\frac{1}{2}$ and $\frac{1}{3}$:



- C5: Aha, I got it. However, why do you use the unit for counting by the difference? Is it occasionally? Can you do that every time? Or, you already knew the difference in fractions with different denominator?
- C6: Not sure, however, if you consider every time, do you think your idea also works every time?
- Teacher: Yes, in mathematics, we develop generally applicable ideas. It must be important point for discussion.

C5: Yes.

- C4: Wow, did you add the denominators in the case that the denominators are the same?
- C5: What?
- C4: For example, how do you think in the case of $\frac{1}{2} + \frac{1}{2}$.

We already learned it is $\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2} = 1$, right?

But if your idea is true, $\frac{1}{2} + \frac{1}{2} = \frac{1+1}{2+2} = \frac{2}{4} = \frac{1}{2}$. It is strange.

- C3,C5: Wow, yes, it is strange. Is it the reason, if the denominators are the same, we add the numerators?
- C4: Yes, now I understand why I did it. That is the objective why I changed the denominators into common denominators! However, how can we produce the common denominator for the addition of fractions in the case of different denominators. I am still not sure of the ways of calculation.
- Teacher: Aha, for considering generally, we checked each proposed way by C3 and C4 on the different denominators by the known case of the same denominator. Then, the way of C4 works and the way of C3 does not. However, the diagram by C5 still looks fine. Why did it produce the inappropriate answer?

- C6: The size of the bottle should be the same. In this case, if we draw the diagram of a 1L milk bottle, it is fine.
- Teacher: Yes, we need to write the size of quantity on the diagram. In this case, we should draw 1L as for the whole bottle size in every diagram. We should use the same size bottle for explaining the addition and subtraction of fractions. In the diagram of C5, we counted different size of fractions. $\frac{1}{2}L$ is counted one and $\frac{1}{3}L$ is counted one. We cannot count one, two because the size is not the same. The diagram by C6 used $\frac{1}{6}L$ for the same counting unit.
- C4: Wow, this is the reason why I used the common denominator. Still not sure of the ways.
- Teacher: Then, from now, we would like to find the shorter and simple way of calculation by considering the common denominator.

Questions for professional development 3

- Q14. In the argument, you may see the same discussion which you read in Chapter 1 and it will be explained by the several terminologies as mentioned in Chapter 2. Let's explain the argument by the terminologies. And explain your appreciation about the terminologies for understanding the difficulty of fraction and what is the necessary content for teaching.
- Q15. In the argument, what kind of explanation did you find? Please read each explanation from the viewpoint of Chapter 1, Q8. Why do we need the diagram? What C4 wished to say? For developing children who learn mathematics by/for themselves, what type of argument you would like to establish in your classroom.
- Q16. If you are not used to draw a diagram for explaining fractions by yourself, the argument might be difficult for you to understand. The difficulty that you recognized is based on the unknown. It means that it is the chance for learning. At first, let's discuss about which part do you feel a difficulty and then, talk about what you shall learn.
- Q17. When you feel a difficulty, your children also feel the same. Please ask your children who knows addition of the same denominator and equivalent fraction, but not yet learned the case of different denominators to read this discussion. And ask them, how do they read? It is also a good chance to learn from children for knowing what is the task for their learning and what is necessary for your preparations.
- Q18. The argument itself was implemented in a class (see Isoda, 1996) and not unusual in Japan. When we compared the classes, major difference is the teaching before the class. What kinds of content shall we teach before this class?

Major Reference and Further readings 3

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