Fraction for Teachers

Knowing What before Planning How to Teach

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Preface

Education is the work to prepare for the future. Developing children who learn mathematics by and for themselves is one of the major issues on mathematics education reforms in the world (See such as Isoda & Katagiri, 2012). After the comparative study of mathematics classroom such as TIMSS video study in 90s, Japanese lesson study is the world-shared methodology as for the tools for professional development because the study indirectly demonstrated the quality of Japanese mathematics teaching and it is established by the lesson study. However, people often misunderstand the lesson study as for the talking about the class rather than studying subject matter. They enjoy the classroom observation likely listening to the music or watching the theatre. However, through listening to the music, and even if we enjoy talking about actors, we cannot prepare the good player ourselves. In Japanese lesson study, most efforts are done for the preparation of the class. The misunderstanding originated due to the limitation of the content guidebook to refer in English. On this reason, I have developed several resources which show the theory for the purpose to improve mathematics education with researches in the world.

For the workshop of SMASE-INSET project under Japan International Cooperation Agency (JICA), Japan and Federal Ministry of Education (FME), Nigeria, this booklet includes the essential theory for enabling teachers to plan the class for developing children who learn mathematics by and for themselves. It focused on the innovation of elementary school mathematics based on the content which is well written in the textbooks in each country and known by teachers. The workshop done in Nigeria was based on the author's experience in Central and South America, South East Asia and Pacific as well as in Japan.

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Chapter 4: When does Fraction become Number?

Number system is the set with mathematical structures such as the relationship of equivalence, greater/less and operations (addition and multiplication). Mathematically, fraction n/m is an element of rational numbers, if n and m are integers and m≠ 0. If n or m are irrational or complex, it becomes a part of a larger number set.

In school mathematics, when we extend numbers we usually discuss existence/necessity/significance, equivalence/larger/smaller, and four arithmetic operations. If fraction is completely learned, we can recognize the fraction as a part of number set: It is a representation of rational numbers which are bridged with decimals. In other words, fraction is not the number until it has been fully completed.

On this context, every teacher should know that the dividing fraction is not the number. It represents the part-whole-relationship actions on the object such as half of a Pizza. Quantity fraction can be arranged on the number line for quantity as well as other numbers such as decimal numbers. Dividing fraction usually begins from the whole and it is not easy to represent on the number line as long as what is the whole which we discussed on Chapter 1.

Equivalent Fraction

$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$... is the equivalent fraction of $\frac{1}{2}$. At the lower grades the dividing fraction, which is represented by paper folding activity, is learned for knowing the half and the quarter, two quarters, and three quarters. It is a necessary activity for knowing the procedure to get the half and the quarter. However, the paper folding activity for dividing fraction is usually missing $\frac{3}{6}$, because it cannot be represented by the folding procedure into half.

Number Line

Number line shows the magnitude/size of numbers up to real numbers as position which is useful for comparison. Number line is scaled by using equally dividing of 0-1 span as dividing fraction and by the unit fraction as operational fraction. As shown on the Figure 3a in Chapter 2, we can divide any string into the number of equal parts. On this basis, we can draw the diagram of number lines which demonstrates the equivalence, larger and smaller (Figure 5). Through showing the position of fraction on the number line, it supports to see the fraction as number as well as others.

The number line includes more than 1 on Figure 5 like a ray. If it is represented by a segment between 0 and 1 without any extra part more than 1, it is just a representation of dividing fraction which shows a part-whole relationship like folding a tape. Here, the whole is 1 without considering the possibility of extension. As discussed in Chapter 2, operational fraction is majored by a unit fraction (remaining part). The number of unit fraction shows a numerator: then extended to improper and mixed fraction more than 1.

Quantity fraction $\frac{2}{3}$m implicates that the unit-quantity is 1m. If quantity is indicated on the number line, the number line shows quantity.

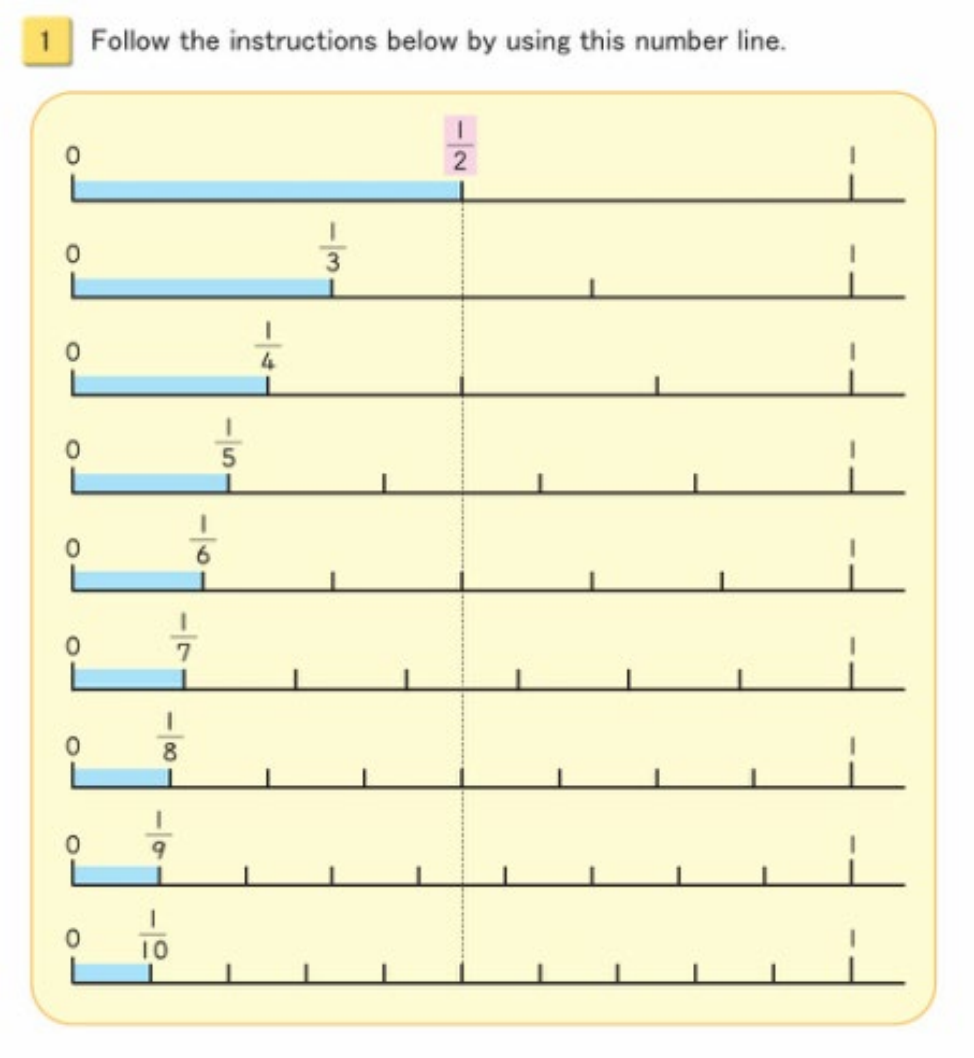


Figure 5. Equivalent fraction on the number line. GakkoTosho
(Grade 5, vol2, p24, 2005; Grade4,Vol2, p78, 2011)

Fraction is Expression: Fraction as Quotient

The answer of division is known as a number. When a fraction can be seen as a division expression, it supports to see fraction as a part of number.

Fraction is related with division which includes both meanings of quotative and dividing fraction on situations (see Chapter 2). In both situations, fraction represents the value of division such as $1 \div 3 = \frac{1}{3}$. We call it **fraction as quotient**. On this definition, fraction is connected with a number as well as the whole number (See figure 6a and 6b) because the answer of division should be a number on the perspective of the permanence of form. In relation to two division situations, $1 \div 3 = \frac{1}{3}$ implicates equally dividing like dividing fractions. $1 \div 3$=0.3333333… in action related with operational fraction by focusing on remainders. However, if we write $0.\overline{3}$ ("—" comes to the top of 3) it is $\frac{1}{3}$.

Exercise. Let's read and answer.

3 Fractions, Decimal Numbers and Whole Numbers

The Quotients of Divisions and Fractions

1 When we divide 2 ℓ of milk among □ students equally, how many liters will each student receive?

2÷□

① Enter the numbers from 1 to 5 in the □ and calculate the answers.

2÷□, 2÷□, 2÷□, 2÷□, 2÷□

② Divide the above equations into 3 groups based on the answers.

ⓐ Answers that are whole numbers.

(　　)

ⓑ Answers that are expressed exactly as decimal numbers.

(　　)

ⓒ Answers that are not expressed exactly as decimal numbers.

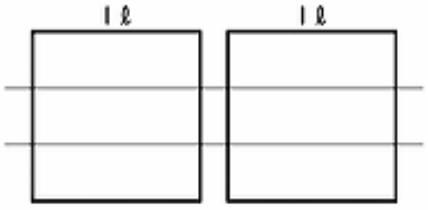
(　　)

2÷3 is 0.666..., so this cannot be expressed exactly as a decimal number because there is no end.

③ When 2 ℓ is divided equally among 3 students, how many liters does each student receive?

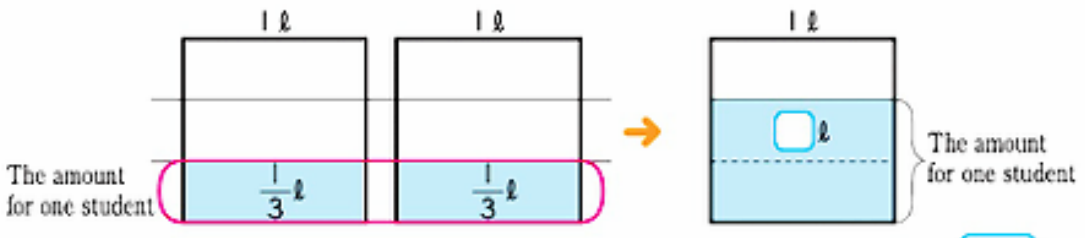
ⓐ Color in the portion for one student.

ⓑ How many liters is one portion?



Let's see how to express the quotient of a division problem when it cannot be expressed exactly as a decimal number.

Figure 6a. Gakko Tosho Grade 5. (vol.2. p29, 2005; vol.1. p128, 2011)



The amount for one student when 1 ℓ is divided into 3 equal parts ··· ☐ ℓ

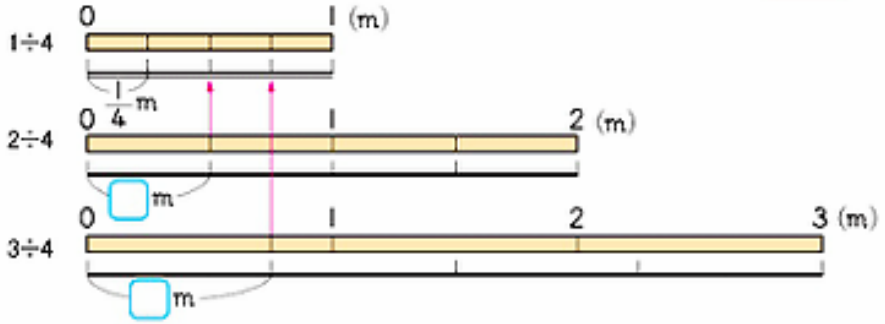
The amount for one student when 2 ℓ is divided to 3 equal parts ··· ☐ ℓ

$2 \div 3 = \frac{\square}{\square}$

2 How many meters is the length of each section when a 3m string is divided into 4 equal parts?

① Write an equation.

② What is the length of one section? $3 \div 4 =$ ☐



The quotient of a division problem in which a whole number is divided by another whole number can be expressed as a fraction.

$\bullet \div \blacksquare = \frac{\bullet}{\blacksquare}$

The quotient can be expressed precisely as a fraction.

Express the quotients in these problems as fractions.

① $1 \div 6$ ② $5 \div 8$ ③ $4 \div 3$ ④ $9 \div 7$

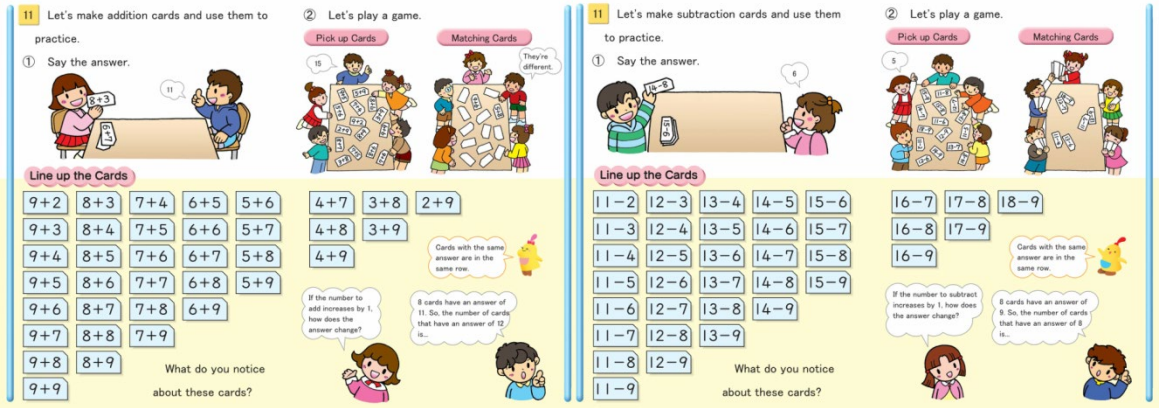
Figure 6b. Gakko Tosho Grade 5. (vol.2. p30, 2005; vol.1. p129, 2011)

If children can see the fraction as quotient, now fraction is a part of number which can be seen as the expression of division such as $\frac{1}{3} = 1 \div 3$. If fraction is given by the expression of division, the meaning of equivalence fraction is also understandable because it is not a mysterious property that only appeared on the fraction. In any arithmetic operation, it has similar properties. When the equivalence of fraction is recognized on the basis of the fraction as quotient, all four operations complete the properties of equivalence on their expressions.

On mathematics, it is the discussion of equivalent class.

Questions for professional development 4

Q19. Let's find the equivalent property of addition and subtraction: Gakkotosho Grade 1.



Q20. In Chapter 2, Q11, there are two meanings of dividing activity. Which activity can you see on the figure 6a and 6b and explain why.

Multiplication Table

multiplicand \ multiplier	1	2	3	4	5	6	7	8	9
row of 1	1	2	3	4	5	6	7	8	9
row of 2	2	4	6	8	10	12	14	16	18
row of 3	3	6	9	12	15	18	21	24	27
row of 4	4	8	12	16	20	24	28	32	36
row of 5	5	10	15	20	25	30	35	40	45
row of 6	6	12	18	24	30	36	42	48	54
row of 7	7	14	21	28	35	42	49	56	63
row of 8	8	16	24	32	40	48	56	64	72
row of 9	9	18	27	36	45	54	63	72	81

Q21. Let's find the same product in the multiplication table.

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