Fraction for Teachers

Knowing What before Planning How to Teach

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Preface for First Edition

Education is the work to prepare for the future. Developing children who learn mathematics by and for themselves is one of the major issues on mathematics education reforms in the world (See such as Isoda & Katagiri, 2012). After the comparative study of mathematics classroom such as TIMSS video study in 90s, Japanese lesson study is the world-shared methodology as for the tools for professional development because the study indirectly demonstrated the quality of Japanese mathematics teaching and it is established by the lesson study. However, people often misunderstand the lesson study as for the talking about the class rather than studying subject matter. They enjoy the classroom observation likely listening to the music or watching the theatre. However, through listening to the music, and even if we enjoy talking about actors, we cannot prepare the good player ourselves. In Japanese lesson study, most efforts are done for the preparation of the class. The misunderstanding originated due to the limitation of the content guidebook to refer in English. On this reason, I have developed several resources which show the theory for the purpose to improve mathematics education with researches in the world.

For the workshop of SMASE-INSET project under Japan International Cooperation Agency (JICA), Japan and Federal Ministry of Education (FME), Nigeria, this booklet includes the essential theory for enabling teachers to plan the class for developing children who learn mathematics by and for themselves. It focused on the innovation of elementary school mathematics based on the content which is well written in the textbooks in each country and known by teachers. The workshop done in Nigeria was based on the author's experience in Central and South America, South East Asia and Pacific as well as in Japan.

May 7, 2013

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Author Masami Isoda, PhD, acknowledges Gakko-Tosho which allowed the author to use the textbooks 'Study with Your Friends MATHEMATICS for Elementary School (2005) for this booklet. The content of this booklet was adopted for the workshop in SMASE-INSET by the author. Author acknowledges the following colleagues who contributed to the workshop in Abuja, Nigeria in May, 2013.

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Further CV and Publications:

Japanese Full

http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/profile.html

English Part
http://www.trios.tsukuba.ac.jp/Profiles/0006/0000997/prof_e.html

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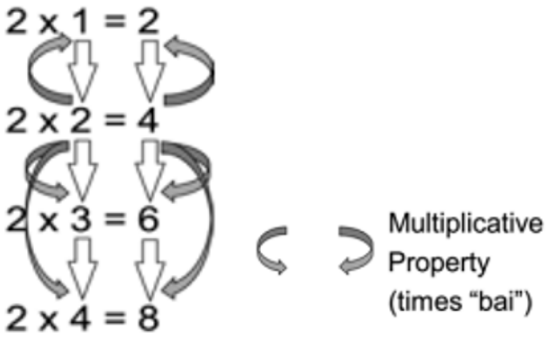
Chapter 7. Multiplication and Division of Fraction (2)

The reasoning for ratio and proportion is already begun from the multiplication at the second grade. Proportional number lines and Rules of three on the table are the representation of the proportionality: Children should study them before learning ratio and proportion for multiplication and division of fraction by and for themselves by using what they already learned.



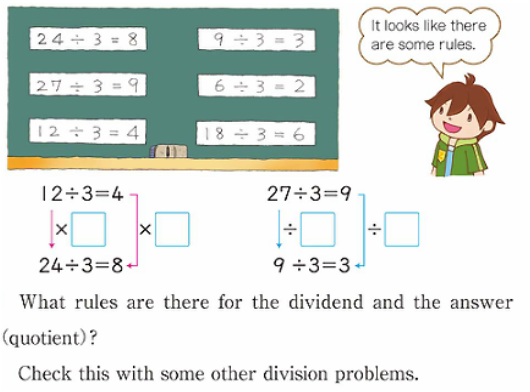
Rules of Multiplication and Division

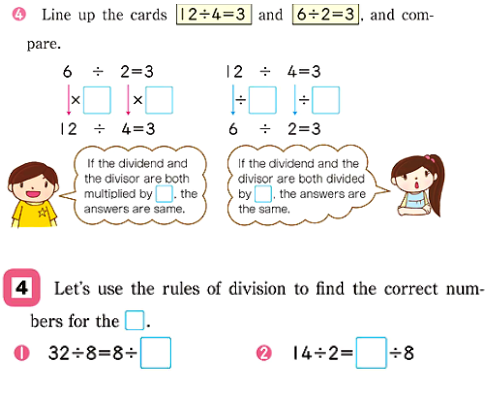
Rules of Multiplication and Rules of Division which appears the comparison of expressions for multiplication and division can be seen as the representation of the proportionality. Both rules are also useful for the extension of multiplication and division into decimals and fractions from the whole number.

Exercise:

The followings are the samples of the rules of multiplication and division:

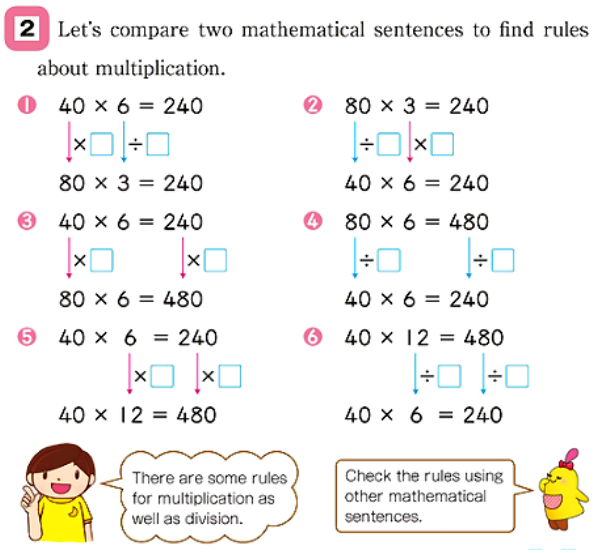
1) Let's find the rules of division.





Gakkotosho G4, vol.1, pp21-24 (2011)

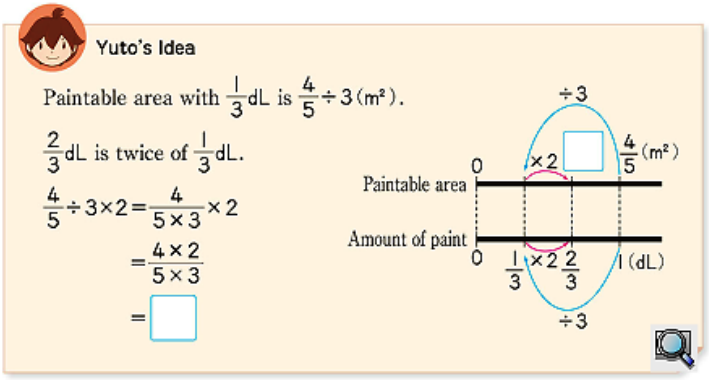
2) Let's find the rules of multiplication:

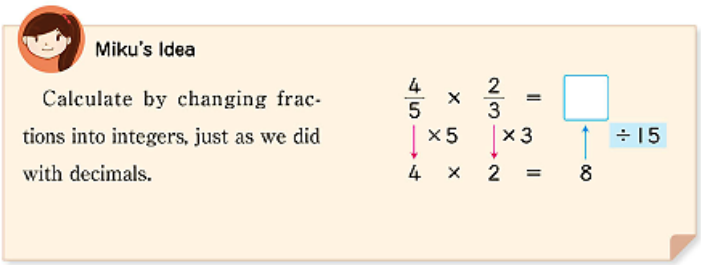


Gakkotosho G4, vol.1, p89 (2011)

3) Let's explain those rules by using four number table for the Rule of three.

4) Rules of multiplication are easier ways to find the answer of multiplication of fractions. Let's compare the following two approaches and explain how to use the rule of multiplication below (see page 49).

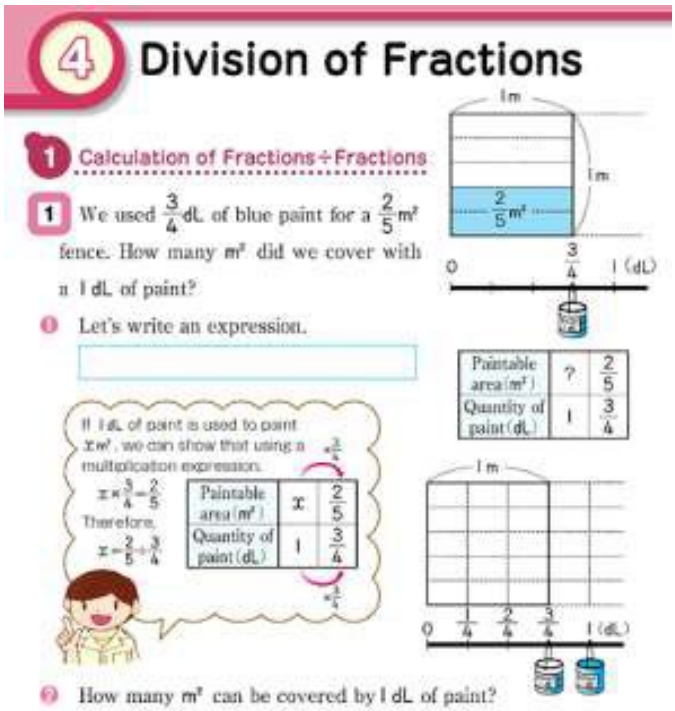




Gakkotosho, G6, vol.1, p36 (2011)

Exercise: Fraction divided by Fractions

1) Let's find the answer of the following task by the proportional number line, rule of three, rule of division, and area diagram.

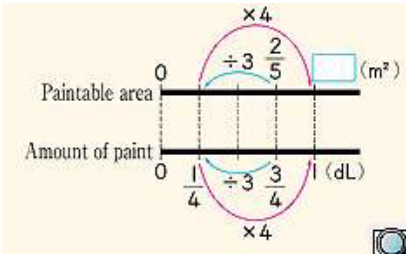


2) Explain the following approaches to obtain the answers.

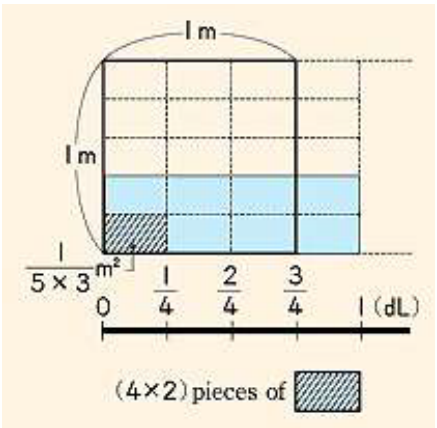
a) Rule of Division

$$\frac{2}{5}\div\frac{3}{4}=\left(\frac{2}{5}\times\frac{4}{3}\right)\div\left(\frac{3}{4}\times\frac{4}{3}\right)$$

b) Proportional Number Line



c) Area Diagram



3) Let's compare the three approaches. Which approach uses the unit fraction for changing the measurement scale? Which approach directly shows the way for the multiplication of inverse fraction? Which approach has the possibility to produce the wrong answer $\frac{8}{20}$?

Major Reference and Further Readings 7

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Summary

As discussed at the exercises of Chapter 1, this textbook is written for developing children who learn mathematics by and for themselves. For developing children, we should set the appropriate task sequence from what they already learned. Difficulty of fraction is related with various meanings of fraction and originated from many teachers who believe that the meaning of fraction is only dividing fraction. This textbook explained that it is the source of misconceptions. It demonstrated various meanings of fractions with terminology and the necessity of teachers considering them for overcoming difficulties.

In Japan, the terminology to distinguish the conceptual difference of fraction and so on were already known in the 1960s, at least, as a result of lesson study since 1873 with surveys of various countries' mathematics education. Terminology has been used to establish curriculum and task sequence on the textbooks. The author learned it from Prof. Tatsuro Miwa on his lectures for undergraduate students in 1981.

Due to difference of curricula sequence and textbooks, teachers who read this textbook may feel uncomfortable to use it. It might be normal as long as keeping the mindset to teach procedure by exercise with minimum explanation by teachers. However, if teachers try to develop children who explain their mathematical ideas by using what they already learned, the terminology might be significant. Indeed, Japanese teachers who show excellent practice use it for clarifying what children already learned and how they use it for thinking, and knowing necessity as for preparation of future learning.

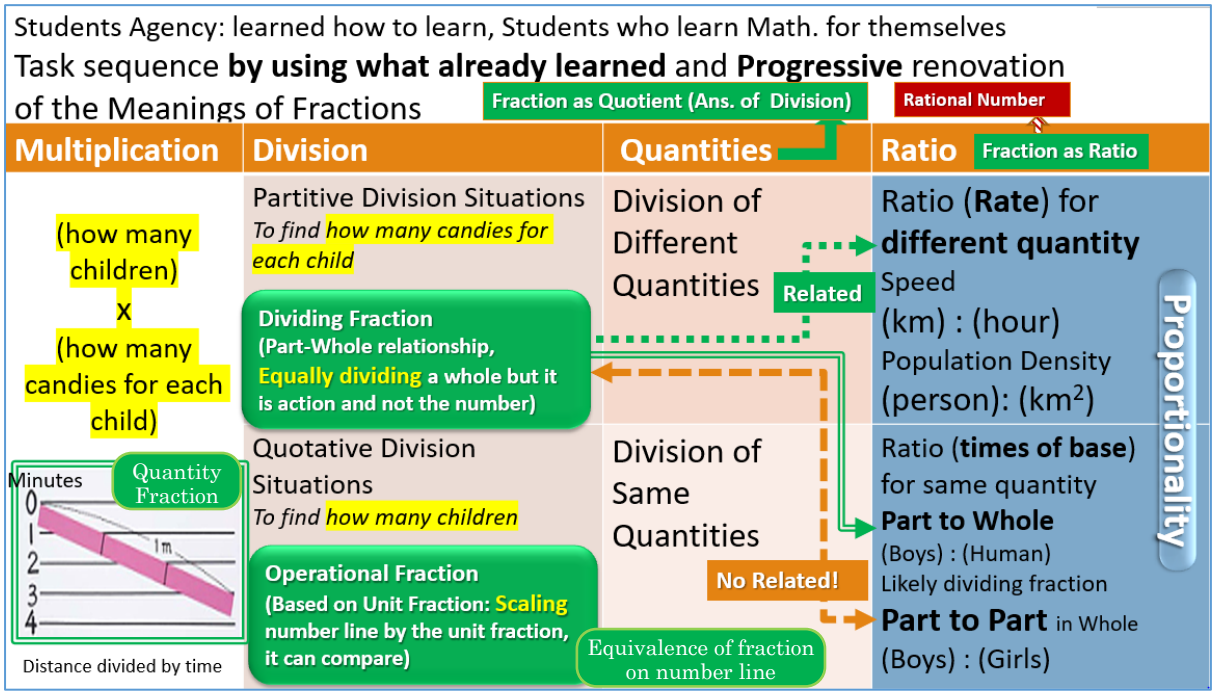

Figure 7. Various meanings of fraction and their development

This textbook used Gakko Tosho textbooks, see QR codes in the reference, which have been written by teachers in Elementary School of the University of Tsukuba which has been leading lesson study since 1873.

For using the terminology of this textbook, the task sequence to develop children who learn mathematics by and for themselves will be considered by Figure 7. Please note, it illustrates the necessary process for conceptual change under the extension and integration up to the fraction as ratio as for the base of rational number.

As long as teachers believe fraction means dividing fraction, dividing fraction may include operational fraction in a broad sense, however, in a narrow sense, the action for dividing into equally corresponds to the action for partitive division and the action for scaling by the unit fraction corresponds to the action for quotative division. On this difference, a narrow meaning of the dividing fraction should be distinguished with operational fraction. To change dividing fraction as comparable fraction, denomination of quantities is necessary for fixing and showing the size of the whole. To recognize fraction as number, equivalence of fraction on the number line and to see fraction as the answer of division are necessary. To consider the multiplication and division of fraction, it is necessary to treat the fraction under proportionality. In relation to proportionality, fraction is ratio. In relation to the meaning of division, there are three types on ratio. At the introductory stage, dividing fraction is a pair of numbers and not a number itself. For extension of number, we should discuss existence, comparison, and operations. After completion of these three discussions, fraction becomes the representation of rational number.

This book focused on fraction. Teaching of decimals can be considered analogically.

Further Readings

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