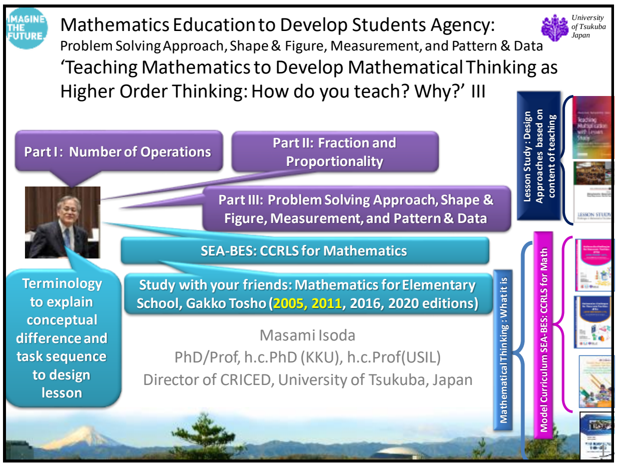
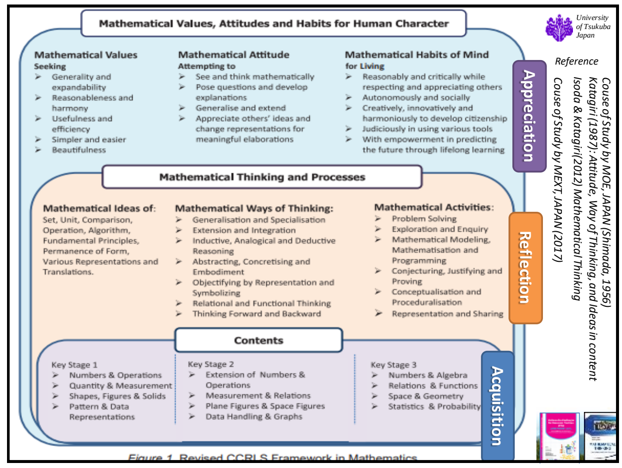
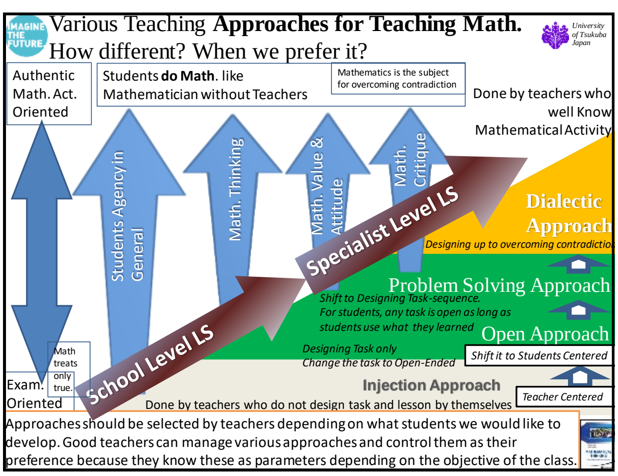

Mathematics Education to Develop Students Agency:
Problem Solving Approach, Shape & Figure, Measurement, and Pattern & Data
'Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why?' III
University of Tsukuba Japan
Part I: Number of Operations
Part II: Fraction and Proportionality
Part III: Problem Solving Approach, Shape & Figure, Measurement, and Pattern & Data
SEA-BES: CCRLS for Mathematics
Terminology to explain conceptual difference and task sequence to design lesson
Study with your friends: Mathematics for Elementary School, Gakko Tosho (2005, 2011, 2016, 2020 editions)
Masami Isoda
PhD/Prof, h.c.PhD (KKU), h.c.Prof(USIL)
Director of CRICED, University of Tsukuba, Japan
Lesson Study: Design Approaches based on content of teaching
Mathematical Thinking : What it is
Model Curriculum SEA-BES: CCRLS for Math

1


Mathematical Values, Attitudes and Habits for Human Character
Mathematical Values
Seeking
Generality and expandability
Reasonableness and harmony
Usefulness and efficiency
Simpler and easier
Beautifulness
Mathematical Attitude
Attempting to
See and think mathematically
Pose questions and develop explanations
Generalise and extend
Appreciate others' ideas and change representations for meaningful elaborations
Mathematical Habits of Mind
for Living
Reasonably and critically while respecting and appreciating others
Autonomously and socially
Creatively, innovatively and harmoniously to develop citizenship
Judiciously in using various tools
With empowerment in predicting the future through lifelong learning
Mathematical Thinking and Processes
Mathematical Ideas of:
Set, Unit, Comparison, Operation, Algorithm, Fundamental Principles, Permanence of Form, Various Representations and Translations.
Mathematical Ways of Thinking:
Generalisation and Specialisation
Extension and Integration
Inductive, Analogical and Deductive Reasoning
Abstracting, Concretising and Embodiment
Objectifying by Representation and Symbolizing
Relational and Functional Thinking
Thinking Forward and Backward
Mathematical Activities:
Problem Solving
Exploration and Enquiry
Mathematical Modeling, Mathematisation and Programming
Conjecturing, Justifying and Proving
Conceptualisation and Proceduralisation
Representation and Sharing
Contents
Key Stage 1
Numbers & Operations
Quantity & Measurement
Shapes, Figures & Solids
Pattern & Data Representations
Key Stage 2
Extension of Numbers & Operations
Measurement & Relations
Plane Figures & Space Figures
Data Handling & Graphs
Key Stage 3
Numbers & Algebra
Relations & Functions
Space & Geometry
Statistics & Probability
Appreciation
Reflection
Acquisition
Figure 1 Revised CCRLS Framework in Mathematics
University of Tsukuba Japan
Reference
Couse of Study by MOE, JAPAN (Shimada, 1956)
Katagiri (1987): Attitude, Way of Thinking, and Ideas in content
Isoda & Katagiri(2012) Mathematical Thinking
Couse of Study by MEXT, JAPAN (2017)

2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University of Tsukuba Japan
Authentic Math. Act. Oriented
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Shift to Designing Task-sequence.
For students, any task is open as long as students use what they learned
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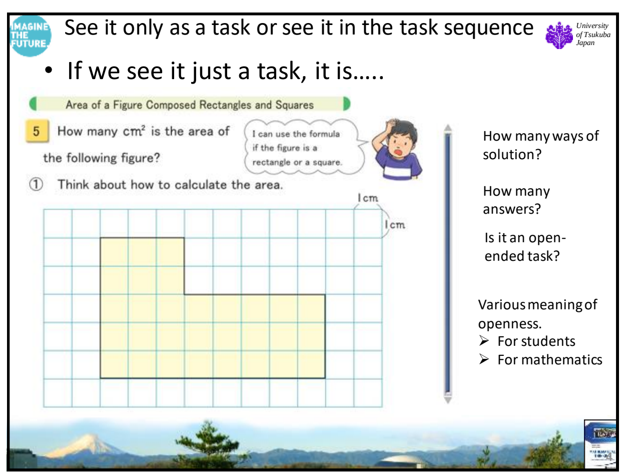
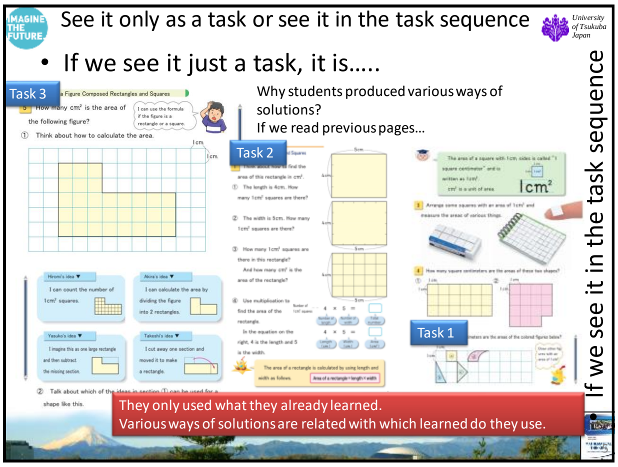
Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.

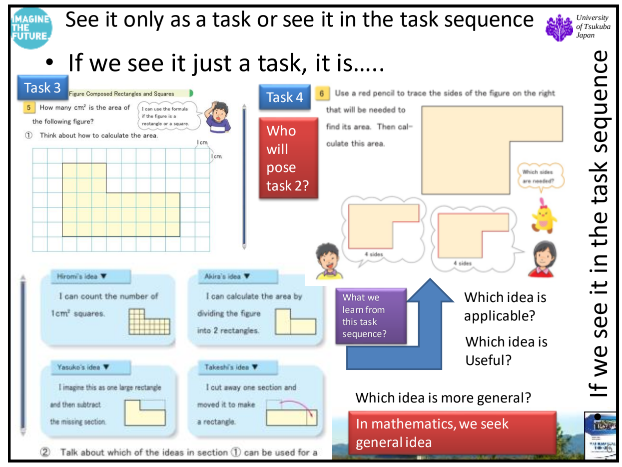
3


See it only as a task or see it in the task sequence
University of Tsukuba Japan
If we see it just a task, it is.....
Area of a Figure Composed Rectangles and Squares
5 How many cm² is the area of the following figure?
I can use the formula if the figure is a rectangle or a square.
① Think about how to calculate the area.
1 cm
1 cm
How many ways of solution?
How many answers?
Is it an open-ended task?
Various meaning of openness.
For students
For mathematics

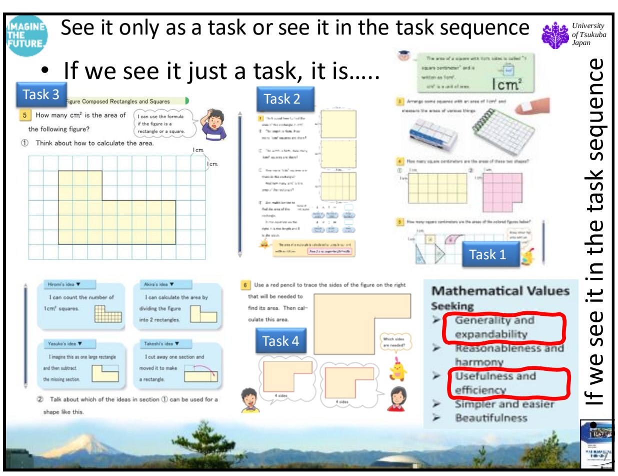
4


See it only as a task or see it in the task sequence
University of Tsukuba Japan
• If we see it just a task, it is…..
Task 3
Why students produced various ways of solutions?
If we read previous pages…
Task 2
Task 1
If we see it in the task sequence
They only used what they already learned.
Various ways of solutions are related with which learned do they use.

5


See it only as a task or see it in the task sequence
University of Tsukuba Japan
• If we see it just a task, it is…..
Task 3
Task 4
Who will pose task 2?
What we learn from this task sequence?
Which idea is applicable?
Which idea is Useful?
Which idea is more general?
In mathematics, we seek general idea
If we see it in the task sequence

6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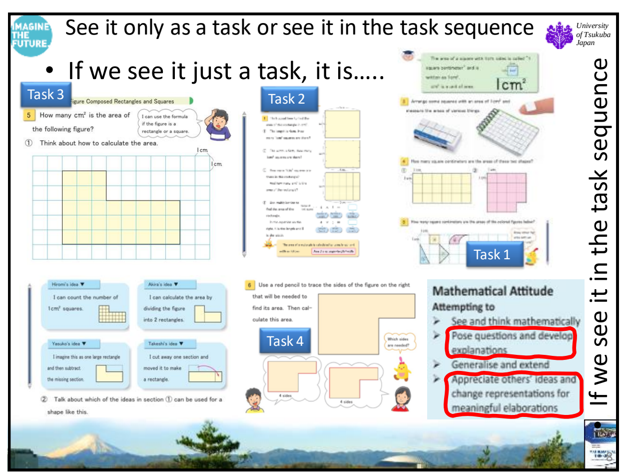
7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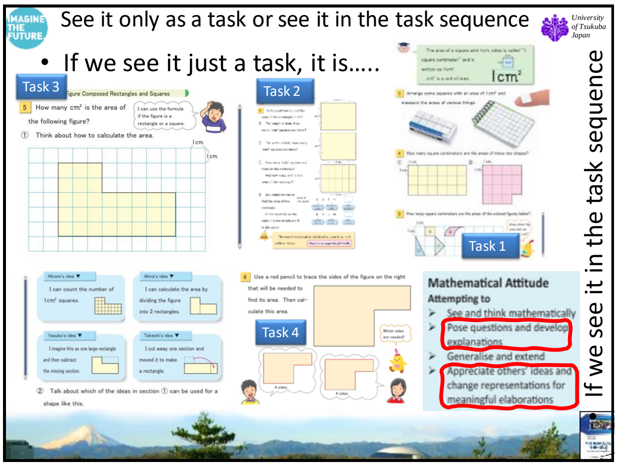
If we see it in the task sequence

8



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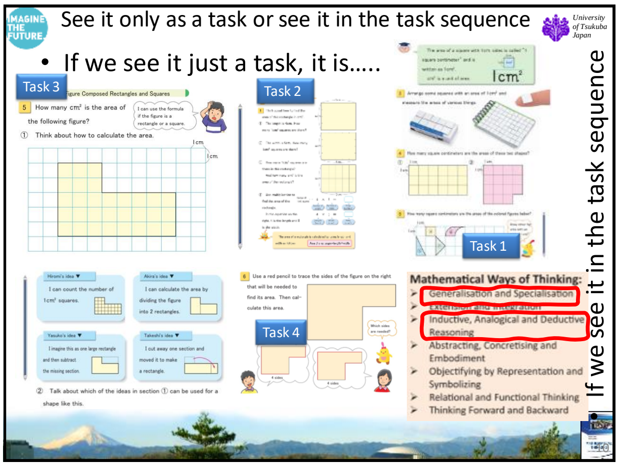

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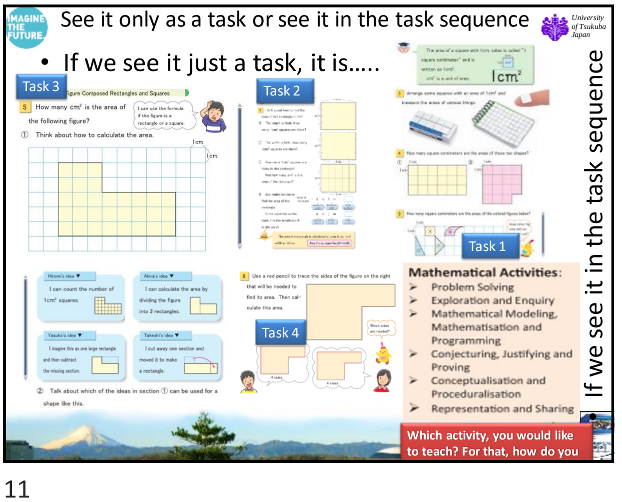

11

In Japan, Open A. is a part of Prob. Solv. A.

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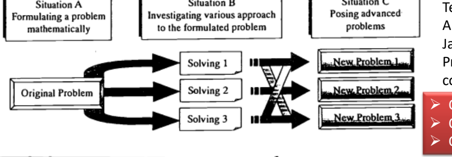
Open A. originated teaching practice in the Secondary School of UT before WWII and the national textbook was published in 1943. Shimada, S. rebirthed it the Open-Ended A. Project at the end of 1960s and published the book (1976).

1) **Theorization**: Nohoda, N. wrote the PhD thesis in 1983.
2) **Extension**: Sawada, T. lead the project for **the problem sequence developped by students**.
3) **Spread to the World**: Becker, J. published English translation of the book in 1996 in USA.

Problem Sequence in Nohoda (1983)

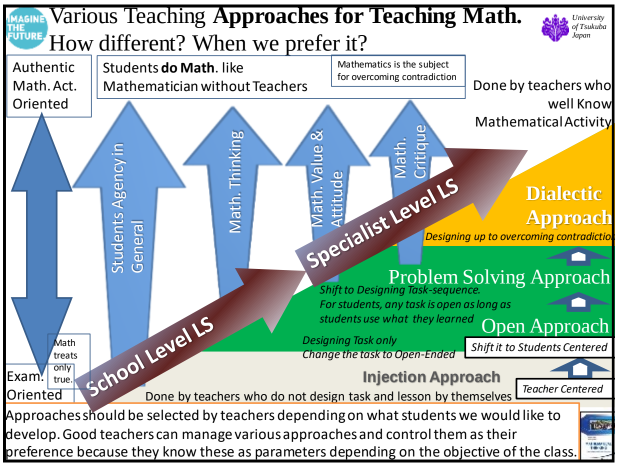


Nohoda, N. (2000). Teaching of Open Approach Methods in Japanese Classroom. In Proceedings of the conference of IGPME.

- Open for students
- Open for mathematics
- Open for teachers

	K. &Ele.	Lower Sec.	Upper Sec.	Tertiary
Total Num. Of Presentation	5246	3586	3213	435
Ratio of "Prob. Solv."	5.4%	2.6%	0.4%	1.1%
Ratio of "Open (+words)"	0.4%	0.9%	0.2%	0

12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