

Mathematics Education to Develop Students Agency
Part III: Measurement
Length and Mass (Weight)

Which one is longest?
 a) 2.3m
 b) 23cm
 c) 230mm
 d) 0.23m
 e) 2m3cm
 f) 2m30cm
 g) 2cm3mm

3. Measurement
 9. How to introduce the unit for measurement
 10. Length and Mass (Weight)
 11. Area and Volume
 12. Time, Angle and Others (rate and ratio)

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Review: Phases to introduce Unit. Why it is necessary?
 We discussed it at the last lesson.

| Phase | Activity | Mathematical Process on each phase | Behind Mathematics | Two types of terminology |
|-------|--|--|---|---|
| P1 | Direct Comparison | Compare two objects on the same conditions ➢ Can compare more objects when we can apply the transitive law: If $a > b$, $b > c$, then $a > c$. ➢ In case $a > b$, $a > c$, then we have to compare b and c : we have to find the algorithm or the possible way to make an order for comparisons. | Axioms for comparison: Mathematical relationships for greater than, less than and equal. | ➢ First one is invariant: Mathematical Thinking |
| P2 | Indirect Comparison (ordering) | Compare any objects on the same conditions by using the alternative material such as a tape. We can demonstrate and record only their order . ➢ We can make an order but not sure the difference , how much more. | + The line/ray only have the origin point and direction. | ➢ Second one is variant which express conceptual differences to explain necessary task sequence. |
| P3 | Arbitrary (non-standardized) Unit (denominated number) | We can specify the difference on the alternative material as long as we can measure by using something as the arbitrary unit on the material. We can record it as data and use . ➢ It works only locally as long as we use the same unit as scale. ➢ If not, we have to seek the sharable scale for measurement: One is seeking smaller scale. Another is seeking other materials for the unit. ➢ It is a kind of denominated number which can be used locally and specifically but does not work as the universal quantity. | + The number line set by the origin, the unit and direction. ➢ As long as we can find greatest common divisor, we can measure with support of Euclidian Algorithm . ➢ Mathematically, irrational number is problematic, however it does not appear practically. | In this lecture, we use both terminology . Instead of preferring the word 'concept.' |
| P4 | Standard Unit (quantity) | Standard unit can be defined universal under the politically acceptance of academies and countries . Recorded data works anywhere as quantity . | | WHY? |

Review: Phases to introduce Unit. Why it is necessary?
 We discussed it at the last lesson, right?

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| P1 | Direct Comparison | Compare two objects on the same conditions ➢ Can compare more objects when we can apply the transitive law: If $a > b$, $b > c$, then $a > c$. ➢ In case $a > b$, $a > c$, then we have to compare b and c : we have to find the algorithm or the possible way to make an order for comparisons. | Axioms for comparison: Mathematical relationships for greater than, less than and equal. | |
| P2 | Indirect Comparison (ordering) | Compare any objects on the same conditions by using the alternative material such as a tape. We can demonstrate and record only their order . ➢ We can make an order but not sure the difference , how much more. | + The line/ray only have the origin point and direction. | Why we do not prefer the word 'concept.' Because the word 'concept' implicates mathematical concept . If you can explain it, exactly, it must be necessary. If not, to explain school curriculum and task sequence, it must be better to use our terminology. |
| P3 | Arbitrary (non-standardized) Unit (denominated number) | We can specify the difference on the alternative material as long as we can measure by using something as the arbitrary unit as data and use . ➢ As long as we can find greatest common divisor, we can measure with support of Euclidian Algorithm . ➢ Mathematically, irrational number is problematic, however it does not appear practically. | + The number line set by the origin, the unit and direction. ➢ As long as we can find greatest common divisor, we can measure with support of Euclidian Algorithm . ➢ Mathematically, irrational number is problematic, however it does not appear practically. | |
| P4 | Standard Unit (quantity) | | | |

Review: Part I Learning the operation of measurement by using what we learned at the number and operation

Numerals in English as a language
 English numeral is NOT the Base 10 system. How about in your language?

Cardinal Number (set number):
 One, two, three, ..., nine, ten, eleven, twelve, ..., nineteen, twenty, twenty-one (20+1), ..., thirty, thirty-one, ... not exact base ten system?

Ordinal Number:
 First, Second, Third, Forth, ..., ninth, tenth, eleventh, ..., nineteenth, twentieth, twenty first, ...

Multiple:
 Once (one time), twice (two times), thrice (three times), ...
 Half, double (twofold), triple (threefold), ...

What is the number?
 We usually teach:
 • Existence and necessity
 • Order/Larger or Greater or Less
 • Operations

In Japan:
 Make sense (understand meaning)
Think about how to calculate/operate/find the easier way to answer
 Acquisition of proficiency
Try to teach how to extend the number
 Change the denominations

Number shows **magnitude** (size) but not show the quantity.
 Denominated number VS Compound (multi-denominated) number

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Necessity of the operations on the measurement.

- Existence and necessity
- Order/Larger or Smaller/ Greater or Less
- Operations

Activity

| | |
|----|--|
| P1 | Direct Comparison |
| P2 | Indirect Comparison (ordering) |
| P3 | Arbitrary (non-standardized) Unit (denominated number) |
| P4 | Standard Unit (quantity) |

Because in our life, we usually use the number as quantity.
We cannot introduce and extend number itself without using denominated number.

Lengths

Who is the longest?

Let's compare yours with a friend?

Let's play a game with rock-paper-scissors.

Do rock-paper-scissors, and if you win with you get A, if you win with you get B, and if you win with you get C.

You stick the tapes together when you get them.

Why we introduce standard unit from this activity?

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Mass

Activity

| | |
|----|---------------------|
| P1 | Direct Comparison |
| P2 | Indirect Comparison |
| P3 | Arbitrary Unit |
| P4 | Standard Unit |

Let's think about ways to compare lengths.

Let's compare yours with a friend?

Let's play a game with rock-paper-scissors.

Do rock-paper-scissors, and if you win with you get A, if you win with you get B, and if you win with you get C.

You stick the tapes together when you get them.

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QR Code

How to Represent Length

Let's measure the width of a pencil using the ruler.

The length of the pencil is 1 centimeter.

1 centimeter is written as 1 cm.

How many cm is the width of the pencil?

1 cm 2 cm 3 cm 4 cm 5 cm 6 cm 7 cm 8 cm 9 cm 10 cm

How to Compare

1. Is Masato's tape really longer than Akira's tape?

2. We put a book and a box of water-color paints on a grid paper.

3. How many squares is the length and width of the book? And the box?

4. Which is longer for each one, the length or the width? By how much?

5. Let's cut the grid paper to make a tool for measuring length. Let's use the tool to measure various objects.

6. 0 1 2 3 4 5 6 7 8 9 10

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Length/Distance are defined by subtraction in operation.

Road Distance is a total of several distances.

It is useful to use a 1m ruler when measuring long objects.

1.5cm = 1m 15cm

How long is the tape by using the 1m scale and 30 cm scale?

1m and remain some.

Four times of 30 cm and short some.

Activity

| | |
|----|---------------------|
| P1 | Direct Comparison |
| P2 | Indirect Comparison |
| P3 | Arbitrary Unit |
| P4 | Standard Unit |

Fill the ☐ with a correct unit.

1. The length of the classroom is ☐.

2. The road distance that we walk in one hour is ☐.

3. The height of the desk is ☐.

4. The height of Mt. Fuji is ☐.

How many meters and centimeters are there at the arrows ☐ on the tape measure?

Which is longer?

1. 2km 50m; 2030m 2. 1580m; 1 km 59m

3. 5km; 4980m 4. 3km 530m - 540m

Let's calculate.

1. 700m + 500m 2. 1 km 900m + 200m

3. 5km 400m + 680m 4. 1 km - 300m

5. 2km 500m - 800m 6. 3km 530m - 540m

Syn goes to the school.

Then, which is the longest road distance through Ryoko's house or Midori's house, and by how long?

Calculate the Length in Vertical Form

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Mass

Activity

| | |
|----|---------------------|
| P1 | Direct Comparison |
| P2 | Indirect Comparison |
| P3 | Arbitrary Unit |
| P4 | Standard Unit |

Let's try comparing weights by using some tools.

Express weight as numbers by using 1 yen coins.

Weight is measured by finding how many units of weight something is equal to.

There is a unit called **gram** that is used to measure weight.

1 gram is written as 1 g.

The weight of a 1 yen coin is 1 g.

What is the weight of a pair of scissors, a compass and a glue, respectively?

Measure the weight of different things using 1 yen coins.

0.1 kg = 100 g

3.1 kg = kg g

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Units of Quantities

10 We have learned the units of length, amount of water and weight. There are units of quantities as follows.

Length mm, cm, m, km

Weight g, kg, t

Amount of water mL, dL, L

1 Fill the with a number.

1 m = mm 1 L = mL

1 km = m 1 kg = g

2 Let's discuss what you found. And write down your notebook.

The bases units of measurement are m, L and g, aren't you?

There are measurement unit which added x or m, aren't you?

1000 of unit develop a new unit, don't you?

Measurement System

International System of Units (Wikipedia)
SI base units

| Symbol | Name | Quantity |
|--------|----------|---------------------------|
| s | second | time |
| m | metre | length |
| kg | kilogram | mass |
| A | ampere | electric current |
| K | kelvin | thermodynamic temperature |
| mol | mole | amount of substance |
| cd | candela | luminous intensity |

All these are the content of learning!

This lecture use SEA-BES: CCRLS to explain objectives

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values

- Generosity and generosity
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

Mathematical Attitude

- Attempting to
- See and think mathematically
- Pose questions and develop explorations
- Generate and extend
- Appreciate others' ideas and change representations for meaningful elaborations

Mathematical Habits of Mind for Living

- Reasonably and critically while respecting and appreciating others' explorations
- Automatically and socially
- Creatively, inventively and humbly to develop citizenship
- Indicatively in using various tools
- With empowerment in predicting the future through lifelong learning

Mathematical Thinking and Processes

Mathematical Ideas of: Set, Unit, Comparison, Operation, Algorithm, Transformation, Reasoning, etc.

Mathematical Ways of Thinking:

- Generalization and Specialization
- Extension and Integration
- Inductive, Analogical and Deductive Reasoning
- Abstracting, Generalizing and Endorsing
- Classifying by Representation and Symbolizing
- Relational and Functional Thinking
- Thinking Forward and Backward

Mathematical Activities:

- Problem Solving
- Explorative and Enquiry
- Mathematical Modeling, Mathematical and Engineering
- Computing, Justifying and Proving
- Generalization and Precorollation
- Representation and Unifying

Contents

Key Stage 1

- Extension of Numbers & Operations
- Measurement & Numbers
- Place Figures & Space Figures
- Data Handling & Graphs

Key Stage 2

- Numbers & Algebra
- Relations & Functions
- Space & Geometry
- Statistics & Probability

APPENDIX B
Terminologies Explained

These terminology explain the process. Thus, if you can imagine concrete materials for each class, you are able to develop it in your classroom. That's why we have been discussing materials.

Mathematical Ideas

After through every mathematical content embedded some ideas, these are essential mathematical ideas which are used in various occasions. Mathematical ideas are not exclusive but functions as complementary. The following are examples of essential mathematical ideas.

| Terminology | Explanation |
|-------------|---|
| Set | A set is a collection of elements based on certain conditions. When the collection of the set changes, result of reasoning related to the set changes too. Sets are compared by one-to-one correspondence. Basically, the idea of set is reflected. |
| Unit | Unit is necessary for counting, measurement, number line, operations and transformation. It is represented as "denomination" for discrete quantity, such as 1 "apple" for situations involving counting, or continuous quantity, such as 1 gram for situations involving measurement. Mathematically, unit is used to indicate a number by mapping it with the quantity in a situation. In a situation, it can be fixed based on the context of comparison, which can either be direct or indirect comparison. In this context, a remainder or a difference from a comparison can be used for fixing a new arbitrary unit for measurement which is a fraction of the original unit. This process of determining a new unit is the application of Euclidean algorithm for finding the greatest common divisor. For the base-10 place value number system, every column is defined by the units such as ones, tens, hundreds and so on. However, in other place value number system such as the binary system, every column is defined by the units such as ones, twos, fours and so on. Therefore, in a place value number system, the unit is not always a multiple of the power of ten. In addition, various other number systems are made up of different units. For the calendar system, the lunar calendar is based on 30 (29.5) days, while the solar |

Figure 1. Revised CCRLS Framework in Mathematics