

**Mathematics Education to Develop Students Agency**  
**Part III: Measurement**  
**Length and Mass (Weight)**

Which one is longest?  
 a) 2.3m  
 b) 23cm  
 c) 230mm  
 d) 0.23m  
 e) 2m3cm  
 f) 2m30cm  
 g) 2cm3mm

Conversion of measuring quantity

Various Representations and Translations.

Appreciate others' ideas and change representations for meaningful elaborations

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3. Measurement

9. How to introduce the unit for measurement

10. Length and Mass (Weight)

11. Area and Volume

12. Time, Angle and Others (rate and ratio)

**Miracle!?**  
 Japanese teacher can demonstrate the problem-solving approach at any country! Why? Because....

He knows Misconception and know how to manage through dialectic approach

At Seoyi Elementary School (감사합니다) in Seoul, Korea

문제  
 먼저 정사각형 을 만들었습니다

Which is larger?

마로모 평면 기하학

**Why Japanese teacher can do problem solving approach?**

**Review Part III Lesson 1~4**

**Mathematical Ways of Thinking:**  
 ➢ Generalisation and Specialisation  
 ➢ Extension and Integration

4. Make a rectangle with 40cm<sup>2</sup> area and 8cm width.  
 What is its length in cm?  
 Let's think about how to find the answer using the formula for the area of a rectangle.

Area = 40cm<sup>2</sup>  
 Width = 8cm  
 Length = ?

Area = Length × Width  
 40 = Length × 8  
 Length = 40 ÷ 8  
 Length = 5

5. What is the area in cm<sup>2</sup> of the following figure?  
 Let's think about how to find the area.

Area = 10 × 5 = 50

**Review:**  
 Two types of terminology  
 First one is invariant: Mathematical Thinking  
 ➢ Second one is variant which express conceptual differences to explain necessary task sequence.

Phase	Activity	Mathematical Process on each phase	Behind Mathematics	Problem Solving Approach
P1	Direct Comparison	Compare two objects on the same conditions ➢ Can compare more objects when we can apply the transitive law: If a > b, b > c, then a > c. ➢ In case a > b, a > c, then we have to compare b and c: we have to find the algorithm or the possible way to make an order for comparisons.	<b>Axioms for comparison:</b> Mathematical relationships for greater than, less than and equal.	<b>On appropriate task sequence</b> ➢ Students use what already learned ➢ Teacher prepare future learning.
P2	Indirect Comparison (ordering)	Compare any objects on the same conditions by using the alternative material such as a tape. We can demonstrate and record <b>only their order</b> . ➢ We can make an order but <b>not sure the difference</b> , how much more.	+ The line/ray only have the origin point and direction.	
P3	Arbitrary (non-standardized) Unit (denominated number)	We can specify the difference on the alternative material as long as we can <b>measure</b> by using something as the arbitrary unit on the material. We can record it as <b>data and use</b> . ➢ It works only locally as long as we use the same unit as scale. ➢ If not, we have to seek the shareable scale for measurement. One is seeking smaller scale. Another is seeking other materials for the unit. ➢ It is a kind of <b>denominated number</b> which can be used locally and specifically but does not work as the universal quantity.	+ The number line set by the origin, the unit and direction. ➢ As long as we can find greatest common divisor, we can measure with support of Euclidian Algorithm. ➢ Mathematically, irrational number is problematic, however it does not appear practically.	<b>In class,</b> ➢ Teacher provide the task ➢ Students find the problematic which coming from unknown
P4	Standard Unit (quantity)	Standard unit can be <b>defined universal</b> under the <b>politically acceptance of academies and countries</b> . Recorded data works anywhere as <b>quantity</b> .		

## Let's explain the reason why this task sequence?

### 12 Area

Which one is larger?

1. We are going to make rectangles and square flower beds with 20 blocks around the edges.

2. What rectangles can we make other than (a), (b), (c) and (d)?

3. Which one has the largest area?

Let's think about how to compare the areas of rectangles and squares and how to represent the areas with numbers.

Compare the areas of (a) with (d).

1. **Daiki's idea**  
Then one on top of the other and then cut the two sections that stick out.

2. **Yumi's idea**  
I drew squares of the same size on the blocks.

3. **Size is the amount of space surrounded by a line. This size expressed as a number is called area.**

4. There are two sheets of colored paper (a) and (b). Which one is larger or how much larger is it? Check by drawing squares with 1cm sides.

## Let's explain task sequence!

### Review Part III Lesson 3

1. What is the area in  $\text{cm}^2$  of the colored figures below?

2. What is the area in  $\text{cm}^2$  of the colored figures below?

3. Area of Rectangles and Squares

1. Let's think about how to find the area of a rectangle on the right in  $\text{cm}^2$ .

2. The width is 4cm. How many 1cm squares can we fit in the width?

3. The length is 6cm. How many 1cm squares can we fit in the length?

4. Let's think about how to find the area of a rectangle.

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## Student Agency who learned how to learn!

### If students use what they learned, they can learn by and for themselves without teacher!

### 13 Area of Figures

1. You made a frame of paper as shown on the right. The frame can change freely by moving. Let's think about the area of quadrilaterals made by the frame.

2. Which shape has the larger area of quadrilaterals (a) or (b)?

3. Let's think about how to calculate the area of quadrilaterals.

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## Where is the height?

### Height can not define on the figure but the plane figure!!

1. Check the lengths of the parallelogram (a) to find the area, and then find the area.

2. Which lengths do you need to find the area of quadrilaterals (b), (c) and (d)?

3. Let's find the area of the parallelogram below.

4. When side BC is the base, find the area by measuring the height.

5. When side CD is the base, find the area by measuring the height.

6. The height depends on the base.

7. Let's find the area of the following parallelograms.

8. Let's think about how to find the area of a parallelogram with side BC as the base.

9. Explain how to find the area by looking at the figures below.


10. What is the area of the parallelogram in  $\text{cm}^2$ ?

11. When side BC is the base, the distance between lines (a) and (b) is the height of parallelogram ABCD.



## Extension of quantity by using remaining part

### Why does Japan use dL?



**8 Amount of Water**

**1** Over the phone, Mitsu and Taro are discussing which of their water bottles can hold more water.

**2** Can we say Taro's water bottle holds more water? Why is that?

**3** How can you compare the amounts of water accurately?

**How to Measure the Amount of Water**

Amount of water can be measured by using a measuring cup.

There is a unit called **1L** to express amount of water. 1 liter is written as 1L.

The two students in the picture above each hold 1L.

**1** For the amount of water contained in two containers, we measure it with measuring cups. How many cups are there?

Plastic bottle      Bucket

**How Many Cups Can This Hold?**

Take a 1L measuring cup and fill it with water, and measure the amount of water in the containers.

**Smaller Measuring Cup**

**1** The amount of water in the glass bottle was measured using a 1L measuring cup. How can we measure the portion that is less than 1L?

To measure the portion less than 1L, we can use a **1 deciliter** measuring cup. How many 1 deciliter measuring cups will we need?

When 1L is divided into 10 equal parts of amount, the amount of one part is called 1 deciliter. 1 deciliter is written as **1dL**.

Deciliter is another unit of amount. **1L = 10dL**.

**2** There is 2L of juice in the glass bottle, and there is 1L of juice in the plastic bottle.

**3** How many L, and dL, are there altogether? Write an expression.

**4** Let's think about how to calculate.

2L 4dL = 2L 4dL  
1L 8dL = 1L 8dL


Let's calculate.

2L 4dL + 1L 8dL = 3L 12dL = 3L 2dL

1L 8dL + 1L 8dL = 2L 16dL = 2L 6dL

## Let's explain task sequence!

### Why they discuss remaining part?



**Volume**

**1** Let's show the amount of rectangular prism and cube on a prepared paper. How can you show the paper?

**2** Which one is the largest among the three?

**3** Let's think about how to calculate.

**4** Let's compare the size of boxes which these children prepared.

**5** How many 1cm cubes are needed for following rectangular prism and cube?

**6** Let's think about how to compare size of boxes.

**7** Let's explain how we can represent the size of rectangular prism and cube.

**8** Let's think about how to compare size of boxes.

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## All these are the content of learning!!

### This lecture use SEA-BES: CCRLS to explain objectives

**Mathematical Values, Attitudes and Habits for Human Character**

**Mathematical Values**

- Generosity and responsibility
- Reasonableness and fairness
- Usefulness and efficiency
- Dispute and error
- Beneficence

**Mathematical Attitude**

- Interest in mathematics
- Love and enthusiasm for mathematics
- Curiosity and eagerness to learn
- Perseverance and determination
- Confidence and self-belief
- Cooperation and teamwork
- Respect for others' opinions
- Open-mindedness and tolerance
- Responsibility and accountability
- Integrity and honesty
- Resilience and perseverance
- Curiosity and eagerness to learn
- Perseverance and determination
- Confidence and self-belief
- Cooperation and teamwork
- Respect for others' opinions
- Open-mindedness and tolerance
- Responsibility and accountability
- Integrity and honesty
- Resilience and perseverance

**Mathematical Habits of Mind for Learning**

- Reasoning and critical thinking
- Problem-solving and decision-making
- Communication and collaboration
- Creativity and innovation
- Self-reflection and evaluation
- Goal-setting and planning
- Time management and organization
- Resource management and utilization
- Adaptability and flexibility
- Resilience and perseverance
- Curiosity and eagerness to learn
- Perseverance and determination
- Confidence and self-belief
- Cooperation and teamwork
- Respect for others' opinions
- Open-mindedness and tolerance
- Responsibility and accountability
- Integrity and honesty
- Resilience and perseverance

**Mathematical Thinking and Processes**

**Mathematical ideas of:**

- Set, Unit, Comparison, Operation, Algebra, Geometry, Probability, Statistics, Measurement, Data Handling, Graphs, Functions, Trigonometry, Calculus, Differential Equations, Vector, Matrix, Complex Numbers, Probability, Statistics, Measurement, Data Handling, Graphs, Functions, Trigonometry, Calculus, Differential Equations, Vector, Matrix, Complex Numbers

**Mathematical Ways of Thinking:**

- Generalization and Specialization
- Inductive and Deductive Reasoning
- Abstracting, Concretizing and Idealizing
- Classifying by Representation and Symbolizing
- Modeling and Functional Thinking
- Thinking Forward and Backward

**Mathematical Activities:**

- Problem Solving
- Exploration and Inquiry
- Mathematical Thinking, Mathematical Modeling and Mathematical Communication
- Reasoning and Critical Thinking
- Problem Solving and Decision Making
- Communication and Collaboration
- Creativity and Innovation
- Self-reflection and Evaluation
- Goal-setting and Planning
- Time management and Organization
- Resource management and Utilization
- Adaptability and Flexibility
- Resilience and Perseverance
- Curiosity and Eagerness to Learn
- Perseverance and Determination
- Confidence and Self-belief
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- Respect for Others' Opinions
- Open-mindedness and Tolerance
- Responsibility and Accountability
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- Resilience and Perseverance

**Contexts**

**Key Stage 1**

- Number & Algebra
- Measurement & Geometry
- Statistics & Probability

**Key Stage 2**

- Number & Algebra
- Measurement & Geometry
- Statistics & Probability

**Figure 1 Revised CCRLS Framework in Mathematics**

**APPENDIX B**

**Terminologies Explained**

**These terminology explain the process. Thus, if you can imagine concrete materials for each class, you are able to develop it in your classroom. That's why we have been discussing materials.**

**Mathematical ideas**

From through above mathematical content, mathematical ideas are shown. These are essential mathematical ideas which are used in various situations. Mathematical ideas are not arbitrary but founded on comprehensive knowledge. The following are examples of typical mathematical ideas.

**Terminology**

**Unit**

Unit is necessary for counting, measurement, number line, operations and transformation. It is represented as "denominator" for division quantity, such as a "apple" for situations involving counting, or continuous quantity, such as 1 gram for situations involving measurement. Mathematically, unit is used to indicate a number by measuring it with the quantity in a situation.

**Base**

In a situation, it can be fixed based on the context of comparison, which can either be direct or indirect comparison. In this context, a remainder or a difference from a comparison can be used for fixing a new arbitrary unit for measurement which is a fraction of the original unit. This process of determining a new unit is the application of Euclidean algorithm for finding the greatest common divisor.

**Scale**

For the base-10 place value number system, every column is defined by the units such as ones, tens, hundreds and so on. However, in other place value number systems such as the binary system, every column is defined by the units such as ones, twos, fours and so on. Therefore, in a place value number system, the unit is not always a multiple of the power of ten.

In addition, various other number systems are made up of different units. For the calendar system, the lunar calendar is based on 30 (29.5) days, while the solar