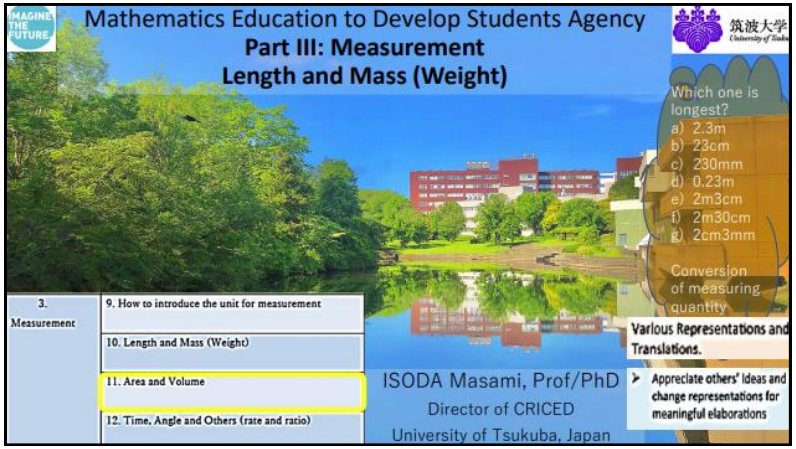
Mathematics Education to Develop Students Agency
Part III: Measurement
Length and Mass (Weight)



Which one is longest?
a) 2.3m
b) 23cm
c) 230mm
d) 0.23m
e) 2m3cm
f) 2m30cm
g) 2cm3mm

Conversion of measuring quantity

3. Measurement	9. How to introduce the unit for measurement
	10. Length and Mass (Weight)
	11. Area and Volume
	12. Time, Angle and Others (rate and ratio)

Various Representations and Translations.

- Appreciate others' Ideas and change representations for meaningful elaborations

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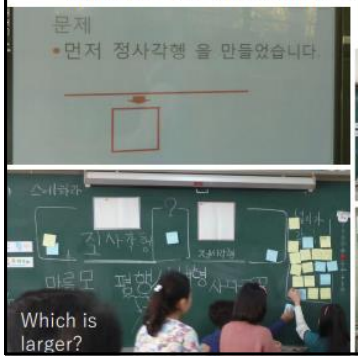
Miracle!?
Japanese teacher can demonstrate the problem-solving approach at any country! Why? Because....

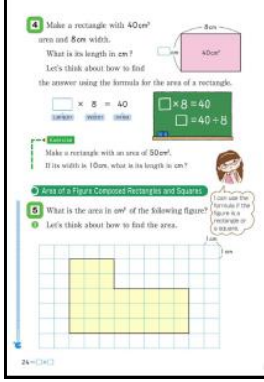
He knows Misconception and know how to manage through dialectic approach

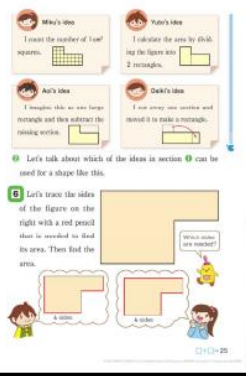


At Seoyi Elementary School (감사합니다) in Seoul, Korea

Which is larger?

Why Japanese teacher can do problem solving approach?





Review
Part III
Lesson 1~4

Mathematical Ways of Thinking:
- Generalisation and Specialisation
- Extension and Integration

Review:
Two types of terminology
First one is invariant: **Mathematical Thinking**

- Second one is variant which express **conceptual differences** to explain necessary task sequence.

Phase	Activity	Mathematical Process on each phase	Behind Mathematics
P1	Direct **Comparison**	Compare two objects on the same conditions ➢ Can compare more objects when we can apply the transitive low: If a>b, b>c, then a>c. ➢ **In case a>b, a>c, then we have to compare b and c**: we have to find the algorithm or the possible way to make an order for comparisons.	**Axioms for comparison:** Mathematical relationships for grater than, less than and equal.
P2	Indirect **Comparison** (ordering)	Compare any objects on the same conditions by using the alternative material such as a tape. We can demonstrate and record **only their order**. ➢ We can make an order but **not sure the difference**, how much more.	+ The **line/ray** only have the origin point and direction.
P3	Arbitrary (non-standardized) **Unit** (denominated number)	We can specify the difference on the alternative material as long as we can **measure** by using something as the arbitrary **unit** on the material. We can record it as **data** and **use**. ➢ It works only locally as long as we use the same unit as scale. ➢ If not, we have to seek the sharable scale for measurement: One is seeking smaller scale. Another is seeking other materials for the unit. ➢ It is a kind of **denominated number** which can be used locally and specifically but does not work as the universal quantity.	+ The **number line** set by the **origin,** the **unit** and **direction**. ➢ As long as we can find greatest common divisor, we can measure with support of **Euclidian Algorithm**. ➢ Mathematically, irrational number is problematic, however it does not appear practically.
P4	Standard **Unit** (quantity)	Standard unit can be defined universal under the **politically acceptance of academies and countries.** Recorded data works anywhere as **quantity**.	

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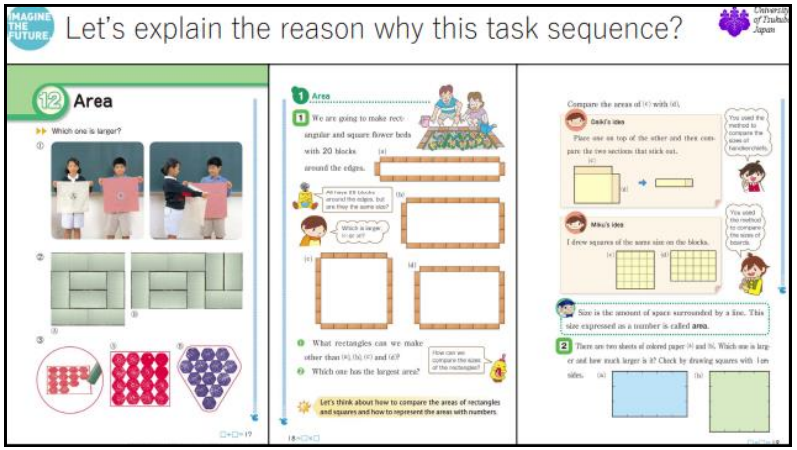
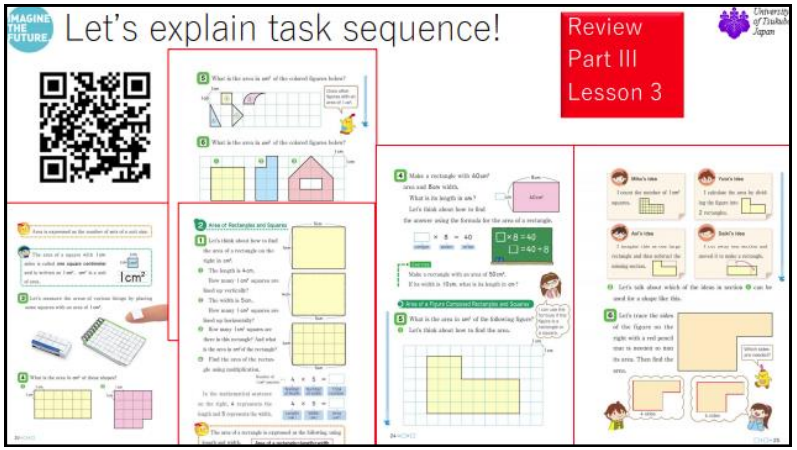
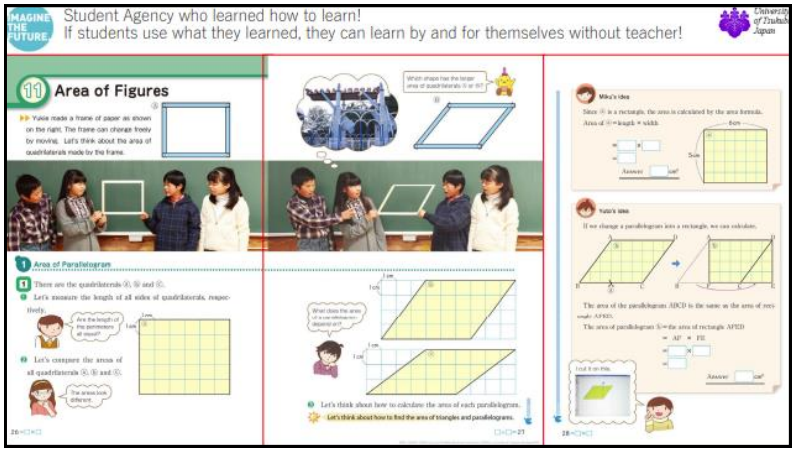
Problem Solving Approach

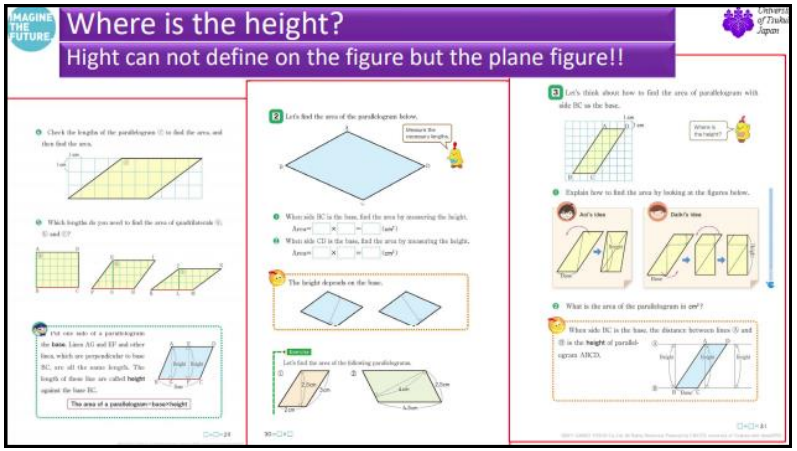
On appropriate task sequence
- **Students use what already learned**
- **Teacher prepare future learning.**

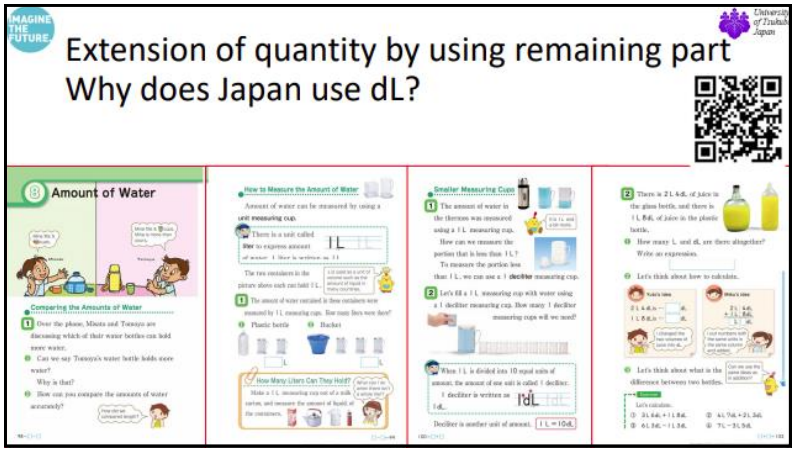
In class,
- **Teacher provide the task**
- **Students find the problematic which coming from un-known**

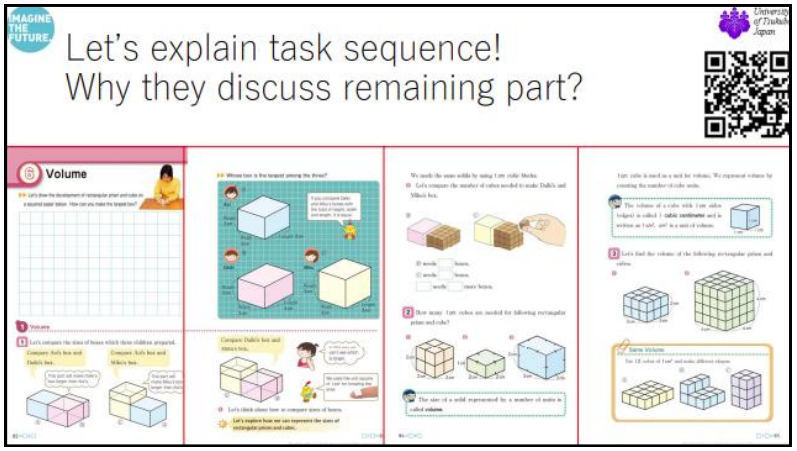

Let's explain the reason why this task sequence?
12 Area
Which one is larger?
1 Area
Let's think about how to compare the areas of rectangles and squares and how to represent the areas with numbers.
Size is the amount of space surrounded by a line. This size expressed as a number is called area.

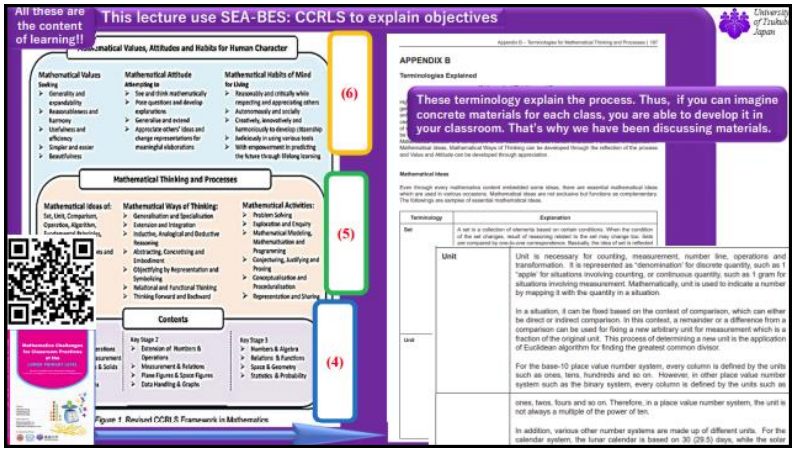

Let's explain task sequence!
Review
Part III
Lesson 3


Student Agency who learned how to learn!
If students use what they learned, they can learn by and for themselves without teacher!
11 Area of Figures
Area of Parallelogram


Where is the height?
Hight can not define on the figure but the plane figure!!


Extension of quantity by using remaining part
Why does Japan use dL?
Amount of Water
Comparing the Amounts of Water
How to Measure the Amount of Water
Smaller Measuring Cups
How Many Liters Can They Hold?


Let's explain task sequence!
Why they discuss remaining part?
Volume


All these are the content of learning!!
This lecture use SEA-BES: CCRLS to explain objectives
Mathematical Values, Attitudes and Habits for Human Character
Mathematical Thinking and Processes
Contexts
(6)
(5)
(4)
APPENDIX B
Terminologies Explained
These terminology explain the process. Thus, if you can imagine concrete materials for each class, you are able to develop it in your classroom. That's why we have been discussing materials.