

**Mathematics Education to Develop Students Agency**  
**Part III: Measurement**  
**Time, Angle and Others (rate and ratio)**

Please pose Task 2 after Task 1 as follows:  
 Task 1 I eat lunch from 12:00 to 12:30. How long did I eat?  
 Conversion of measuring quantity

**Various Representations and Translations.**  
 Appreciate others' ideas and change representations for meaningful elaborations

ISODA Masami, Prof/PhD  
 Director of CRICED  
 University of Tsukuba, Japan

3. Measurement  
 9. How to introduce the unit for measurement  
 10. Length and Mass (Weight)  
 11. Area and Volume  
 12. Time, Angle and Others (rate and ratio)

**Historical ideas of time and angle**

**Ancients' astronomy and Navigation**

**4000 years ago: Mesopotamia Lunisolar Calendar**

- One month on Luna is around 30 (29.5) days, thus a month became 30 days or 29 days. Then, a year, 12 month, is 354 days (12 less month) which provided base-60 system. A year on Sun 365 days + B. Around 360 days.
- They used to refer the tables for calculation.  
 $60 = 2^2 \times 3^2 \times 5^1$   
 60, 30, 15, 10, 6, 5, 4, 3, 2, 1  
 Prime number: 2, 3, 5, 2, ...
- Universe followed God? It follows Luna & Sun and Mathematics

**Why 12 o'clock?**

Task 1  
 6 equal parts of the circumference

Task 2  
 12 equal parts of the circumference

Longitude is given by time

Navigation: **Marine Chronometer**

Calculator of University of California

Why 1 hour = 60 min, & 1 min = 60 seconds?  
 First decimal place  
 Second decimal place

**Time is introduced by the schedule of the day.**

English usage of time: *How English is complicated language to learn Math?*

- Ten forty-five
- Quarter to eleven
- Fifteen before eleven, also, 7 fifteen minutes to eleven hours?
- Time is ~~quarter past~~ but 10:30 is a half past 10 or half to 11
- 10:45 hours? However, 09:00-09:40 HRS. (NY)
- 10:45 hours or 9:45 hours?
- Is it 10 hours, 27 minutes and 0 seconds or 10 hours 45 minutes?

In English

- 10 o'clock 45 minutes is wrong. Why ten forty-five.
- If we see hours as first decimal place on the day, it will be reasonable; it is my understanding, I could not find this explanation on the web.
- Why clock is limited to use? Because it express 'not by sundial'.

MIDDLE LOW GERMAN: *ikok* (see table in English)

MIDDLE DUTCH: *ikok* (see table in English)

**Time**

- 10:00
- 9:10
- 10:45

Difficulty for children:

- For reading the clock, short and long hands
- 12 numbers show how many times on count by 5.
- Even though almost 12 o'clock on the short hand, we have to read smaller number: it is not same as other measurement.
- Duration is defined by the difference of times: Time cannot add but subtraction of time is duration and the representation of time and duration are the same.
- On the operation, we have to convert based by the base 60, 12 (light or even) and 24 as well as 7 days, week, month and year. Years and days are base 10 system.

**Time and duration**

**Duration and Time**

Duration is defined by the difference of times: Time cannot add but subtraction of time is duration and the representation of time and duration are the same.

On the operation, we have to convert based by the base 60, 12 (light or even) and 24 as well as 7 days, week, month and year. Years and days are base 10 system.

10:00, 9:10, 10:45

