

**Mathematics Education to Develop Students Agency**  
**Part III: Measurement**  
**Time, Angle and Others (rate and ratio)**

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3. Measurement  
 9. How to introduce the unit for measurement  
 10. Length and Mass (Weight)  
 11. Area and Volume  
 12. Time, Angle and Others (rate and ratio)

Please pose Task 2 after Task 1 as follows:  
 Task 1  
 I eat lunch from 12:00 to 12:30. How long did I eat?  
 Conversion of measuring quantity  
 Various Representations and Translations.  
 Appreciate others' ideas and change representations for meaningful elaborations

**Historical ideas of time and angle**

Ancients' astronomy and Navigation

4000 years ago: Mesopotamia Lunisolar Calendar

One month on Luna is around 30 (29.5) days, thus a month became 30 days or 29 days. Then, a year, 12 month, is 354 days (approx month which provided base-60 system. A year on Sun 365 days + B. Around 360 days).

They used to refer the tables for calculation.  
 Divisors of 60:  $60 = 2^2 \times 3^2 \times 5^1$   
 60, 30, 15, 12, 6, 5, 4, 3, 2, 1  
 Prime number: 2, 3, 5, 7, ...

Universe followed God? It follows Luna & Sun and Mathematics

Why 12 o'clock?  
 Task 1  
 6 equal parts of the circumference  
 2 equal parts of ... No problem  
 4 equal parts of ...  
 Mathematical Problems!  
 5 equal parts of ...  
 7 equal parts of ...  
 Trisector of angle

Why 1 hour = 60 min, & 1 min = 60 seconds?  
 First decimal place  
 Second decimal place

Navigation: Marine Chronometer

**Time is introduced by the schedule of the day.**

English usage of time: *How English is complicated language to learn Math?*

- Ten forty-five
- Quarter to eleven
- Fifteen before eleven, also, 7 fifteen minutes to eleven hours?
- Time is not past-40 but 10:30 is a half past 10 or half to 11
- 10:45 hours? However, 09:00-09:40 HRS. (NY)
- 10:45 hours or 9:45 hours?
- Is it 10 hours, 27 minutes and 0 seconds or 10 hours 45 minutes?

In English

- 10 o'clock 45 minutes is wrong. Why ten forty-five.
- If we see hours as first decimal place on the day, it will be reasonable; it is my understanding, I could not find this explanation on the web.
- Why clock is limited to use? Because it express 'not by sundial'.

MIDDLE LOW GERMAN: o'clock  
 MIDDLE DUTCH: o'clock  
 OLD ENGLISH: o'clock

Time

10:00, 9:10, 10:45

Difficulty for children:

- For reading the clock, short and long hands
- 12 numbers show how many times on count by 5.
- Even though almost 12 o'clock on the short hand, we have to read smaller number: it is not the same as other measurement.
- Duration is defined by the difference of times: Time cannot add but subtraction of time is duration and the representation of time and duration are the same.
- On the operation, we have to convert based by the base 60, 12 (light or even) and 24 as well as 7 days, week, month and year. Years and days are base 10 system.

**Time and duration**

Duration and Time

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