Mathematics Education to Develop Students Agency:

Part III: 1. Problem Solving Approach,

Lesson 4. Dialectic Approach and Task Sequence

Topic	Title of Lesson
1. Problem Solving Approach	1. Variation: Teaching Approaches for What
	2. Open Approach: Case of Thailand
	3. Problem Solving Approach and Task Sequence
	4. Dialectic Approach and Task Sequence 'If your saying is true, what will happen?'

Motto of the course:
- By using what we learned, we continue further learning

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Director of CRICED, University of Tsukuba, Japan

Lesson Study : Design Approaches based on content of teaching

Mathematical Thinking : What it is

Model Curriculum SEA-BES: CCRLS for Math

Task sequence on the textbook
Part I. Number and Operations: Lesson 2~10

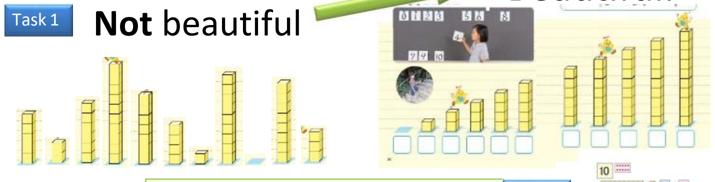
Review of task sequence for PSA

Mathematical Ideas of:
Set, Unit, Comparison

Mathematical Values
Seeking
- Generality and expandability
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

If we make it in order, it becomes beautiful.

Task 1 **Not beautiful** → **Beautiful!**



Repetition by using what students learned. Task 2

Appreciate learned, again and again....

In case, we do not have the textbook which has appropriate task sequence: **teachers** need **to enjoy** the activity to **design task sequence** for which students can think mathematically by themselves.

First Step: Local manner for preparing tomorrow's class
- Enjoy problem solving on your textbook by yourself for knowing how we can solve it by **only using what already taught.**
- Imagine your students can apply it or not. If not, **set Task 1 for reviewing the complementary of what they should already learned.** That is a most necessary preparation for today's class.
- Then, **plan to do discussion by using students' solution of the task 2 (problem) which is existed on your**

Values, Attitudes and Habits for Human Character

Mathematical Attitude — Mathematical Habits of Mind

How students are able to **reflect** and **appreciate** by and for themselves?

Appreciation

Mathematical Thinking and Processes

Through **repetitions of appropriate** (well designed) **task sequence**

Thinking Forward and Backward — Representation and Sharing

Reflection

Contents

Japanese Textbook (Gakkko Tosho) is an exemplar which provide the appropriate task sequence.

Acquisition

Further Step: General manner
Design unit plan or curriculum via using terminology

Reference
Couse of Study by MEXT, JAPAN (2017)

Design PSA's task sequence

Dialectic Approach (since Isaoda 1991): For Overcoming Contradiction which is a part of Problem Solving Approach

Authentic Math. Act. Oriented

Students **do Math.** like Mathematician without Teachers

Mathematics is the subject for overcoming contradiction

Done by teachers who well Know Mathematical Activity

If your saying is true, what will happen?

Objective oriented

Students Agency in General — Math. Thinking and Ideas — Math. Value & Attitude — Math. Critique

Specialist Level LS

Dialectic Approach

Designing up to overcoming contradictio

Problem Solving Approach

Shift to Designing Task-sequence.

For students, any task is open as long as students use what they learned

Open Approach

Designing Task only
Change the task to Open-Ended

Shift it to Students Centered

School Level LS

Exam. Oriented

Math treats only true.

Injection Approach

Teacher Centered

Done by teachers who do not design task and lesson by themselves

Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.

Mathematics should not have the contradiction;
However, **School Mathematics Curricula have huge number of contradictions from the perspective of learners.**
That is *a **difficulty** of learning mathematics for students!*

Ex. the extension of numbers
- Multiplication increase the number! → **NO**! Not always!
- Subtract smaller number from lager number! → **NO**! Any number can subtract!

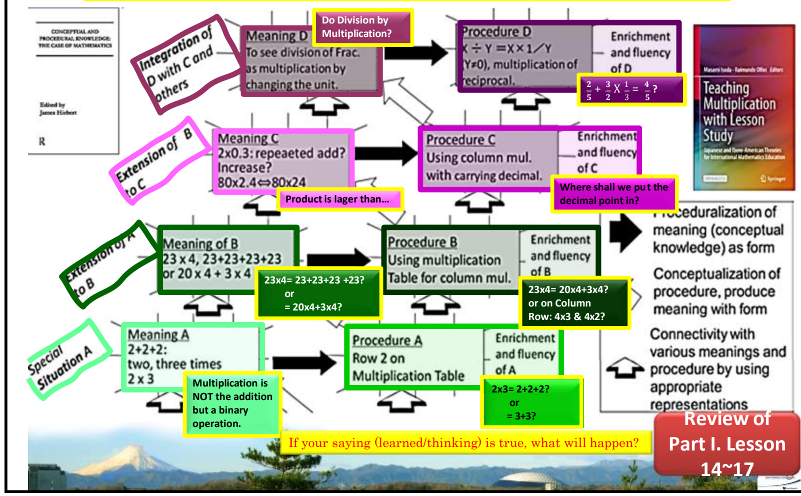
Could we teach these conclusions (no parts) from the beginning? We could not, right?
What we can do is to **develop students who challenge these contradictions**!
It is the nature for mathematics which was known as a Hegelian, Imre Lakatos (**1976**) on his book 'Proofs and Refutations.'

Dialectic which has been known by Ancients Greece: it begins from "**If your saying (conclusion) is true, what will happen?**" In **mathematics**, it is a kind of **action through analysis** and embedded into mathematics system as the **proof by contradiction** (reductive absurdum/reductio ad absurdum) and the **proof by contraposition**. In German **didactics**, it is **Socratic method** of eliciting ideas latent in the mind of another by a series of questions and answers, called the **maieutic method**.

Thus, the **principle of curricula** is necessary such as:
- **Mathematization** (as reinvention by Freudental, H. **1973**):
  Reorganize the experience in real world and in mathematics: the means of the thinking on the lower level become the subject matter of the thinking on the upper level.
- **Extension and Integration** in Japanese curriculum **since 1968** to develop mathematical thinking.

The **principles positively treat the contradictions** as a part of mathematical activity in classroom **for the opportunity to develop mathematical thinking.**

Process of Mathematization and Extension & Integration under the perspective of Conceptual and Procedural Knowledge
(Isoda & Olfos, 2021; In Japanese Isoda, 1991): Learners met the difficulties because they could only think by using what they already learned as long as we ask students to use what they learned.



① 2.3 + 1.25

Situation	Meaning	Procedure	Explanation	Appropriateness
I Introduction of calculation in vertical notation using whole numbers (integer)	Decimal notation system meaning	(A) Write 23 + 5	The meaning of a decimal notation system is based on the procedure of keeping decimal points in alignment. *(The meaning and procedure match)*	Appropriate

Originated from
Extension from Whole Number to Decimal Number

Programmed Emergence of Misconception in Curriculum/Teaching
Anyone cannot avoid: It is an Epistemological Obstacle.

① 2.3 + 1.25 ② 23 + 125

Where does it come from?② explains ①.
Schoenfeld, A (1986)
Isoda, M.(1991, 1996)

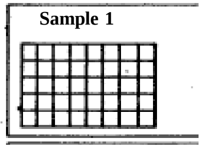
If your saying is true, what will happen? For example....

Situation	Meaning	Procedure	Explanation	Appropriateness
I Introduction of calculation in vertical notation using whole numbers (integer)	Decimal notation system meaning	(A) Write 23 + 5	The meaning of a decimal notation system is based on the procedure of keeping decimal points in alignment. *(The meaning and procedure match)*	Appropriate
II Becoming proficient in whole numbers	? (Forgotten) (C)	(B) Align to the right and write	When children become proficient, they no longer need to think about the reason they follow that procedure. As a result, the procedure is simplified from the alignment of the decimal points to one of right-side alignment.	Valid by Proficiency *(Procedure itself has meaning in some cases)*
III Application to the decimal numbers	(No meaning)	Align to the right and write	The procedure for whole numbers is generalized for decimal numbers.	Inappropriate *(Contradiction)*

Background

To introduce parallel, Mr. Masaki started by drawing a sample lattice pattern. The following process shows how students emagent the idea of parallel in case of the no instruction of the definition of parallel. (Isoda, 1991, 1996)

Task 1. Let's draw the sample 1 lattice pattern



Synthesis: Define the parallel line based on the difference

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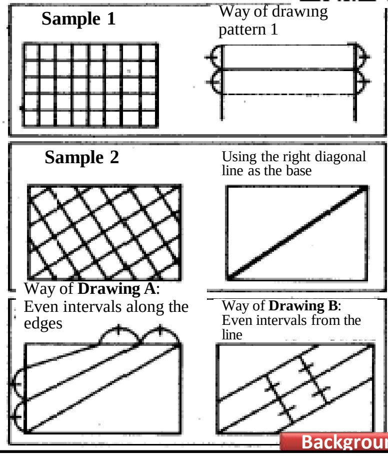
Task 1. Let's draw the sample 1 lattice pattern

Task 2. Let's draw the sample 2 lattice pattern

Dialectic Discussion: "What? ""Why?"



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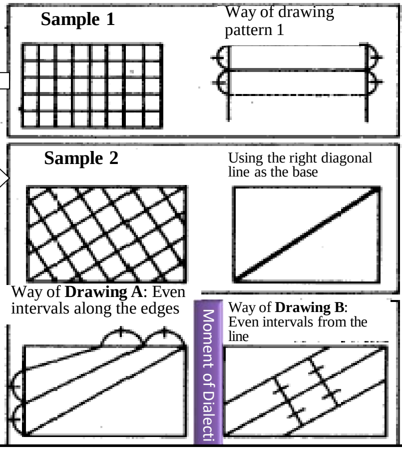
Way of drawing 1: **Procedure a**
→Way of **Drawing A**; Task 2
If you want to draw the model, draw lines spread evenly apart from the top edge of the paper.

Way of drawing 1: **Procedure b**
→Way of **Drawing B**; Task 2
If you want to draw the model, draw lines spread evenly apart from diagonal.



Even if teacher explained many times, there are diversity of children's understanding and ways of utilizing learned ideas.
Why? Because not yet learned children can not distinguish special ideas and general ideas.

A lot of examples

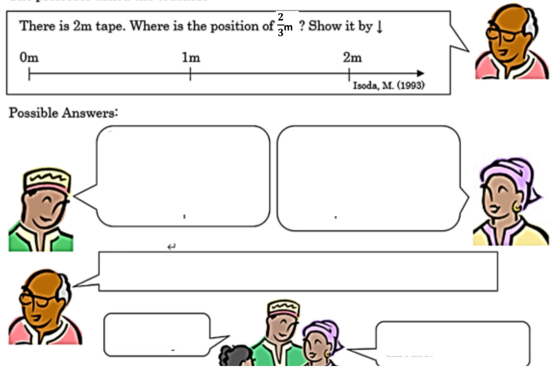
Review Part II. Lesson 1. What is fraction?

University of Tsukuba Japan

In mathematics, if we find the contradiction, we have to think

Chapter 1: What is fraction?

The professor asked the teachers:



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Mathematical Attitude
Attempting to
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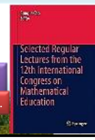
- Pose questions and develop explanations
- Generalise and extend
- Appreciate others' ideas and change representations for meaningful elaborations



Isoda, M. (2015). Dialectic on the Problem Solving Approach: Illustrating Hermeneutics as the Ground Theory for Lesson Study in Mathematics Education. DOI 10.1007/978-3-319-17187-6_21

Read Appendix

If your saying is true, what will happen? For example....

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Chapter 1: What is fraction?

The professor asked the teachers:

There is 2m tape. Where is the position of $\frac{2}{3}$m ? Show it by ↓

0m 1m 2m

Isoda, M. (1993)

Possible Answers:

Som: 0m 1m 2m ↑

Any: 0m 1m 2m ↑

What do you want to do next? Isoda&Katagiri (2012)

I would like to ask why?

Yes, we would like to discuss!

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Read Appendix

Background

Selected Regular Lectures from the 12th International Congress on Mathematical Education

Review Part II. Lesson 3. : Addition with different denominator

University of Tsukuba Japan

The professor asked the teachers:

In addition of the fractions, we teach the case of the same denominator at first and then we introduce the case of different denominator.

In the case of $\frac{1}{2}+\frac{1}{3}$:

We have two bottles with $\frac{1}{2}$L and $\frac{1}{3}$L of milk. How much L in total? Can you imagine the student's answer? Isoda (1996)

Possible Answers:

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Possible Answers:

Som: Yes, we have children who calculate as follows:

$\frac{1}{2}+\frac{1}{3}=\frac{1+1}{2+3}=\frac{2}{5}$

Any: Yes, we have. However we have to teach:

$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

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Board writing (Bansho): Sample

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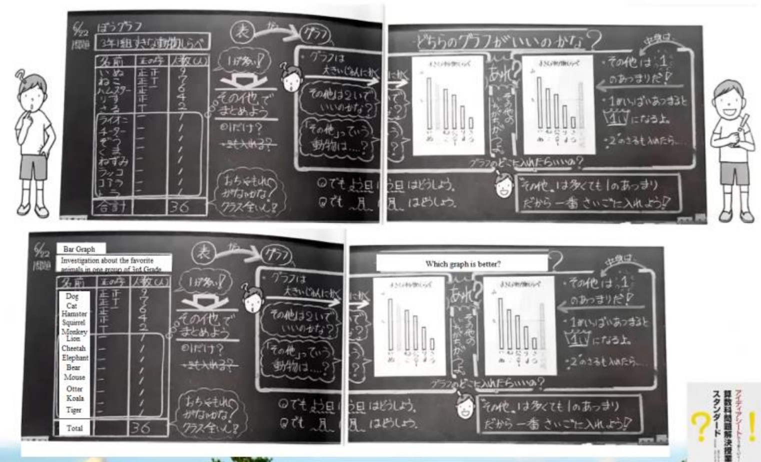

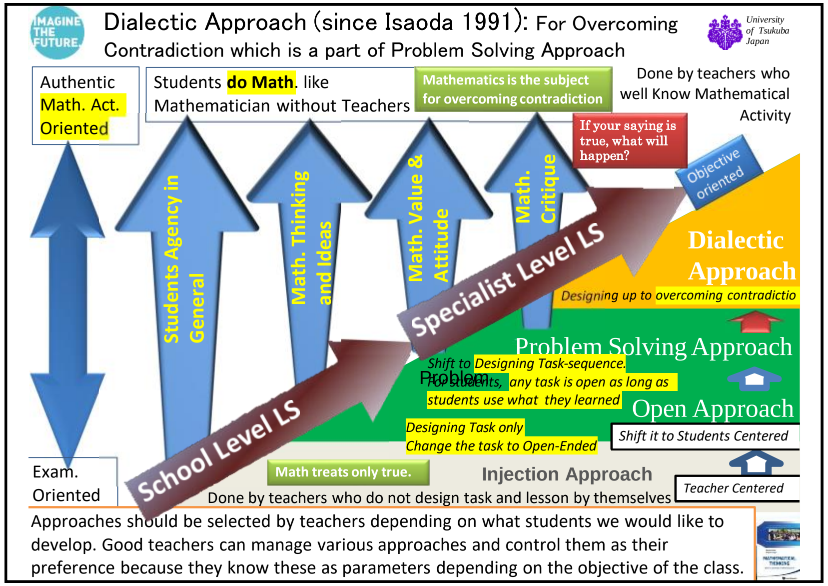
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Shift to Designing Task-sequence.
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Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.