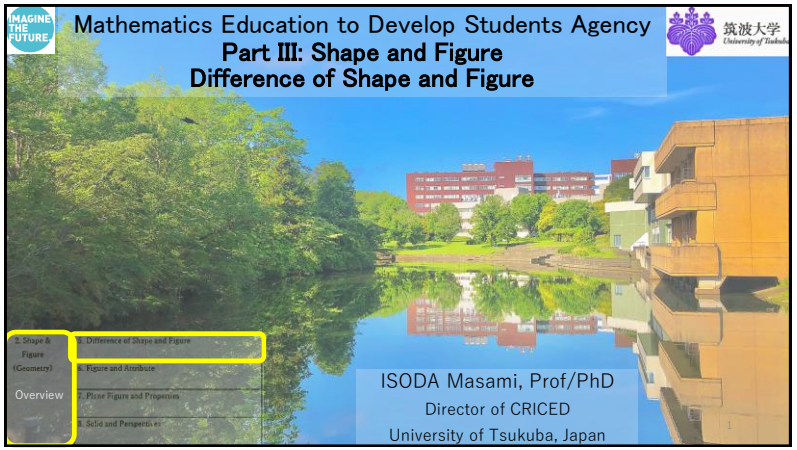
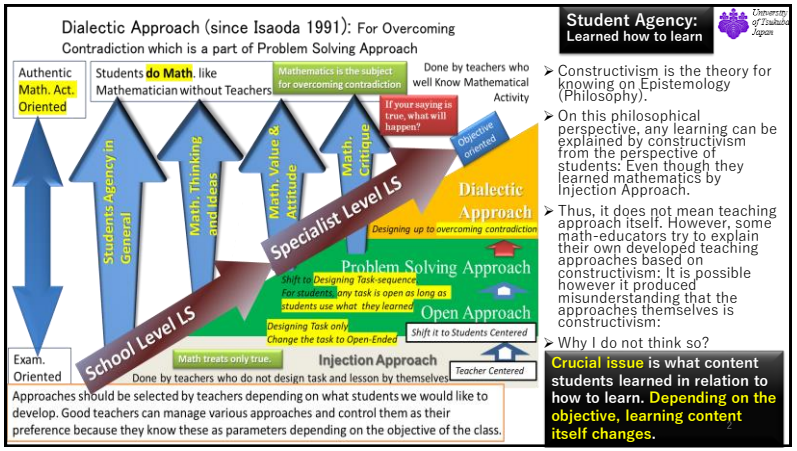
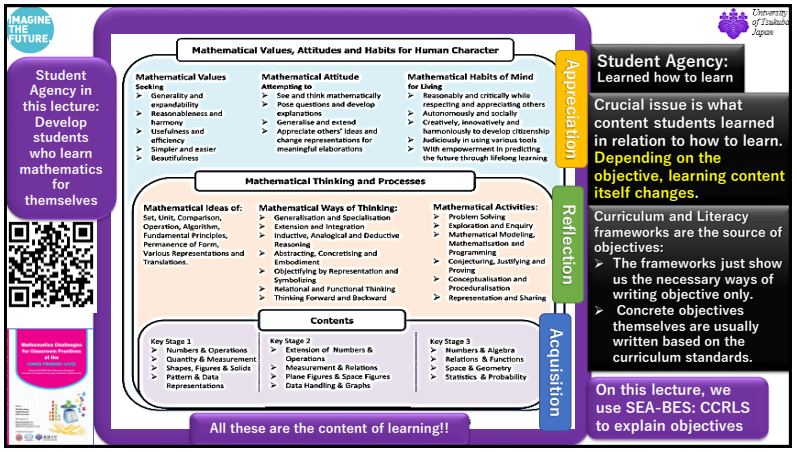
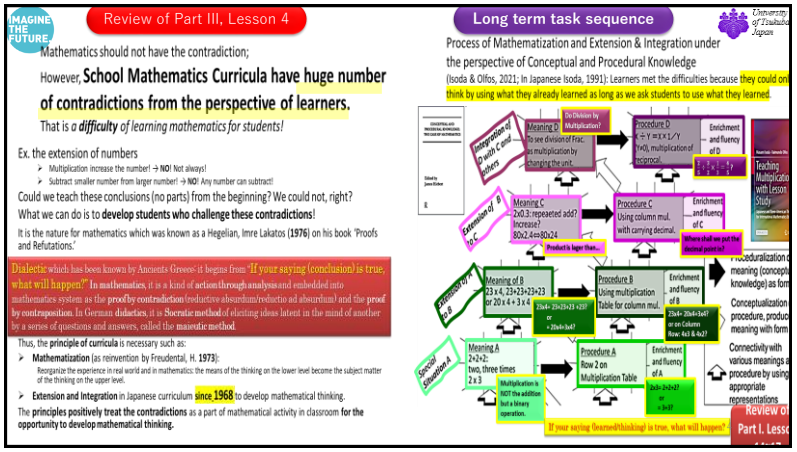

Mathematics Education to Develop Students Agency
Part III: Shape and Figure
Difference of Shape and Figure
筑波大学
University of Tsukuba
2. Shape & Figure (Geometry)
3. Difference of Shape and Figure
4. Figure and Attribute
5. Plane Figure and Properties
6. Solid and Perspective
Overview
ISODA Masami, Prof/PhD
Director of CRICED
University of Tsukuba, Japan


Dialectic Approach (since Isaoda 1991): For Overcoming Contradiction which is a part of Problem Solving Approach
Authentic Math. Act. Oriented
Students do Math. like Mathematician without Teachers
Mathematics is the subject for overcoming contradiction
Done by teachers who well Know Mathematical Activity
If your saying is true, what will happen?
Objective oriented
Students Agency in General
Math. Thinking and ideas
Math. Value & Attitude
Math. Critique
Specialist Level LS
Dialectic Approach
Designing up to overcoming contradiction
Problem Solving Approach
Shift to Designing Task-sequence
For students, any task is open as long as students use what they learned
Open Approach
Designing Task only
Change the task to Open-Ended
Shift it to Students Centered
School Level LS
Math treats only true.
Injection Approach
Teacher Centered
Exam. Oriented
Done by teachers who do not design task and lesson by themselves
Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.
Student Agency: Learned how to learn
University of Tsukuba Japan
Constructivism is the theory for knowing on Epistemology (Philosophy).
On this philosophical perspective, any learning can be explained by constructivism from the perspective of students: Even though they learned mathematics by Injection Approach.
Thus, it does not mean teaching approach itself. However, some math-educators try to explain their own developed teaching approaches based on constructivism; It is possible however it produced misunderstanding that the approaches themselves is constructivism:
Why I do not think so?
Crucial issue is what content students learned in relation to how to learn. Depending on the objective, learning content itself changes.


Student Agency in this lecture: Develop students who learn mathematics for themselves
Mathematical Values, Attitudes and Habits for Human Character
Mathematical Values Seeking
Generality and expandability
Reasonableness and harmony
Usefulness and efficiency
Simpler and easier
Beautifulness
Mathematical Attitude Attempting to
See and think mathematically
Pose questions and develop explorations
Generalise and extend
Appreciate others' ideas and change representations for meaningful elaborations
Mathematical Habits of Mind for Living
Reasonably and critically while respecting and appreciating others
Autonomously and socially
Creatively, innovatively and harmoniously to develop citizenship
Judiciously in using various tools
With empowerment in predicting the future through lifelong learning
Appreciation
Mathematical Thinking and Processes
Mathematical Ideas of:
Set, Unit, Comparison, Operation, Algorithm, Fundamental Principles, Permanence of Form, Various Representations and Translations.
Mathematical Ways of Thinking:
Generalisation and Specialisation
Extension and Integration
Inductive, Analogical and Deductive Reasoning
Abstracting, Concretising and Embodiment
Objectifying by Representation and Symbolising
Relational and Functional Thinking
Thinking Forward and Backward
Mathematical Activities:
Problem Solving
Exploration and Enquiry
Mathematical Modeling, Mathematisation and Programming
Conjecturing, Justifying and Proving
Conceptualisation and Proceduralisation
Representation and Sharing
Reflection
Contents
Key Stage 1
Numbers & Operations
Quantity & Measurement
Shapes, Figures & Solids
Pattern & Data Representations
Key Stage 2
Extension of Numbers & Operations
Measurement & Relations
Plane Figures & Space Figures
Data Handling & Graphs
Key Stage 3
Numbers & Algebra
Relations & Functions
Space & Geometry
Statistics & Probability
Acquisition
All these are the content of learning!!
Student Agency: Learned how to learn
Crucial issue is what content students learned in relation to how to learn. Depending on the objective, learning content itself changes.
Curriculum and Literacy frameworks are the source of objectives:
The frameworks just show us the necessary ways of writing objective only.
Concrete objectives themselves are usually written based on the curriculum standards.
On this lecture, we use SEA-BES: CCRLS to explain objectives


Review of Part III, Lesson 4
Mathematics should not have the contradiction;
However, School Mathematics Curricula have huge number of contradictions from the perspective of learners.
That is a difficulty of learning mathematics for students!
Ex. the extension of numbers
Multiplication increase the number! → NO! Not always!
Subtract smaller number from larger number! → NO! Any number can subtract!
Could we teach these conclusions (no parts) from the beginning? We could not, right?
What we can do is to develop students who challenge these contradictions!
It is the nature for mathematics which was known as a Hegelian, Imre Lakatos (1976) on his book 'Proofs and Refutations.'
Dialectic which has been known by Ancients Greeces it begins from "If your saying (conclusion) is true, what will happen?" In mathematics, it is a kind of action through analysis and embedded into mathematics system as the proof by contradiction (reductive absurdum/reductio ad absurdum) and the proof by contraposition. In German didactics, it is Socratic method of eliciting ideas latent in the mind of another by a series of questions and answers, called the maieutic method.
Thus, the principle of curricula is necessary such as:
Mathematization (as reinvention by Freudental, H. 1973):
Reorganize the experience in real world and in mathematics: the means of the thinking on the lower level become the subject matter of the thinking on the upper level.
Extension and Integration in Japanese curriculum since 1968 to develop mathematical thinking.
The principles positively treat the contradictions as a part of mathematical activity in classroom for the opportunity to develop mathematical thinking.
Long term task sequence
Process of Mathematization and Extension & Integration under the perspective of Conceptual and Procedural Knowledge
(Isoda & Olfos, 2021; In Japanese Isoda, 1991): Learners met the difficulties because they could only think by using what they already learned as long as we ask students to use what they learned.
Integration of Grade C and Others
Meaning D
Do Division by Multiplication?
Procedure D
Enrichment and fluency of D
Extension of B to C
Meaning C
Procedure C
Using column mul. with carrying decimal.
Enrichment and fluency of C
Extension of A to B
Meaning of B
Procedure B
Using multiplication Table for column mul.
Enrichment and fluency of B
Special Situation A
Meaning A
2×2+2; two, three times
Procedure A
Row 2 on Multiplication Table
Enrichment and fluency of A
Multiplication is NOT the addition but a binary operation.
If your saying (learned/thinking) is true, what will happen?
Review of Part I. Lesson

Curriculum Sequence under Mathematization Principle (Freudenthal, 1973)

Freudenthal defined Mathematization by the re-organization of (mathematical) experience via mathematical means.

I first introduced my discovery in the following way (van Hiele, 1955, p. 289):

> You can say somebody has attained a higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these operations on new objects. The attainment of the new level cannot be effected by teaching, but still, by a suitable choice of exercises the teacher can create a situation for the pupil favorable to the attainment of the higher level of thinking. Van Hiele, P. M. (1986). Structure and Insight, Accademic Press. p39.

van Hiele Levels (Level of Thinking by Freudenthal): **Teaching is the activity for students to be able to think on upper level.**

Level	Object	Operation/Means	Nature of language: ways of thinking
L1	Concrete O.	Shape (形)	Concrete object is treated by the hidden attribute of each shape
L2	Figure(図形)	attribute	Attribute belonging each shape is treated by the properties of figure
L3	Properties	Proposition	Figures are recognized/operated by properties.
L4	Proposition	Proof	Proposition are explained by proof (local theory)
L5	Proof	Logic	Each proposition become a part of Axiomatic system (general theory)

Mathematization (Freudenthal, 1973) and van Hiele Levels (van Hiele,1986)

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Level	Object	Operation/ Means	Nature of language: ways of thinking	Contradiction between levels
L1	Concrete Object	**Shape** (形)	Concrete object is treated by the **hidden** attribute of each shape	Round shape is not a circle (as figure). Fold round shapes and find a circle when it was folded just overlap. Circle is drawn by compass with center and radius.
L2	Figure(図形)	**attribute**	Attribute belonging each figure is treated as the properties of figure	Square, rhombus and rectangle are different figure (shape). Properties of figure are attribute of figure (shape).
L3	Properties (平面図形:plane figure)	**Proposition**	Figures are recognized by properties.	Square is rhombus. Square is rectangle. Rhombus is square if … . Rectangle is rhombus if … .

Contradiction is the opportunity to think and discuss mathematically

How can we provide our students for the opportunity to think by and for themselves, and learning how to?

Mathematical Activities: Problem Solving; Exploration and Enquiry; Mathematical Modeling, Mathematisation and Programming; Conjecturing, Justifying and Proving; Conceptualisation and Proceduralisation; Representation and Sharing

Mathematical Ways of Thinking: Generalisation and Specialisation; Extension and Integration; Inductive, Analogical and Deductive Reasoning; Abstracting, Concretising and Embodiment; Objectifying by Representation and Symbolising; Relational and Functional Thinking; Thinking Forward and Backward

Mathematical Values Seeking: Generality and expandability; Reasonableness and harmony; Usefulness and efficiency; Simpler and easier; Beautifulness

By Isoda such as 2015 and 2018

Mathematization from L1 to L2

- Japanese had already developed the task sequence of it from 1934 textbook up to 1950s curriculum reform because they had been distinguished shape, figure and plane figure in Japanese at the beginning of school system in 1870s.
- If we use appropriate terminology to distinguish conceptual differences, we can produce appropriate task sequence for students' activity.
- **What terminology is the key on those two pages' activity?**

Shape, Attribute via collecting, touching, stacking and making

6 Shapes

Mathematical Ideas of: Set, Unit, Comparison, Operation, Algorithm,

Let's play with shapes.

Collecting Similar Shapes

Stacking Boxes

Guessing Shapes

Making Things from Boxes

Mathematical Ways of Thinking:
- Generalisation and Specialisation
- Abstracting, Concretising and Embodiment

Why this activity is necessary?

Let's explain by using terminology.

Did you use?

Shape, Attribute via tracing, cutting and drawing

1 Let's trace shapes and draw a picture.

Mathematical Values
Seeking
- Generality and expandability
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

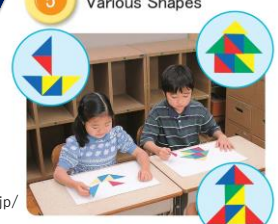
Mathematical Attitude
Attempting to
- See and think mathematically

Mathematical Activities:
- Conceptualisation and Proceduralisation
- Representation and Sharing

Those pages are NOT independently existed. It has task sequence beyond chapters and grades as well as other content-strands.

https://support.gakuto.co.jp/r2s_sansu_translation/

5 Various Shapes



1 Make various shapes by using 2 sheets of colored cardboard shaped like this

2 Make shapes by using 4 sheets of colored cardboard shaped like this

① Make the shape shown in ⓐ.

② From ⓐ, make ⓑ, ⓒ, ⓓ and ⓔ.

3 Make the following shapes from colored cardboard shaped like this

15 Triangles and Quadrangles

Straight Lines

1 Use string to make cat's cradles as shown above.

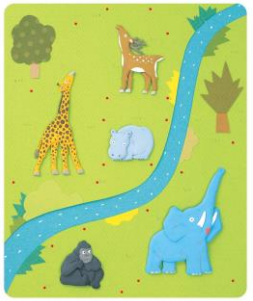
2 Make a straight line.

① Stretch an elastic band.

② Fold a piece of paper.

The line that is like a piece of string pulled tight is called "a straight line."

3 Draw a straight line using a ruler and check that the line is really straight by comparing it to a tight string.



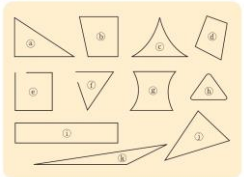
Triangles and Quadrangles

1 Connect the points using straight lines to enclose each animal.

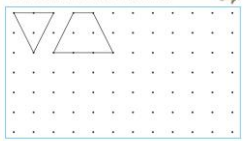
The shape made using 3 straight lines is called "a triangle."

The shape made using 4 straight lines is called "a quadrangle."

3 Find the triangles and quadrangles.



4 Draw various triangles and quadrangles by connecting points with straight lines.



5 Cut the paper to make the following shapes.

① 2 triangles.

② 2 quadrangles.

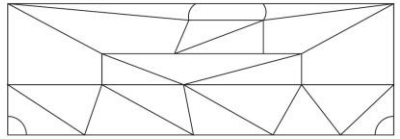
③ A triangle and a quadrangle.

6 Look for things that are shaped like a triangle or a quadrangle.

IMAGINE THE FUTURE

Which level is it?

1 Color the triangles red and the quadrangles blue.



2 Cut colored paper in the same shapes and make patterns.

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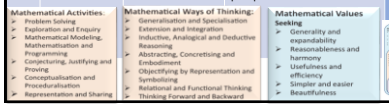

By Isoda such as 2015[13] and 2018