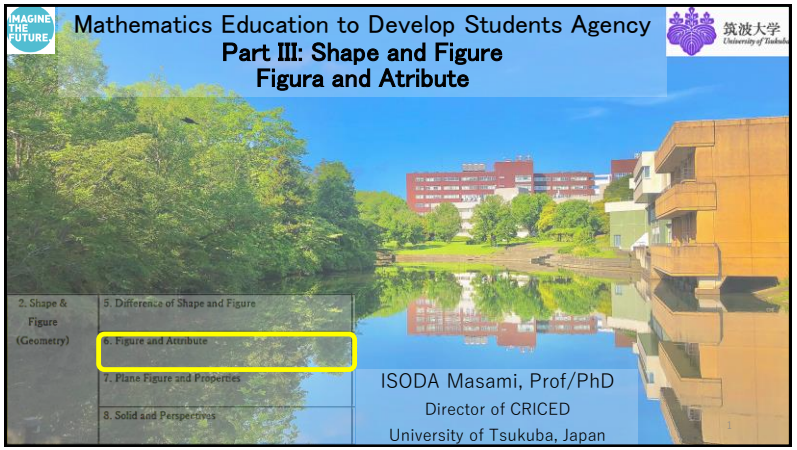
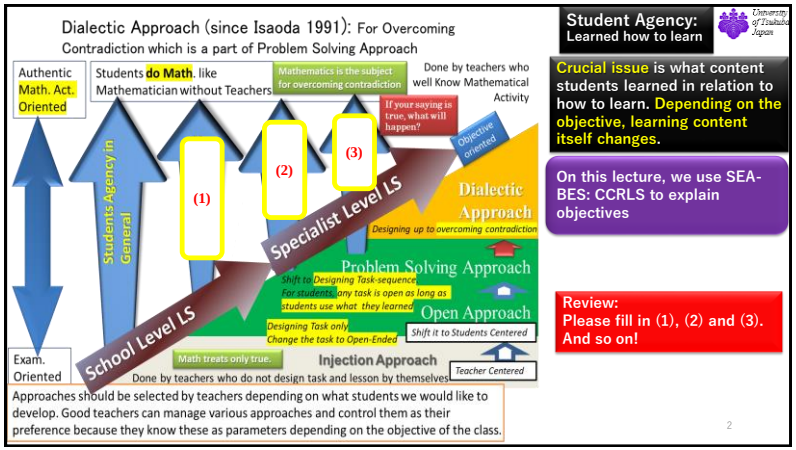
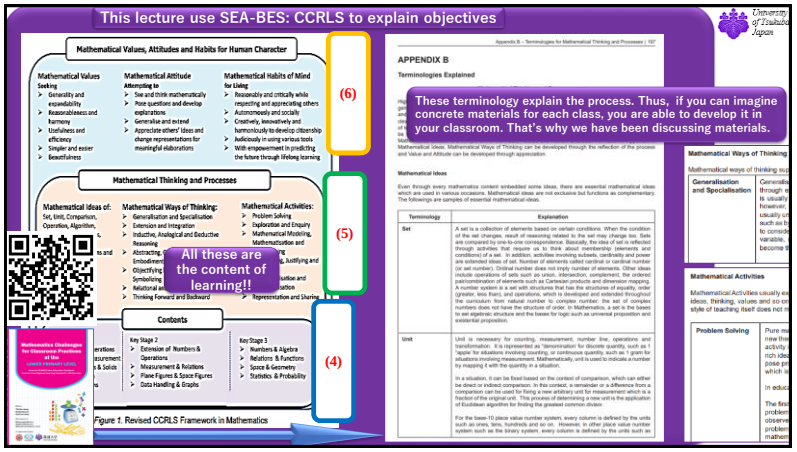
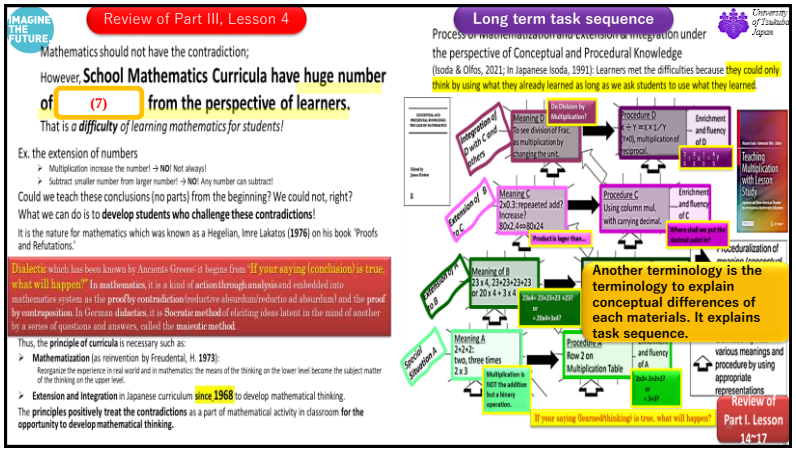

Mathematics Education to Develop Students Agency
Part III: Shape and Figure
Figura and Atribute
筑波大学
University of Tsukuba
2. Shape & Figure (Geometry)
5. Difference of Shape and Figure
6. Figure and Attribute
7. Plane Figure and Properties
8. Solid and Perspectives
ISODA Masami, Prof/PhD
Director of CRICED
University of Tsukuba, Japan


Dialectic Approach (since Isaoda 1991): For Overcoming Contradiction which is a part of Problem Solving Approach
Student Agency: Learned how to learn
Authentic Math. Act. Oriented
Students do Math. like Mathematician without Teachers
Mathematics is the subject for overcoming contradiction
Done by teachers who well Know Mathematical Activity
If your saying is true, what will happen?
Objective oriented
Students Agency in General
(1)
(2)
(3)
Specialist Level LS
Dialectic Approach
Designing up to overcoming contradiction
Problem Solving Approach
Shift to Designing Task-sequence
For students, any task is open as long as students use what they learned
Open Approach
Designing Task only
Change the task to Open-Ended
Shift it to Students Centered
School Level LS
Exam. Oriented
Math treats only true.
Injection Approach
Teacher Centered
Done by teachers who do not design task and lesson by themselves
Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.
Crucial issue is what content students learned in relation to how to learn. Depending on the objective, learning content itself changes.
On this lecture, we use SEA-BES: CCRLS to explain objectives
Review:
Please fill in (1), (2) and (3).
And so on!


This lecture use SEA-BES: CCRLS to explain objectives
Mathematical Values, Attitudes and Habits for Human Character
Mathematical Thinking and Processes
Contents
(6)
(5)
(4)
These terminology explain the process. Thus, if you can imagine concrete materials for each class, you are able to develop it in your classroom. That's why we have been discussing materials.
All these are the content of learning!!
Figure 1. Revised CCRLS Framework in Mathematics
APPENDIX B
Terminologies Explained


Review of Part III, Lesson 4
Mathematics should not have the contradiction;
However, School Mathematics Curricula have huge number of (7) from the perspective of learners.
That is a difficulty of learning mathematics for students!
Ex. the extension of numbers
Could we teach these conclusions (no parts) from the beginning? We could not, right?
What we can do is to develop students who challenge these contradictions!
It is the nature for mathematics which was known as a Hegelian, Imre Lakatos (1976) on his book 'Proofs and Refutations.'
Mathematization
Extension and Integration
The principles positively treat the contradictions as a part of mathematical activity in classroom for the opportunity to develop mathematical thinking.
Long term task sequence
Process of Mathematization and Extension & Integration under the perspective of Conceptual and Procedural Knowledge
(Isoda & Olfos, 2021; In Japanese Isoda, 1991): Learners met the difficulties because they could only think by using what they already learned as long as we ask students to use what they learned.
Another terminology is the terminology to explain conceptual differences of each materials. It explains task sequence.
various meanings and procedure by using appropriate representations
Review of Part I. Lesson 14~17

Mathematization (Freudenthal, 1973) and van Hiele Levels (van Hiele,1986)

Freudenthal defined Mathematization by the re-organization of (mathematical) experience by mathematical means.

van Hiele Levels (Level of Thinking by Freudenthal): **Teaching is the activity for students to be able to think on upper level.**

Level	Object	Operation/ Means	Nature of language: ways of thinking	Contradiction between levels
L1	Concrete Object	**Shape** (形)	Concrete object is treated by the **hidden** attribute of each shape	Round shape is not a circle (as figure). Fold round shapes and find a circle when it was folded just overlap. Circle is drawn by compass with center and radius.
L2	Figure(図形)	**attribute**	Attribute belonging each figure is treated as the properties of figure	Square, rhombus and rectangle are different figure (shape). Properties of figure are attribute of figure (shape).
L3	Properties (平面図形:plane figure)	**Proposition**	Figures are recognized by properties.	How can we provide our students for the opportunity to think by and for themselves, and learning how to?

Contradiction is the opportunity to think and discuss mathematically

(8)

By Isoda such as 2015 and 2018

Mathematization from L1 to L2

6 Shapes

- Japanese had already developed the task sequence of it from 1934 textbook up to 1950s curriculum reform because they had been distinguished shape, figure and plane figure in Japanese at the beginning of school system in 1870s.
- If we use appropriate terminology to distinguish conceptual differences, we can produce appropriate task sequence for students' activity.
- **What terminology is the key on those two pages' activity?**

(9)

Mathematical Ideas of: Set, Unit, Comparison, Operation, Algorithm,

Mathematical Ways of Thinking:
- Generalisation and Specialisation
- Abstracting, Concretising and Embodiment

Why this activity is necessary?

Let's explain by using terminology.

Did you use?

Shape, Attribute via (10)

1 Let's trace shapes and draw a picture.

Mathematical Values
Seeking
- Generality and expandability
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

Mathematical Attitude
Attempting to
- See and think mathematically

Mathematical Activities:
- Conceptualisation and Proceduralisation
- Representation and Sharing

Various Shapes

Those pages are NOT independently existed. It has task sequence beyond chapters and grades as well as other content-strands.

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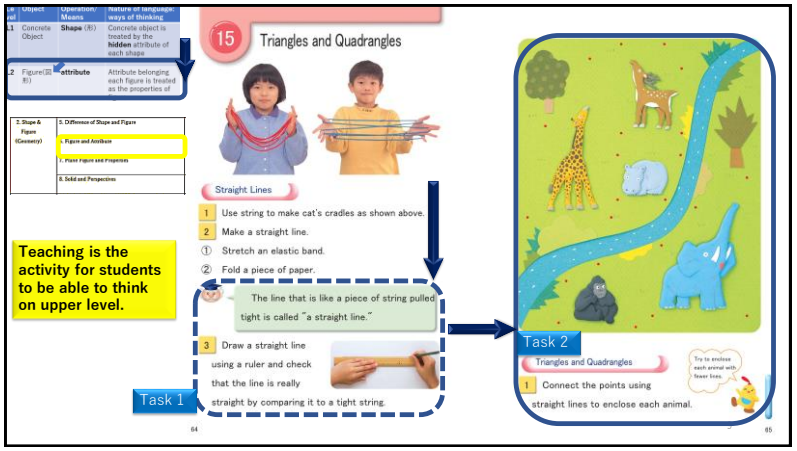
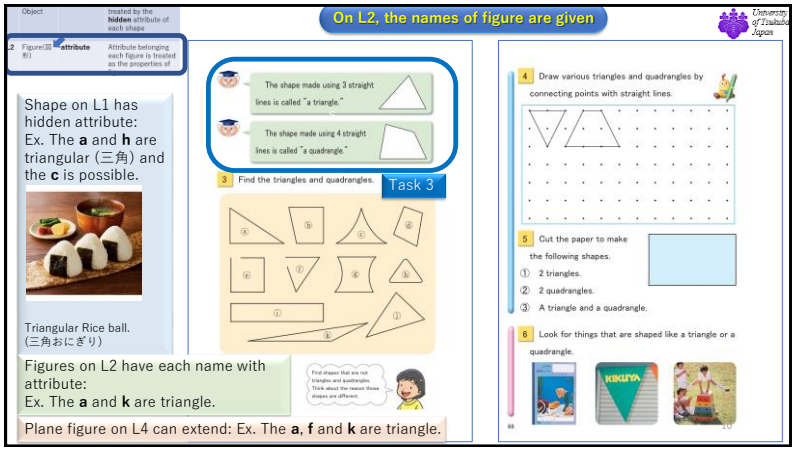
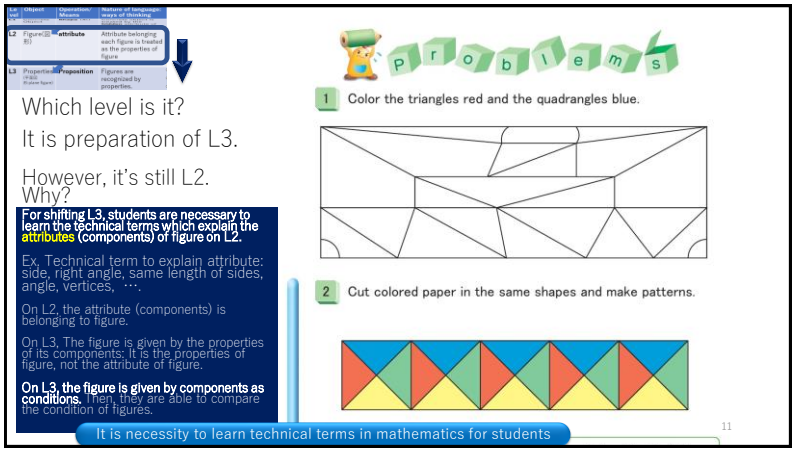
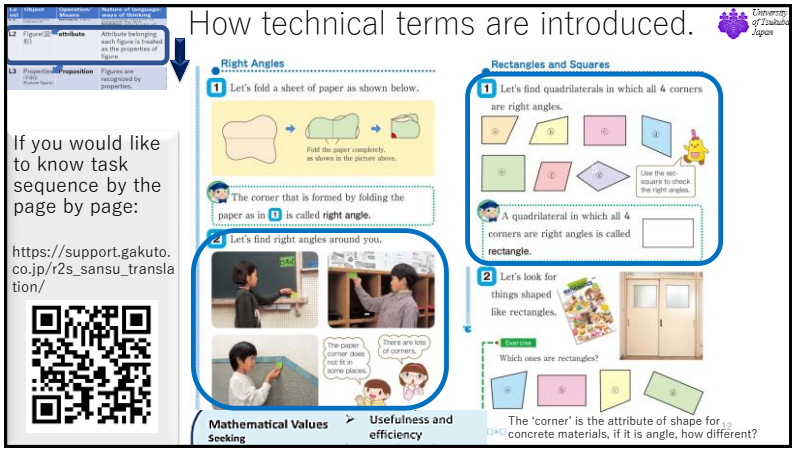
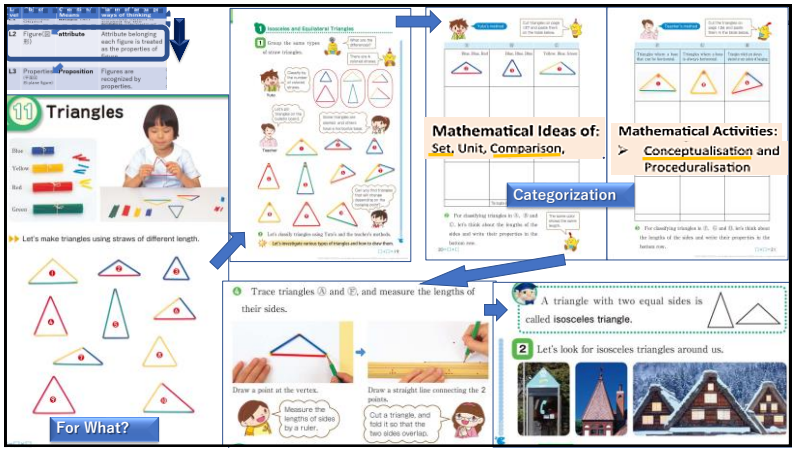
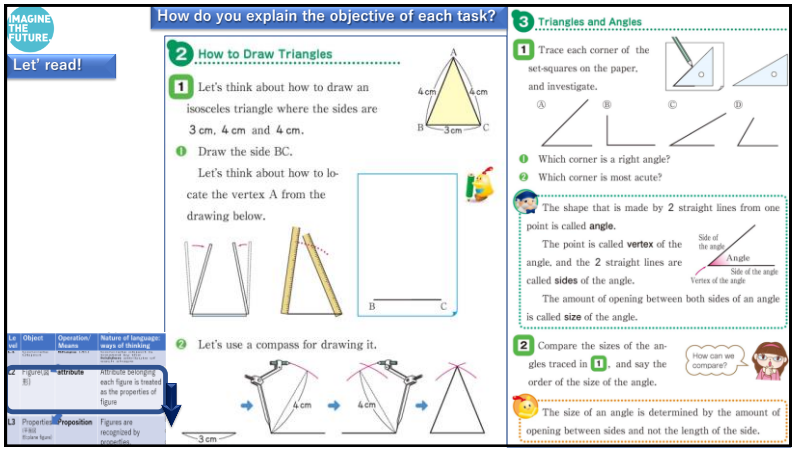

15 Triangles and Quadrangles
Straight Lines
1 Use string to make cat's cradles as shown above.
2 Make a straight line.
① Stretch an elastic band.
② Fold a piece of paper.
The line that is like a piece of string pulled tight is called "a straight line."
3 Draw a straight line using a ruler and check that the line is really straight by comparing it to a tight string.
Task 1
Task 2
Triangles and Quadrangles
1 Connect the points using straight lines to enclose each animal.
Teaching is the activity for students to be able to think on upper level.

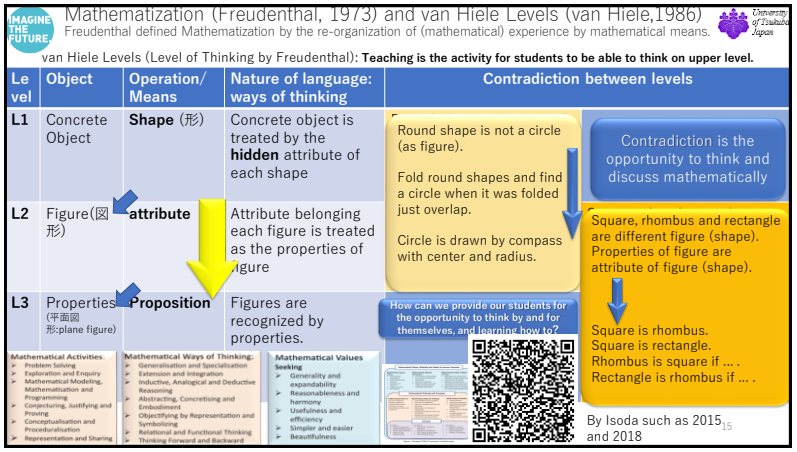

On L2, the names of figure are given
Shape on L1 has hidden attribute:
Ex. The a and h are triangular (三角) and the c is possible.
Triangular Rice ball.
(三角おにぎり)
The shape made using 3 straight lines is called "a triangle."
The shape made using 4 straight lines is called "a quadrangle."
3 Find the triangles and quadrangles.
Task 3
Figures on L2 have each name with attribute:
Ex. The a and k are triangle.
Plane figure on L4 can extend: Ex. The a, f and k are triangle.
4 Draw various triangles and quadrangles by connecting points with straight lines.
5 Cut the paper to make the following shapes.
① 2 triangles.
② 2 quadrangles.
③ A triangle and a quadrangle.
6 Look for things that are shaped like a triangle or a quadrangle.


Which level is it?
It is preparation of L3.
However, it's still L2.
Why?
For shifting L3, students are necessary to learn the technical terms which explain the attributes (components) of figure on L2.
Ex, Technical term to explain attribute: side, right angle, same length of sides, angle, vertices, ….
On L2, the attribute (components) is belonging to figure.
On L3, The figure is given by the properties of its components; it is the properties of figure, not the attribute of figure.
On L3, the figure is given by components as conditions. Then, they are able to compare the condition of figures.
Problems
1 Color the triangles red and the quadrangles blue.
2 Cut colored paper in the same shapes and make patterns.
It is necessity to learn technical terms in mathematics for students


How technical terms are introduced.
If you would like to know task sequence by the page by page:
https://support.gakuto.co.jp/r2s_sansu_translation/
Right Angles
1 Let's fold a sheet of paper as shown below.
Fold the paper completely, as shown in the picture above.
The corner that is formed by folding the paper as in 1 is called right angle.
2 Let's find right angles around you.
Rectangles and Squares
1 Let's find quadrilaterals in which all 4 corners are right angles.
Use the set-square to check the right angles.
A quadrilateral in which all 4 corners are right angles is called rectangle.
2 Let's look for things shaped like rectangles.
Exercise
Which ones are rectangles?
Mathematical Values
Seeking
Usefulness and efficiency
The 'corner' is the attribute of shape for concrete materials, if it is angle, how different?


11 Triangles
Let's make triangles using straws of different length.
For What?
Mathematical Ideas of:
Set, Unit, Comparison,
Categorization
Mathematical Activities:
Conceptualisation and Proceduralisation
Trace triangles Ⓐ and Ⓕ, and measure the lengths of their sides.
Draw a point at the vertex.
Measure the lengths of sides by a ruler.
Draw a straight line connecting the 2 points.
Cut a triangle, and fold it so that the two sides overlap.
A triangle with two equal sides is called isosceles triangle.
Let's look for isosceles triangles around us.


IMAGINE THE FUTURE.
Let' read!
How do you explain the objective of each task?
2 How to Draw Triangles
Let's think about how to draw an isosceles triangle where the sides are 3 cm, 4 cm and 4 cm.
Draw the side BC.
Let's think about how to locate the vertex A from the drawing below.
Let's use a compass for drawing it.
3 Triangles and Angles
Trace each corner of the set-squares on the paper, and investigate.
Which corner is a right angle?
Which corner is most acute?
The shape that is made by 2 straight lines from one point is called angle.
The point is called vertex of the angle, and the 2 straight lines are called sides of the angle.
The amount of opening between both sides of an angle is called size of the angle.
Compare the sizes of the angles traced in 1, and say the order of the size of the angle.
How can we compare?
The size of an angle is determined by the amount of opening between sides and not the length of the side.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University of Tsukuba Japan
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attribute
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Fold round shapes and find a circle when it was folded just overlap.
Circle is drawn by compass with center and radius.
Contradiction is the opportunity to think and discuss mathematically
Square, rhombus and rectangle are different figure (shape).
Properties of figure are attribute of figure (shape).
Square is rhombus.
Square is rectangle.
Rhombus is square if ... .
Rectangle is rhombus if ... .
How can we provide our students for the opportunity to think by and for themselves, and learning how to?
Mathematical Activities:
Mathematical Ways of Thinking:
Mathematical Values Seeking
By Isoda such as 2015 and 2018
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