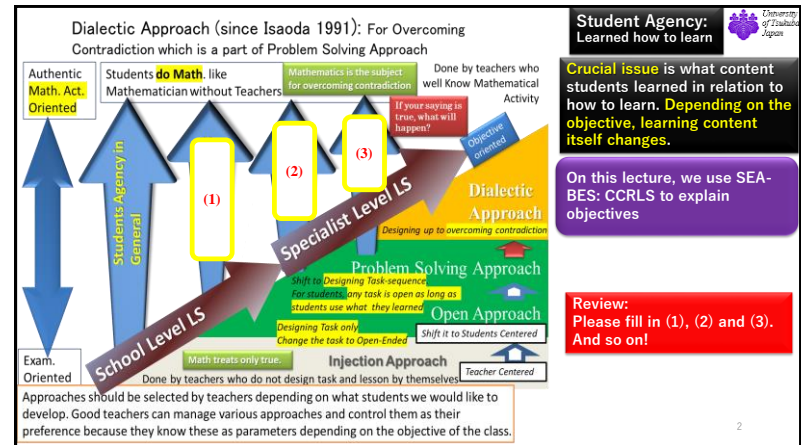


Mathematics Education to Develop Students Agency
Part III: Shape and Figure
Plane Figure and Properties

ISODA Masami, Prof/PhD
 Director of CRICED
 University of Tsukuba, Japan

2. Shape & Figure (Geometry)
 5. Difference of Shape and Figure
 6. Figure and Attribute
 7. Plane Figure and Properties
 8. Solid and Perspectives



This lecture use SEA-BES: CCRLS to explain objectives

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Thinking and Processes

Contents

Key Stage 1

Key Stage 2

Key Stage 3

Figure 1: Revised CCRLS Framework in Mathematics

Appendix B: Terminologies for Mathematical Thinking and Processes

These terminology explain the process. Thus, if you can imagine concrete materials for each class, you are able to develop it in your classroom. That's why we have been discussing materials.

Review:

Mathematization (Freudenthal, 1973) and van Hiele Levels (van Hiele, 1986)

Freudenthal defined Mathematization by the re-organization of (mathematical) experience by mathematical means.


van Hiele Levels (Level of Thinking by Freudenthal): Teaching is the activity for students to be able to think on upper level.

Level	Object	Operation/Means	Nature of language: ways of thinking	Contradiction between levels
L1	Concrete Object	Shape (形)	Concrete object is treated by the hidden attribute of each shape	Round shape is not a circle (as figure). Fold round shapes and find a circle when it was folded just overlap.
L2	Figure (図形)	Attribute	Attribute belonging each figure is treated as the properties of figure	Circle is drawn by compass with center and radius.
L3	Properties (平面図形-plane figure)	Proposition	Figures are recognized by properties.	How can we provide our students for the opportunity to think by and for themselves, and learning how to?

Contradiction is the opportunity to think and discuss mathematically


Review:

By Isoda such as 2015 and 2018

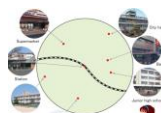


IMAGINE
THE
FUTURE

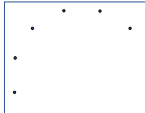
How to use each of these for the discussions of parallel line and perpendicular line?




University
of Tsukuba
Japan




① Parallel lines



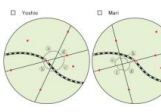
② Perpendicular lines



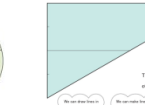
③ Parallel lines




④ Perpendicular lines



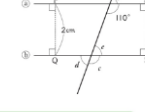
⑤ Parallel lines




⑥ Perpendicular lines




⑦ Parallel lines




⑧ Perpendicular lines




⑨ Parallel lines



⑩ Perpendicular lines




⑪ Parallel lines




⑫ Perpendicular lines

⑬ Parallel lines



⑭ Perpendicular lines



⑮ Parallel lines

⑯ Perpendicular lines

⑰ Parallel lines

⑱ Perpendicular lines

⑲ Parallel lines

⑳ Perpendicular lines

㉑ Parallel lines

㉒ Perpendicular lines

㉓ Parallel lines

㉔ Perpendicular lines

㉕ Parallel lines

㉖ Perpendicular lines

㉗ Parallel lines

㉘ Perpendicular lines

㉙ Parallel lines

㉚ Perpendicular lines

㉛ Parallel lines

㉜ Perpendicular lines

㉝ Parallel lines

㉞ Perpendicular lines

㉟ Parallel lines

㊱ Perpendicular lines

㊲ Parallel lines

㊳ Perpendicular lines

㊴ Parallel lines

㊵ Perpendicular lines

㊶ Parallel lines

㊷ Perpendicular lines

㊸ Parallel lines

㊹ Perpendicular lines

㊺ Parallel lines

㊻ Perpendicular lines


㊼ Parallel lines

㊽ Perpendicular lines


㊾ Parallel lines


㊿ Perpendicular lines


㋀ Parallel lines




How do we use each of these for the discussions of parallel line and perpendicular line?









One line is already drawn.
Think about how to draw the other 3 lines.


We can draw lines in

We can make lines even

We can draw lines

Objectifying by Representation and Symbolizing

Conceptualisation and Proceduralisation



Object (Figure) **Operation/Mean** (Figure) **Nature of language: ways of thinking** (Figure)

L2 Figure (Figure) **Attribute** (Figure) **Proposition** (Figure)

Discuss figure by components.
Discuss figures by the relationships.

Angles of Triangles

1. Look at the sum of the 3 angles of a triangle in various ways.

2. Draw a triangle and measure the angles with a protractor. The sum of the 3 angles is \square° .

3. Cut out the 3 angles and place them together as shown below.

Since the 3 angles together make a straight line, the sum of these angles is \square° .

4. Put together triangles with the same shape and size to make a continuous pattern without any gaps.

Since 3 angles at points A and B make a straight line, their sum is \square° .

5. Fold a triangle to connect the 3 angles.

Since the 3 angles make a straight line, the sum is \square° .

Tessellation

Produce Plane by Figure

page 150.

Can you tessellate?

University of Tsukuba Japan

Mathematization (Freudenthal, 1973) and van Hiele Levels (van Hiele, 1986)

Freudenthal defined Mathematization by the re-organization of (mathematical) experience by mathematical means.

van Hiele Levels (Level of Thinking by Freudenthal): **Teaching is the activity for students to be able to think on upper level.**

Level	Object	Operation/Mean	Nature of language: ways of thinking	Contradiction between levels
L1	Concrete Object	Shape (形)	Concrete object is treated by the hidden attribute of each shape	Round shape is not a circle (as figure). Fold round shapes and find a circle when it was folded just overlap. Circle is drawn by compass with center and radius.
L2	Figure (図形)	Attribute	Attribute belonging each figure is treated as the properties of figure	How can we provide our students for the opportunity to think by and for themselves, and learning how to?
L3	Properties (平面図形: plane figure)	Proposition	Figures are recognized by properties.	Contradiction is the opportunity to think and discuss mathematically. Square, rhombus and rectangle are different figure (shape). Properties of figure are attribute of figure (shape). Square is rhombus. Square is rectangle. Rhombus is square if Rectangle is rhombus if

Mathematical Activities:

- Problem Solving
- Exploration and Enquiry
- Mathematical Modeling, Mathematical and
- Programming
- Conjecturing, Justifying and Proving
- Conceptualization and Proceduralization
- Representation and Operation

Mathematical Ways of Thinking:

- Generalization and Specialization
- Extension and Integration
- Inductive, Analogical and Deductive Reasoning
- Abstracting, Concretizing and Embedding
- Objectifying by Representation and Symbolizing
- Relational and Functional Thinking

Mathematical Values Seeking:

- Generality and expandability
- Reasonableness and harmony
- Usefulness and efficiency
- Simpler and easier
- Beautifulness

By Isoda such as 2015₁₄ and 2018.

University of Tsukuba Japan