


Mathematics Education to Develop Students Agency
Part III: Shape and Figure
Solid and Perspectives



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2. Shape & Figure (Geometry)	3. Difference of Shape and Figure
	4. Figure and Attribute
	7. Plane Figure and Properties
	8. Solid and Perspectives

Van Hiele Levels are discussed
How about the case of 3D-shapes and space

van Hiele Levels (Level of Thinking by Freudenthal): Teaching is the activity for students to be able to think on upper level.

Level	Object	Operation/Meaning	Nature of language: ways of thinking
L1	Physical O.	Shape (形)	Physical/concrete object is treated by the hidden attribute of each shape
L2	Figure(図形)	attribute	Attribute belonging each shape is treated by the properties of figure
L3	Properties (平面図形)	Proposition	Figures (plane figure) are recognized/operated by properties.
L4	Proposition (平面幾何)	Proof	Proposition are explained by proof (local theory)

In English: Triangular and triangle, rectangular and rectangle, circular and circle can be used to explain the difference of shape and figure. However, I am not sure how can I explain it in the case of solid. Here, I distinguish as follows.

Level	Object	Operation/Meaning	The case of 3D
L1	Physical O.	Shape	Physical/concrete object is treated by the hidden attribute
L2	Solid(立体)	attribute	Attribute of each solid is treated by the components. Solids are usually considered with figures/shapes.
L3	Properties (空間図形)	Proposition	Space solids are discussed with properties of figures. Space solids are usually considered with manner of plane figure.

On this meaning, 3D-shapes and space are discussed with various operations of shape, figure and plane figure.

Level	Object	Operation/Meaning
L1	Physical O.	Shape
L2	Figure(図形)	attribute
L3	Properties (平面図形)	Proposition

L1 includes the exploration of shapes





- Collecting, Touching, Stacking, Making, Tracing, Cutting and drawing




By Legend:
TANAKA Hiroshi,
Elementary School
at Tsukuba who
has been known as
a teacher to
develop
mathematical
thinking via
'conjunction' and
'considering others'

Lower/bottom level
activity provide various
bases for upper levels
(Freudenthal, 1973)




- Tanaka's video: How many?

Level	Object	Operations/Means
1	Physical O. Shape	
2	Diagram	 
3	Properties (問題性質)	 




④ What a ball.

 An object that looks like a circle from any direction is called sphere.


⑤ Look for things shaped like a sphere.






⑥ What is the shape of the cross-section of a sphere?
Where should we cut to make the largest cross-section from a sphere?







Which level is it?
the solid (object of thinking) by the figure/shape (operation/means).


 When a sphere is cut in half, the radius and the diameter of the circle of the cross-section are called **center**, **radius** and **diameter** of the sphere.



⑦ How can we find the diameter of a sphere?

⑧ Let's use the giraffe ruler on pages 125 and 126 for measuring the diameter of spheres.



University of Tokyo
Department of Applied Physics

Learn (対象) **Object** **Question** (質問) **Statement** (文)

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Tsubota's video

By Legend:
TSUBOTA
Kozo,
Elementary
School at
Tsukuba,
who had
been known
as Open
Approach

This lecture use SEA-BES: CCRLS to explain objectives

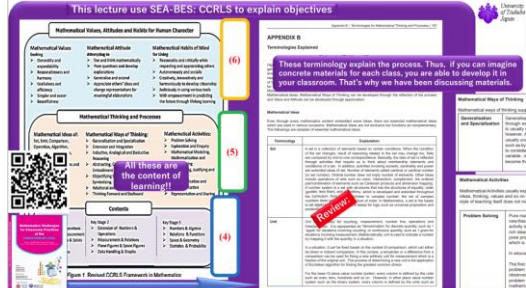


Figure 1 Revised CCRLS Framework in Mathematics