Slide 1

Mathematics Education to Develop Students Agency
Part III: Measurement
How to introduce the unit for measuremt

筑波大学
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3. Measurement	9. How to introduce the unit for measurement
	10. Length and Mass (Weight)
	11. Area and Volume
	12. Time, Angle and Others (rate and ratio)

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Slide 2

1 Which is longer?

How can you compare?
- Let's critique the ways to compare!
- To compare we should set the same conditions such as the starting points and the directions.

Comparing length and width...

Direct Comparison   Indirect Comparison

Slide 3

13 Comparing Sizes

What activity they are engaging in.

Isoda & Murata (2011). Study with Your Friends Mathematics for Elementary School 1st Grade。Gakko Tosho.

Comparing Length

1 Which is longer? Let's talk about how to compare.
1 Jump rope
2 Postcard   3 Book

Let's compare length and width.

2 Let's compare different lengths using a tape.

Let's Compare

Measure the lengths of opened arms and compare.
Ayumi
Satoshi

Direct Comparison   Indirect Comparison

Slide 4

What activity they are engaging in.

Isoda & Murata (2011). Study with Your Friends Mathematics for Elementary School 1st Grade。Gakko Tosho.

Arbitrary Unit

3 Which is longer? And by how much?
1 Length and width of the desk.

Length ·········· 3 pencils
Width ·········· 4 pencils

Length [ ] erasers
Width [ ] erasers

2 Pen and pencil

Let's Compare Amount of Water
Which container can hold more juice?

1 Let's think about how to compare the amount of juice.

Indirect Comparison

Direct Comparison   Arbitrary Unit

Slide 1

Phases	Activity
P1	Direct Comparison
P2	Indirect Comparison
P3	Arbitrary Unit
P4	Standard Unit

The manner to introduce the measurement unit.

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Why we need this manner?
- Provide the necessity for the measurement unit
- Experience the activity to create necessary measurement unit.
- Task sequence to provide the pre-concept before mathematical definition

⑬ Comparing Sizes

Comparing Length

❶ Which is longer? Let's talk about how to compare.
- ❶ Jump rope — How do we compare?
- ❷ Postcard
- ❸ Book

Let's compare length and width.

② Let's compare different lengths using a tape.

My waist is... Can this desk go through the doorway?

I counted by pencils. I counted by erasers.

Let's Compare

Measure the lengths of opened arms and compare.
Ayumi
Satoshi

Comparing lengths on page 114.

③ Which is longer? And by how much?
- ❶ Length and width of the desk.
  - Length .... 3 pencils
  - Width .... 4 pencils
  - Length .... ____ erasers
  - Width .... ____ erasers

Can I compare by counting the number of □?

- ❷ Pen and pencil

Can I compare by counting the number of □?

Slide 2

Dialectic Approach (since Isoda 1991): For Overcoming Contradiction which is a part of Problem Solving Approach

Student Agency: Learned how to learn

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Authentic Math. Act. Oriented

Students **do Math.** like Mathematician without Teachers

Mathematics is the subject for overcoming contradiction

Done by teachers who well Know Mathematical Activity

If your saying is true, what will happen?

Students Agency in General

Math. Thinking and Ideas

Math. Value & Attitude

Math. Critique

Specialist Level LS

Objective oriented

Dialectic Approach

Designing up to overcoming contradiction

Problem Solving Approach

Shift to Designing Task-sequence For students, any task is open as long as students use what they learned

Open Approach

Designing Task only Change the task to Open-Ended

Shift it to Students Centered

School Level LS

Exam. Oriented

Math treats only true.

Injection Approach

Done by teachers who do not design task and lesson by themselves

Teacher Centered

Approaches should be selected by teachers depending on what students we would like to develop. Good teachers can manage various approaches and control them as their preference because they know these as parameters depending on the objective of the class.

- Globalization is going through standardization:
  - Technological Innovation
  - Politics for standardization
  - Business innovation
- It is a story of business control with politicians: small traditional shops/companies are gone for innovative convenience of mass.
- How to create the measurement unit beyond/before how to measure. It means to produce new standard for new welfare depending on what people want to be.
- If measurement strand only means how to measure, your students can use existed technology and follows others' business. It is necessary however they could not lead innovation itself.

Slide 3

Phases	Activity	Mathematical Nature of each phase	Behind mathematics
P1	Direct Comparison	Compare two objects on the same conditions ➤ Can compare more objects when we can apply the transitive low: If a>b, b>c, then a>c. ➤ **If a>b, a>c, then we have to compare a and b**: we have to set the algorithm to make an order of comparisons.	**Axioms for comparison:** Mathematical relationships for grater than, less than and equal.
P2	Indirect Comparison	Compare any objects on the same conditions by using the alternative material such as a tape. ➤ We can make an order but **not sure the difference**, how much more.	+The **line/ray** only have the origin point and direction. +The **number line** set by the **origin**, the **unit** and **direction**.
P3	Arbitrary Unit	We can specify the difference **as long as we can measure** by using the arbitrary **unit** on the alternative material. ➤ It works only locally as long as we use the same unit. ➤ If not, we have to seqe the new scale for measurement.	➤ As long as we can find greatest common divisor, we can measure with support of **Euclidian Algorithm**. ➤ Mathematically, **irrational number** is problem however it does not appear practically.
P4	Standard Unit	Standard unit is universal and can use only at the **politically accepted countries**.	

Slide 4

Euclidean Algorithm: Review of Part II-Lesson 2, Fraction for Teachers (Isoda, 2012)

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⑭ Fractions

We cut a tape with a length that is equal to the length of the blackboard and measured it with a 1 m stick.
The length is 1 m and a remaining part. How many meters is the remaining part?

1 m stick

remaining part

The length of the remaining part is less than 1m.

Can we express it without using decimals?

Produce the unit by using remaining part

Euclidian Algorithm

For example, Greatest Common Divisor of 18 and 42 is: 42=6×7, 18=3×6, 3 and 7 are prime numbers, then 6 is GCD of 18 and 42. It will be found using the remainders as follows.

$$42=2×18+6 \qquad → \qquad 18=3×6$$

42 divided by 18, the remainder is 6.

18 divided by the remainder

The rectangular diagram is the better way to demonstrate Euclidian Algorithm.

Exercise:
Let's find the GCD of (41, 18) by the rectangular diagram.