


 Free Program for SEAMEO School Network
 from the University of Tsukuba, Affiliate Member of SEAMEO
**Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
 How do you teach? Why?**

Lesson 10: How to extend Subtraction

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Adopting a 21st
Century Curriculum



Revitalizing
Teacher Education

What is the number? Review

We usually teach:

- Existence and necessity
Cardinal (Set) Number
- Order/Larger or Smaller/
Greater or Less
- Operations

How do you teach?
Make sense?
Acquisition of proficiency?

For what?
Number sense?

In Japan:
Make sense (understand
meaning)

**I think about how to
calculate/operate/find the
easier way to answer**

Acquisition of proficiency

**Try to teach how to extend
the number**

How do we teach the number more than ten?

How do we teach addition more than ten?

How do we teach subtraction more than ten?

Review

① How many acorns did Toshio pick up?

2 boxes of 10 and 8 more

twenty eight

For 28, the figure in tens place is and the figure in ones place is

② How many acorns did Hiroko pick up?

3 boxes of 10 and no extra ones

thirty

For 30, the figure in tens place is and the figure in ones place is

How each task chose the number and set the task sequence.

Extend the number by using the idea of the base ten place value system: Permanence of Form, Analogical Reasoning based on what children already learned

New way of representation of cardinal number for 21-45

New way of representation of ordinal number for 21-45

Write the correct numbers in the

① Write the following numbers.

② If the figure in tens place is 1 and the figure in ones place is 0, then the number is

③ 5 tens and 8 ones is

④ How many are there?

⑤ Write the correct numbers in the

Necessity of using 0 as for showing place value

Review Using what you already knew on the past 8 lessons!

| | | | |
|---|--|---------------------------------------|--|
| TOPIC 1: INTRODUCTION | L1: Introductory discussion to develop mathematical thinking | | |
| TOPIC 2: NUMBERS | L2: How to introduce number | L3: What is number | |
| | L4: How to introduce addition | L5: What is addition | |
| TOPIC 3: ADDITION AND SUBTRACTION | L6: How to introduce subtraction | L7: What is subtraction | |
| | | | |
| TOPIC 4: EXTEND NUMBER WITH ADDITION AND SUBTRACTION USING COLUMN FORM | L8: How to extend number to more than 10 | L9: How to extend addition | L10: How to extend subtraction |
| | L11: How to extend number to more than 100 | L12: How to introduce column addition | L13: How to introduce column subtraction |
| | | | |

□ Participants need to consider **what's new**.

□ Participants of this program are able to **imagine the ways of learning from the past process of learning.**

What is the representation/model for the base ten place value system

How children shift new way of counting?

Review Which number is really base ten place value system?

Terminology!

Number Arabic digit, Numerical, Concrete object, Semi-concrete object, Denumerated number, Base ten system, Base ten place value system.

Numbers Larger than 10

① How many dragonflies are there?

② How many are there? Grouping for 10

Count by 10: 10 is a unit for counting

Review

① How many are there?

② Write the answers in the

③ 3 tens and 7 ones make

④ 4 tens and ones make 46.

⑤ 40 is tens.

For what?

Which is More?

Are there more dogs or more cats?

Are there more ● or more ○?

What is the best way to compare?

Necessity of re-grouping/re-arrangement under base ten place value system to show (cardinal) number

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values:
Generality and Expandability
Reasonableness and Harmony
Usefulness and Efficient
Simpler and Easier
Beautifulness

Mathematical Ideas for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Attitude attempting to:
See and think mathematically
Pose question and develop explanation such as why and when
Generalize and extend
Appreciate others' idea and change representation to conceptualize

Mathematical Thinking and Processes

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization and Proceduralization
Representation and Sharing

Habits of mind for Citizen to live:
Reasonably and critically with respecting and appreciating others
Autonomously Creatively and innovatively in harmony
Judiciously using tools such as ICT
Empower in imagining the future through lifelong learning

Those terminology distinguish tasks and explain task sequence for the preparation of future learning.

Content

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations
- Extension of Number and Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs
- Number & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability

Curriculum Standards: SEABES-CRCLS (by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017))

When we should teach count by two and five, here?

Why 28 instead of 21?

Review Where and when did children learn it, already?

① Let's count.

② Write the answers in the

③ Which number is larger?

④ Where should you put these cards?

⑤ Up to what number did the frog and rabbit go?

Mathematical Ideas for:
Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Thinking and Processes

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization and Proceduralization
Representation and Sharing

Review

Look for Numbers

Let's look for pairs around us.

Let's look for groups of 3 and 4.

Let's count things in school.

For what?

Number can be seen under the various units by using attribute of concrete object on situations.

It is a preparation of multiplication.

By using what children learned!
Task sequence for preparation of future

Number

- Existence and necessity
- Order
- Larger or Smaller
- Greater or Less
- Operations

In Japan: Make sense (understand meaning)
Think about how to calculate/operate/ find the easier way to answer
Try to teach how to extend the number

Contents

Numbers

- Numbers up to 10 8
- How Many? 26
- Numerical Order 32
- Addition(1) 34
- Subtraction(1) 46
- Numbers Larger than 10 64
- Which is More? 73
- Look for Numbers 77
- Addition(2) 77
- Addition for Japanese Background 85
- Shapes 60

Story Problems
Algebraic Situation A, B, C
Concrete Object
Illustration
Explanation by words

Mathematical Modeling
Explanation with blocks
Semi Concrete Object

Math expression
Addition, operation
Math sentence
addition

hapes

Review

Extension and Integration (MOE-JP, 1968)
Conceptualization and Proceduralization
(Isoda, 1991; Isoda&Ofitos 2021)

(Set/ Cardinal)Number up to 10: Counting unit is one, compose & de-compose, and beautiful pattern by arrangement

Extension to unknown

(Set/ Cardinal)Number more than 10: Counting unit is one, ten, and composite & de-composite, and beautiful pattern by arrangement, line and table of numbers (start from 0)

Integration new on unknown with known

A theory for **conceptual change** for learning trajectory: (Isoda, 1991;Isoda&Ofitos, 2021)

Situation A
for Number up to 10

Meaning A
1-2 Composite of Numbers

Procedure A
Mental Calculation

Extension B
Number more than 10

Meaning B
12-9, 11-3, 10s or 1s of minuend shall we reduce?

Procedure B
Mental Calculation

Subtraction
cards up to 10

Subtraction
cards more than 10

Proceduralization of meaning (conceptual knowledge) as form
Conceptualization of procedure, produce meaning with form
Connectivity with various meanings and procedure by using appropriate representations

Extension and Integration (MOE-JP, 1968)
Conceptualization and Proceduralization
(Isoda, 1991; Isoda&Ofitos 2021)

(Set/ Cardinal)Number up to 10: Counting unit is one, compose & de-compose, and beautiful pattern by arrangement

Extension to unknown

(Set/ Cardinal)Number more than 10: Counting unit is one & ten, and composite & de-composite, and beautiful pattern by arrangement, line and table of numbers (start from 0)

Integration new on unknown with known

A theory for **conceptual change** for learning trajectory: (Isoda, 1991;Isoda&Ofitos, 2021)

Situation A
for Number up to 10

Meaning A
5-2, de-composite of Numbers

Procedure A
Mental Calculation

Extension B
Number more than 10

Meaning B
12-9, 11-3, 10s or 1s of minuend shall we reduce?

Procedure B
Mental Calculation

Subtraction
cards up to 10

Subtraction
cards more than 10

Proceduralization of meaning (conceptual knowledge) as form
Conceptualization of procedure, produce meaning with form
Connectivity with various meanings and procedure by using appropriate representations

What's new? **Change the meaning of addition!** How?

Learned:
Addition situation
☐ Altogether
☐ Increase
Addition was composite of numbers

Counting by using BTPVS.
Mathematical Sentence is given.
Think about how to calculate/operate

Make 10 by de-composite of augend. And then, composite.

De-composite-augend method

Make 10 by de-composite of addend. And then, composite.

De-composite-addend method

Why this order: 9+4 → 3+9 → 8+6?

Subtraction was de-composite of numbers

Counting back by using BTPVS.

Mathematical Sentence is given.

Think about how to calculate/operate

Use 10s on minuend for subtraction and add remaining.

Subtraction-Addition method

Use 1s on minuend by de-composing subtrahend and subtract remaining from 10s on minuend

Subtraction-subtraction method

10s or 1s, which part is easier to reduce at first.

Why this order: 12-9 → 11-3 → 14-6?

Could you explain the learning process for extension of subtraction more than 10 by using terminology?

Learned:
Subtraction situation
☐ left
☐ difference
Subtraction was de-composite of numbers

Counting back by using BTPVS.
Mathematical Sentence is given.
Think about how to calculate/operate

Use 10s on minuend for subtraction and add remaining.

Subtraction-Addition method

Use 1s on minuend by de-composing subtrahend and subtract remaining from 10s on minuend

Subtraction-subtraction method

10s or 1s, which part is easier to reduce at first.

Why this order: 12-9 → 11-3 → 14-6?

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Mathematical Thinking: How to Develop it in the Classroom

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Mathematics Challenges for Classroom Practice at the Lower Secondary Level

Mathematics Challenges for Classroom Practice at the Upper Primary Level

Mathematics Challenges for Classroom Practice at the Upper Secondary Level

Isoda, M., Tani, D. (2019). Junior High School Mathematics Textbook, Gakko Tosho

How do you explain by using terminology?

Think about how to

- Make 10
- De-composite of Augend
- De-composite of Addend

World of addition: Beautifulness of pattern (structure)

Math expression

Math sentence addition

Using blocks for BTPVS

Think about how to calculate?

Proceduralization

Beautifulness of additions

Think about how to

- Use 10s or 1s on minuend
- Subtraction-addition method
- Subtraction-subtraction method

World of Subtraction: Beautifulness of pattern (structure)

Math expression

Math sentence Subtraction

Using blocks for BTPVS

Think about how to calculate?

Proceduralization

Beautifulness of Subtraction

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