筑波大学
University of Tsukuba

Free Program for SEAMEO School Network
from the University of Tsukuba, Affiliate Member of SEAMEO

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
How do you teach? Why?

Lesson 10: How to extend Subtraction

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Adopting a 21st Century Curriculum — Revitalizing Teacher Education

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What is the number? We usually teach:

Review

- Existence and necessity
  Cardinal (Set) Number
- Order/Larger or Smaller/ Greater or Less

How do we teach the number more than ten?

- Operations

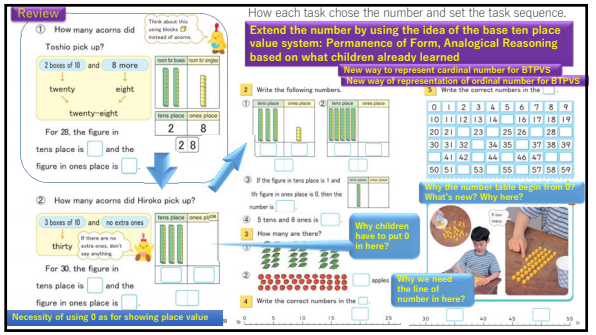
How do we teach addition more than ten?
How do we teach subtraction more than ten?

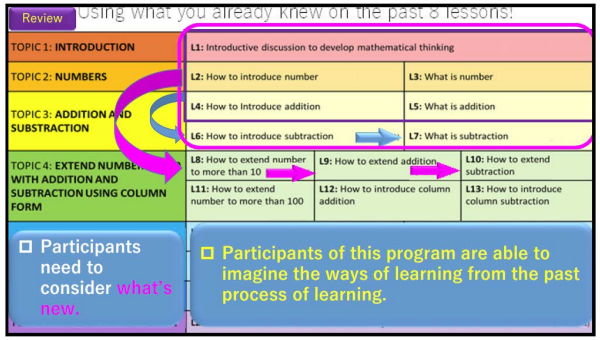
How do you teach?
Make sense?
Acquisition of proficiency?

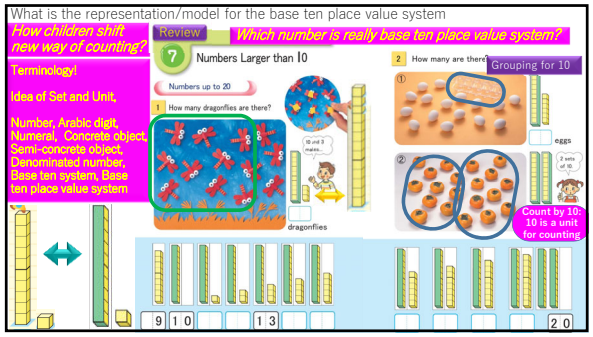
For what?
Number sense?

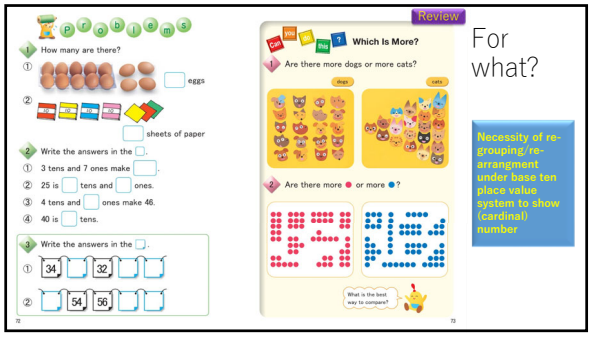
In Japan:
Make sense (understand meaning)
Think about how to calculate/operate/find the easier way to answer
Acquisition of proficiency

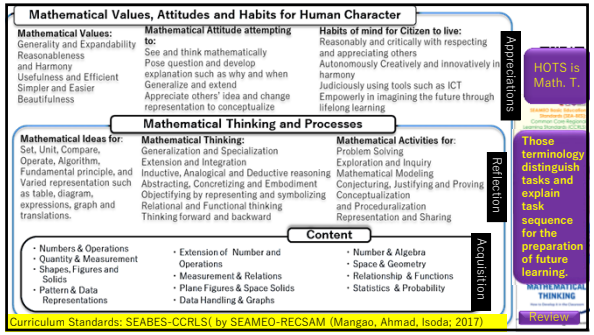
Try to teach how to extend the number

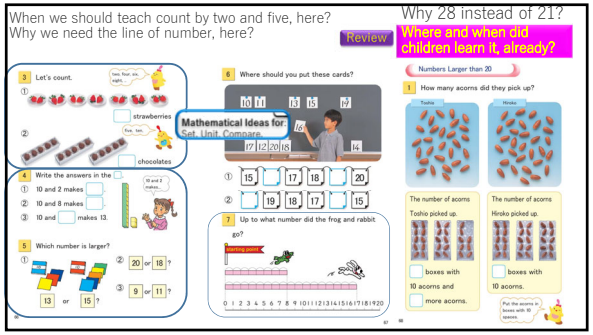


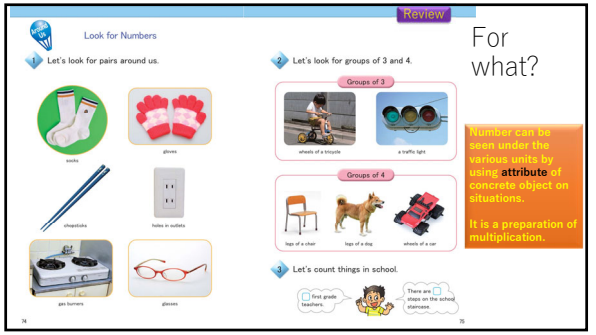


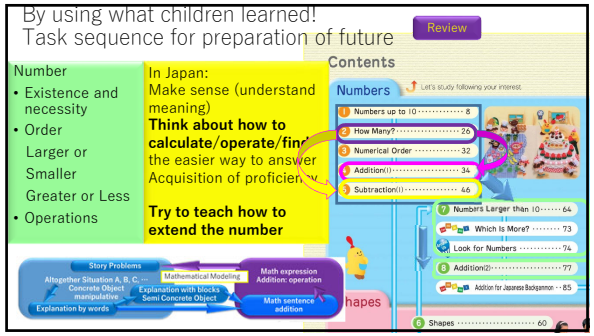


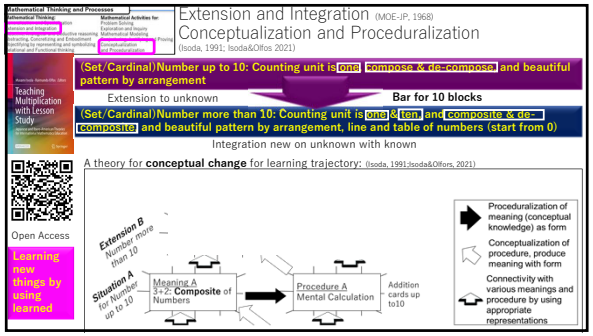


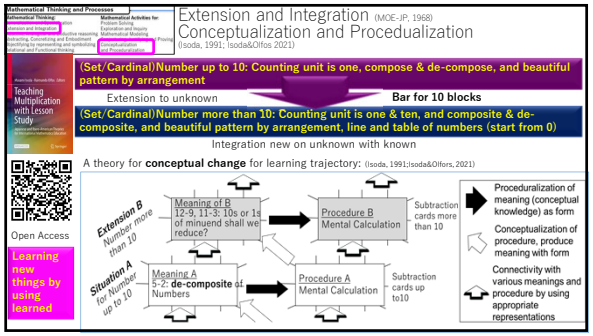







By using what children learned!
Task sequence for preparation of future
Review
Contents
Number
• Existence and necessity
• Order
Larger or Smaller
Greater or Less
• Operations
In Japan:
Make sense (understand meaning)
Think about how to calculate/operate/find the easier way to answer
Acquisition of proficiency
Try to teach how to extend the number
Numbers
Numbers up to 10
How Many?
Numerical Order
Addition(1)
Subtraction(1)
Numbers Larger than 10
Which Is More?
Look for Numbers
Addition(2)
Addition for Japanese Backgammon
Shapes
Story Problems
Mathematical Modeling
Math expression
Addition operation
Concrete Object manipulative
Explanation with blocks
Semi Concrete Object
Explanation by words
Math sentence addition


Extension and Integration (MOE-JP, 1968)
Conceptualization and Procedualization
(Isoda, 1991; Isoda&Olfos 2021)
(Set/Cardinal)Number up to 10: Counting unit is one, compose & de-compose, and beautiful pattern by arrangement
Extension to unknown
Bar for 10 blocks
(Set/Cardinal)Number more than 10: Counting unit is one & ten, and composite & de-composite, and beautiful pattern by arrangement, line and table of numbers (start from 0)
Integration new on unknown with known
A theory for conceptual change for learning trajectory: (Isoda, 1991;Isoda&Olfors, 2021)
Open Access
Learning new things by using learned
Extension B
Number more than 10
Situation A
for Number up to 10
Meaning A
3+2: Composite of Numbers
Procedure A
Mental Calculation
Addition cards up to10
Proceduralization of meaning (conceptual knowledge) as form
Conceptualization of procedure, produce meaning with form
Connectivity with various meanings and procedure by using appropriate representations


Extension and Integration (MOE-JP, 1968)
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Open Access
Learning new things by using learned
Extension B
Number more than 10
Meaning of B
12-9, 11-3: 10s or 1s of minuend shall we reduce?
Procedure B
Mental Calculation
Subtraction cards more than 10
Situation A
for Number up to 10
Meaning A
5-2: de-composite of Numbers
Procedure A
Mental Calculation
Subtraction cards up to10
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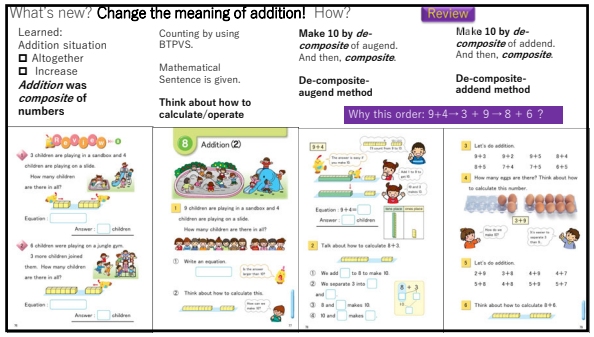
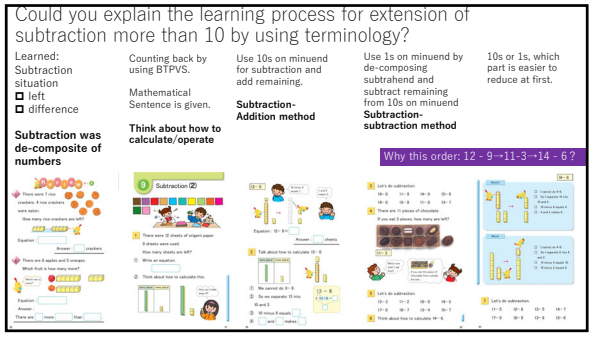
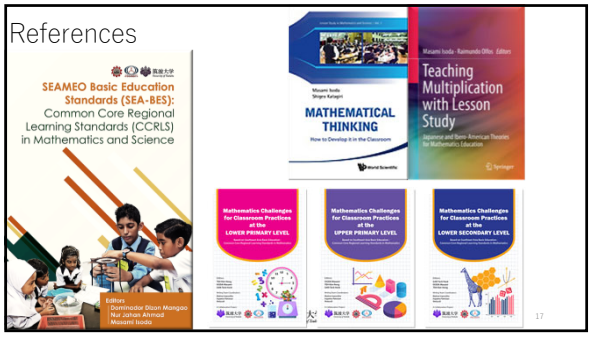

What's new? Change the meaning of addition! How?
Review
Learned:
Addition situation
Altogether
Increase
Addition was composite of numbers
Counting by using BTPVS.
Mathematical Sentence is given.
Think about how to calculate/operate
Make 10 by de-composite of augend. And then, composite.
De-composite-augend method
Make 10 by de-composite of addend. And then, composite.
De-composite-addend method
Why this order: 9+4→3 + 9→8 + 6 ?
Addition (2)


Could you explain the learning process for extension of subtraction more than 10 by using terminology?
Learned:
Subtraction situation
left
difference
Subtraction was de-composite of numbers
Counting back by using BTPVS.
Mathematical Sentence is given.
Think about how to calculate/operate
Use 10s on minuend for subtraction and add remaining.
Subtraction-Addition method
Use 1s on minuend by de-composing subtrahend and subtract remaining from 10s on minuend
Subtraction-subtraction method
10s or 1s, which part is easier to reduce at first.
Why this order: 12 - 9→11-3→14 - 6 ?
Subtraction (2)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Editor: Dominador Dixon Mangao
Nur Jahan Ahmad
Masami Isoda
Teaching Multiplication with Lesson Study
MATHEMATICAL THINKING
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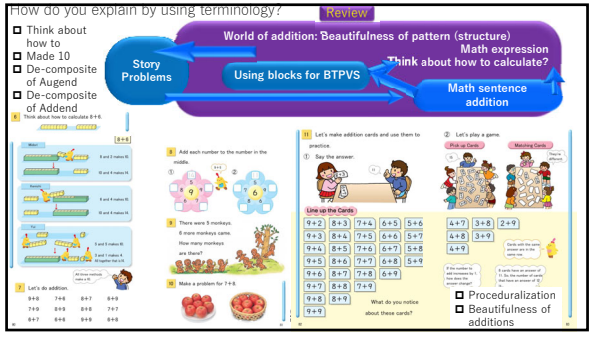
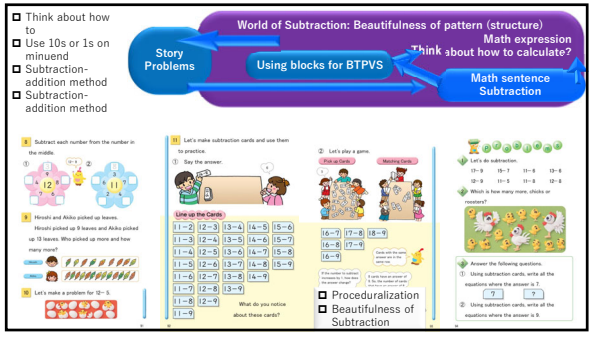
Mathematics Challenges for Classroom Practices at the UPPER PRIMARY LEVEL
Mathematics Challenges for Classroom Practices at the LOWER SECONDARY LEVEL


How do you explain by using terminology?
Review
Think about how to
Made 10
De-composite of Augend
De-composite of Addend
World of addition: Beautifulness of pattern (structure)
Math expression
Think about how to calculate?
Story Problems
Using blocks for BTPVS
Math sentence addition
Proceduralization
Beautifulness of additions


Think about how to
Use 10s or 1s on minuend
Subtraction-addition method
Subtraction-addition method
World of Subtraction: Beautifulness of pattern (structure)
Math expression
Think about how to calculate?
Story Problems
Using blocks for BTPVS
Math sentence Subtraction
Proceduralization
Beautifulness of Subtraction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