Slide 1

筑波大学
University of Tsukuba

Free Program for SEAMEO School Network
from the University of Tsukuba, Affiliate Member of SEAMEO

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
How do you teach? Why?

Lesson 15: How to introduce Multiplication and It's Table (Rows)

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Adopting a 21st Century Curriculum — 7 — Revitalizing Teacher Education

Slide 2

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values:	Mathematical Attitude attempting to:	Habits of mind for Citizen to live:
Generality and Expandability Reasonableness and Harmony Usefulness and Efficient Simpler and Easier Beautifulness	See and think mathematically Pose question and develop explanation such as why and when Generalize and extend Appreciate others' idea and change representation to conceptualize	Reasonably and critically with respecting and appreciating others Autonomously Creatively and innovatively in harmony Judiciously using tools such as ICT Empowery in imagining the future through lifelong learning

Appreciations

HOTS is Math. T.

Mathematical Thinking and Processes

Mathematical Ideas for:	Mathematical Thinking:	Mathematical Activities for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.	Generalization and Specialization Extension and Integration Inductive, Analogical and Deductive reasoning Abstracting, Concretizing and Embodiment Objectifying by representing and symbolizing Relational and Functional thinking Thinking forward and backward	Problem Solving Exploration and Inquiry Mathematical Modeling Conjecturing, Justifying and Proving Conceptualization and Proceduralization Representation and Sharing

Reflection

Those terminology distinguish tasks and explain task sequence for **the preparation of future learning.**

MATHEMATICAL THINKING

Content

· Numbers & Operations · Quantity & Measurement	· Extension of Number and Operations	· Number & Algebra · Space & Geometry

Acquisition

We are reading the JP textbook by using terminology to explain to distinguish every content of learning and sequence of content for knowing the way to develop children who learn and think by and for themselves through the preparation of future learning.

Curriculum Standards: SEABES-CCRLS(by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017)

Review

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Using what you already knew on the past 14 lessons!

TOPIC 1: INTRODUCTION	L1: Introductive discussion to develop mathematical thinking	
TOPIC 2: NUMBERS	L2: How to introduce number	L3: What is number
TOPIC 3: ADDITION AND SUBSTRACTION	L4: How to Introduce addition	L5: What is addition
	L6: How to introduce subtraction	L7: What is subtraction
TOPIC 4: EXTEND NUMBER TO 100 WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10	L9: How to extend addition	L10: How to extend subtraction
	L11: How to extend number to more than 100	L12: How to introduce column addition and subtraction	L13: How to extend column addition and subtraction
TOPIC 5: MULTIPLICATION	L14: How to introduce multiplication	L15: How to develop multiplication table
	L16: What is the multiplication table	L17: How to introduce column multiplication
☐ Participants need to consider what's new.	☐ Participants of this program are able to imagine the ways of learning from the past process of learning.	

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What is the number?
We usually teach:

Review

• Existence and necessity

• Order/Larger or Smaller/ Greater or Less

• Operations

How do you teach?
Make sense?
Acquisition of proficiency?

For what?
Number sense?

In Japan:
Make sense (understand meaning)

Think about how to calculate/operate/find the easier way to answer

Acquisition of proficiency

Try to teach how to extend the number

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What is multiplication?
How do you introduce it?

Review

☐ Multiplicative situations (Vergnaud)
  ✓ Isomorphism of measure
  ✓ Product of measure
  ✓ Unique measure space
☐ Binominal operation
  ✓ Multiplier x Multiplicand
  ✓ Factor x Factor
☐ Repeated Addition VS Attribute
☐ Definition by measurement: **to get the total quantity when the unit quantity and the number (amount) of unit are given.**
☐ Area
☐ Cartesian Product
☐ Combinatory
☐ Tree Diagram
☐ Splitting
☐ Operator
  ✓ Multiple from two side

$4 \text{ (dishes)} \times 3 \text{ (apples/dish)} = 12 \text{ (apples)}$
$\neq 3 \text{ (apples/dish)} + 3 \text{ (apples/dish)} + 3 \text{ (apples/dish)} + 3 \text{ (apples/dish)}$
$\neq 12 \text{ (apples/dish)}, \text{ or} \neq (12 \text{ apples})/(4 \text{ dishes}) = 3 \text{ (apples/dish)}$
However, in mathematics textbooks, it will be as follows.

$3 \text{ (apples)} + 3 \text{ (apples)} + 3 \text{ (apples)} + 3 \text{ (apples)} = 12 \text{ (apples)}$

4 (dishes)

Descartes 1637

Open Access

Teaching Multiplication with Lesson Study

BE:BC = BD:BA, then BE × BA = BC × BD.
If BA is a unit, it can be seen as BE = BC × BD.

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Problem of English Notation

Review

Which one is operator in English?
Add **a** to b: b + **a**, b is addend and a is **augend**
Subtract **a** from b: b − **a**, b is minuend and a is **subtrahend**
Multiply **a** by b: **a** x b, a is **multiplier** and b is multiplicand
'a times b': a x b looks '(a x) b'
Divide a by **b**: a ÷ **b**, a is dividend and b is **divisor**
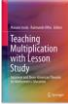
Find the answers
$1 \rightarrow (x2) \rightarrow$
$1 \rightarrow (2x) \rightarrow$
$1 \rightarrow (\div 2) \rightarrow$
$1 \rightarrow (x \frac{1}{2}) \rightarrow$
$1 \rightarrow (2 \div) \rightarrow$
$1 \rightarrow (\frac{1}{2} x) \rightarrow$

Give me the situation (story problem) for the following compound expression
1) $(3+2)$ x 2 ÷ 5
2) 2 x $(3+2)$ ÷ 5

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Problem of English Notation 2

Review

23 x 7: Multiplier 23, Multiplicand 7

Model A	The repeated addition for Model A		vs	Model B	Row 2 for the addition on Model B
	1x2	=2)	vs		2x1 (=1+1)
	2x2	=2+2)	vs		2x2 (=2+2)
	3x2	=2+2+2)	vs		2x3 (=3+3)
	4x2	=2+2+2+2)	vs		2x4 (=4+4)
	5x2	=2+2+2+2+2)	vs		2x5 (=5+5)
	6x2	=2+2+2+2+2+2)	vs		2x6 (=6+6)
	7x2	=2+2+2+2+2+2+2)	vs		2x7 (=7+7)
	8x2	=2+2+2+2+2+2+2+2)	vs		2x8 (=8+8)
	9x2	=2+2+2+2+2+2+2+2+2)	vs		2x9 (=9+9)

$$\begin{array}{r} 2\ 3 \\ \times\ 7 \\ \hline 1\ 6\ 1 \end{array}$$

Multiplicand?
Multiplier?

How do you explain?

$$\begin{array}{r} 2\ 3 \\ \times\ 7 \\ \hline 1\ 6\ 1 \end{array}$$

Multiplicand *for row 7*
Multiplier *of row 7*

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Problem of English Notation 2 (Continued)

Review

Reasonable for English

Every term increases by 1
3x1=1+1+1
(o) (o) (o)
3 x 2=2+2+2
(oo) (oo) (oo)
3 x 3=3+3+3
(ooo) (ooo) (ooo)

Invariant of the number of terms

Row of three (idea)
3x1=1+1+1
3x2=2+2+2
3x3=3+3+3
3x4=4+4+4
3x5=5+5+5
3x6=

Grammatical Difference

In English, 3x2 means 3 times 2.

In Japanese, 3x2 means 3, 2 times.

Contradiction

Japanese Common

Accumulation of same numbers
3x1=3
(ooo)
3 x 2=3+3
(ooo) (ooo)
3 x 3=3+3+3
(ooo) (ooo) (ooo)

Invariant of the number for accumulation

Row of three (most countries)
3x1=3
3x2=3+3
3x3=3+3+3
3x4=3+3+3+3
3x5=3+3+3+3+3
3x6=

In English, there is no contradiction but it is not accumulation

In English, multiplication is easier to remember but it is a contradiction. Commutativity is necessary

In Japanese, multiplication is accumulation, easier to remember; no contradiction

- In Japanese there are no problem between Japanese explanation and notation.

Why there are four chapters? Why this order?

These Multiplication Chapters are a best chapter for knowing how Japanese try to develop students who learn mathematics for themselves based on what they learned.

Numbers and Calculations

2nd grade vol.1

- Thinking about Calculations
- Various Shapes
- Vertical Addition
- Telling Time
- Tables and Graphs
- Thinking about Calculations
- Numbers up to 1000
- Addition and Subtraction(1)
- Subtraction in Vertical Form
- Length(1)

Grade 1
Number up to 100

9 Multiplication(1)·········· 2
Express as a Multiplication Equation···· 12

10 Multiplication(2)·········· 13
The 2×□ Multiplication Table···· 13
The 5×□ Multiplication Table···· 15
The 3×□ Multiplication Table···· 17
The 4×□ Multiplication Table···· 19
Card Game·················· 21
Let's Use the Multiplication Table that You Have Studied···· 25

Grade 2

11 Multiplication(3)·········· 27
The 6×□ Multiplication Table···· 27
The 7×□ Multiplication Table···· 29
The 8×□ Multiplication Table···· 31
The 9×□ Multiplication Table···· 33
The 1×□ Multiplication Table···· 35
Which Calculation··············· 36

12 Multiplication(4)·········· 39
Multiplication Table··············· 39
Multiplication Game··············· 42
Making Multiplication Patterns···· 45
Let's Look for Examples of Multiplication in Town··· 46

Numbers Larger than 1000··· 55

Addition and Subtraction(2)···71

For Knowing it, we should review what we learned until last 15 lessons!

Shapes

The representation/model for the base ten place value system: Count by 10

How children shift new way of counting?

Review **Which number is really base ten place value system**

Terminology!

Idea of Set and Unit,

Number, Arabic digit, Numeral, Concrete object, Semi-concrete object, Denominated number, Base ten system, Base ten place value system

7 Numbers Larger than 10

Numbers up to 20

1 How many dragonflies are there?

2 How many are there? Grouping for 10

10 and 3 makes...

eggs

2 sets of 10

Count by 10: 10 is a unit for counting

dragonflies

9 1 0   1 3   2 0

When we should teach count by two and five, here?
Why we need the line of number, here?
Count by 2, 5, and 10 through Grouping   Review

Why 28 instead of 21?
Where and when did children learn it, already?

3 Let's count.

strawberries

chocolates

6 Where should you put these cards?

Mathematical Ideas for: Set, Unit, Compare

17 12 20 18   14

4 Write the answers in the □
① 10 and 2 makes
② 10 and 8 makes
③ 10 and □ makes 13

10 and 2 makes...

① 15 ⬜ 17 18 ⬜ 20
② ⬜ 19 18 17 ⬜ 15

7 Up to what number did the frog and rabbit go?

5 Which number is larger?
① 20 or 18 ?
② 13 or 15 ?   9 or 11 ?

Numbers Larger than 20

1 How many acorns did they pick up?

Toshio   Hinoko

The number of acorns Toshio picked up.
The number of acorns Hinoko picked up.

boxes with 10 acorns and more acorns.   boxes with 10 acorns.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Review

Look for Numbers

1 Let's look for pairs around us.

socks   gloves

2 Let's look for groups of 3 and 4.

Groups of 3

wheels of a tricycle   a traffic light

chopsticks   holes in outlets

Groups of 4

legs of a chair   legs of a dog   wheels of a car

3 Let's count things in school.

gas burners   glasses

first grade teachers   There are □ steps on the school staircase.

For what?

A Part of Number Sense:

Number can be seen under the various units by using attribute of concrete object on situations.

It is a preparation of multiplication.

Use the attribute for one things to see any number as a unit

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Slide 1

- See the situation for multiplication
- Multiplicative Situation A, B, C, …
- Pictures & Diagrams
- by using dishes & boxes
- A Set of Group and a number of set
- Properties of Rows
- Rows of Multiplication are binary operations.
- Tape Diagram & Times
- Proceduralization
- Repeated addition
- A Set of Group x a number of set

Multiplication(3)

Slide 2

Write down what you have learned from the multiplication table. pages 13~20

Multiplicand \ Multiplier	1	2	3	4	5	6	7	8	9
row of 2	2		4						
row of 3	3				15				
row of 4	4						28		
row of 5	5		15						

① Change the order of the row numbers and make another multiplication table.

Multiplicand \ Multiplier	1	2	3	4	5	6	7	8	9
row of 3	3								
row of 4	4								
row of 2	2								
row of 5	5								

② Change the order of the row numbers yourself and try.

Conjecturing, Justifying and Proving

Slide 3

(same diagram as Slide 1)

- Conjecturing, Justifying and Proving
- Distribution in action

Slide 4

What is the next Lesson? Can you imagine?

2nd grade vol.2 Contents

How many meanings?

For Learning How to Extend the Rows

Slide 5

References

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MATHEMATICAL THINKING — How to Develop it in the Classroom

Teaching Multiplication with Lesson Study

Slide 6

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