

 Free Program for SEAMEO School Network
 from the University of Tsukuba, Affiliate Member of SEAMEO
**Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking.
 How do you teach? Why?**

Lesson 17: How to introduce column multiplication

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Adopting a 21st Century Curriculum



Revitalizing Teacher Education

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values: Generality and Expandability Reasonableness and Harmony Usefulness and Efficient Simpler and Easier Beautifulness	Mathematical Attitude attempting to: See and think mathematically Pose question and develop explanation such as why and when Generalize and extend Appreciate others' idea and change representation to conceptualize	Habits of mind for Citizen to live: Reasonably and critically with respecting and appreciating others Autonomously Creatively and innovatively in harmony Judiciously using tools such as ICT Empowerly in imagining the future through lifelong learning
Mathematical Thinking and Processes		
Mathematical Ideas for: Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.	Mathematical Thinking: Generalization and Specialization Extension and Integration Inductive, Analogical and Deductive reasoning Abstracting, Concretizing and Embodiment Objectifying by representing and symbolizing Relational and Functional thinking Thinking forward and backward	Mathematical Activities for: Problem Solving Exploration and Inquiry Mathematical Modeling Conjecturing, Justifying and Proving Conceptualization Proceduralization and Representation and Sharing
Content		
• Numbers & Operations • Quantity & Measurement	• Extension of Number and	• Number & Algebra

We are preparing the IP textbook by using terminology to explain to students with every content of learning and sequence of content for knowing the way to develop children who learn and think by and for themselves through the preparation of future learning.

Curriculum Standards: SEABES-CCRLS (by SEAMEO-RECSAM (Mango, Ahmad, Isoda, 2017))

Appreciations

Reflection

Acquisition

HOTS is Math. T. Those terminology distinguish tasks and explain task sequence for the preparation of future learning.

Review Using what you already knew on the past 16 lessons!

TOPIC 1: INTRODUCTION	L1: Introductory discussion to develop mathematical thinking		
TOPIC 2: NUMBERS	L2: How to introduce number	L3: What is number	
TOPIC 3: ADDITION AND SUBTRACTION	L4: How to introduce addition	L5: What is addition	
TOPIC 3: ADDITION AND SUBTRACTION	L6: How to introduce subtraction	L7: What is subtraction	
TOPIC 4: EXTEND NUMBER WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10	L9: How to extend addition	L10: How to extend subtraction
	L11: How to extend number to more than 100	L12: How to introduce column addition and subtraction	L13: How to extend column addition and subtraction
	L14: How to introduce multiplication	L15: How to develop multiplication table	
TOPIC 5: MULTIPLICATION	L16: What is the multiplication table	L17: How to introduce column multiplication	

Participants need to consider what's new.

Participants of this program are able to imagine the ways of learning from the past process of learning.

What is column multiplication?

USA:

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$$

Thailand:

$$\begin{array}{r} 23 \times \\ \times 7 \\ \hline 161 \end{array}$$

23 x 7: Multiplier 23, Multiplicand 7.
It is (20+3)x7=20x7+3x7=2x7x10+3x7
However,
↑ indicates 7 x 3: Multiplier 7, Multiplicand 3.
↑ indicates 7 x 20: Multiplier 7, Multiplicand 20.

Chile:

$$\begin{array}{r} 2 \\ 23 \times 7 \\ \hline 161 \end{array}$$


Netherlands (Freudenthal Institute):

$$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$$

$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$

$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$

$\begin{array}{r} 23 \\ \times 7 \\ \hline 161 \end{array}$

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See you next class! How to invent column multiplication?

Extension of A to B

Special Situation A

Meaning A: 2+2+2; two, three times

Meaning B: ?

Procedure A: Row 2 on Multiplication Table

Enrichment and fluency of A

Enrichment and fluency of B


Procedure B: ?

Enrichment and fluency of B

Fig. 1.1 Simplified extension and integration process of multiplication (*mul.*) in the task sequence detailed in the textbooks, which is explained by conceptual and procedural knowledge (Isoda, 2009)

Japan:

$$\begin{array}{r} 23 \quad 23 \quad 23 \\ \times 7 \quad \times 7 \quad \times 7 \\ \hline 140 \quad 21 \quad 21 \\ \hline 161 \quad 161 \quad 161 \end{array}$$



Preparation of Column multiplication

2. Let's find the various ways for the equation that has the same answer to 7x8.

① What numbers go in the below

7x8=□ □

7x□=□ □

□x8=□ □

② How much larger is the answer to 7x8 than the answer to 7x5?

7x8=□x5+□ □

③ How much smaller is the answer to 7x8 than the answer to 7x7?

7x8=□x7-□ □

④ Think about 7x8 by separating the multiplicand and the multiplier.

Let's separate the multiplicand into two parts.

$$7 \times 8 = 7 \times (5 + 3) = 7 \times 5 + 7 \times 3$$

Let's separate the multiplier into two parts.

$$7 \times 8 = (7 + 1) \times 8 = 7 \times 8 + 1 \times 8$$

In multiplication, we can calculate by separating the multiplier and multiplying and adding the answers.

Write the correct numbers in the below.

① 8x1=□ □

② 8x4 is larger than 4x5 by □ □

③ 8x3 is smaller than 8x7 by □ □

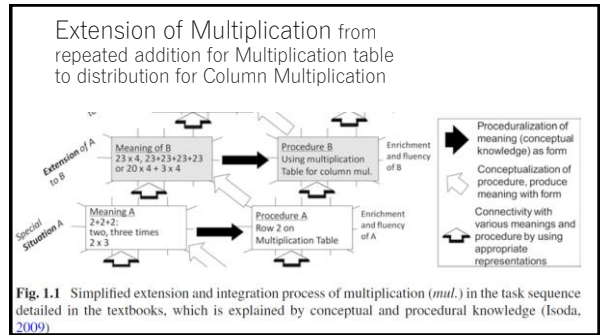
Commutativity

In multiplication, answers are the same even if the multiplicand and the multiplier are reversed.

□ is called "equal". This symbol means that the left side and right side are equal in size.

Distribution

Think about how to calculate



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