Free Program for SEAMEO School Network
from the University of Tsukuba, Affiliate Member of SEAMEO

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
How do you teach? Why?

Lesson 19: How to extend division with remainder

Isoda, Masami. Prof., Faculty of Human Sciences
Director of CRICED, The University of Tsukuba, Japan
With collaborations of
Nguyen Chi Thanh and Teh Kim Hong

Adopting a 21st Century Curriculum — Revitalizing Teacher Education

---

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values:
Generality and Expandability
Reasonableness and Harmony
Usefulness and Efficient
Simpler and Easier
Beautifulness

Mathematical Attitude attempting to:
See and think mathematically
Pose question and develop explanation such as why and when
Generalize and extend
Appreciate others' idea and change representation to conceptualize

Habits of mind for Citizen to live:
Reasonably and critically with respecting and appreciating others
Autonomously Creatively and innovatively in harmony
Judiciously using tools such as ICT
Empowerly in imagining the future through lifelong learning

Appreciations

Mathematical Thinking and Processes

Mathematical Ideas for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization and Proceduralization
Representation and Sharing

Reflection

Content

- Numbers & Operations
- Quantity & Measurement
- Extension of Number and
- Number & Algebra

Acquisition

We are reading the JP textbook by using terminology to explain to distinguish every content of learning and sequence of content for knowing the way to develop children who learn and think by and for themselves through the preparation of future learning.

Curriculum Standards: SEABES-CCRLS(by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017)

HOTS is Math. T.

Those terminology distinguish tasks and explain task sequence for **the preparation of future learning.**

MATHEMATICAL THINKING

Review

Review — Using what you already knew on the past 18 lessons!

TOPIC 1: INTRODUCTION	L1: Introductive discussion to develop mathematical thinking		
TOPIC 2: NUMBERS	L2: How to introduce number	L3: What is n	
TOPIC 3: ADDITION AND SUBSTRACTION	L4: How to Introduce addition	L5: What is a	
	L6: How to introduce subtraction	L7: What is s	
TOPIC 4: EXTEND NUMBER TO 100 WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10	L9: How to extend addition	
	L11: How to extend number to more than 100	L12: How to introduce column addition	L13: How to introduce column subtraction
TOPIC 5: MULTIPLICATION	L14: How to introduce multiplication	L15: How to develop multiplication table	
	L16: What is the multiplication table	L17: How to introduce column multiplication	
TOPIC 6: DIVISION	L18: How to introduce division	L19: How to extend division with remainder	
TOPIC 7: REFLECTIVE DISCUSSION	L20: Panel-Reflective discussion for summary		

Participants of this program ar able to imagine the ways of learning from th past process of learning.

Participants nee to consider wha new.

What is division with remainder?

Q1. **Which situation do you use to teach remainder?:**

Partitive Division situation

12 candies distribute 4 children equally. How many candies each child will receive.

12 ÷ 4 = 3, Ans. 3 candies for each child: 4 x 3 =12 in English
(3 x 4 =12 in Japan)

Quotative Division situation

There is 12 candies and each child receive 4 candies equally. How many children will receive it.

12 ÷ 4 = 3, Ans. 3 children: 3 x 4 =12 in English
(4 x 3 =12 in Japan)

Q2. Please answer (- 5) ÷ 3

Have you ever discuss the remainder?

(- 5) ÷ 3 ? … Remainder is -2 or +1?

□	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
Answer for □÷3	3	2	2	2	1	1	1	0	0	0	0	0	-1	-1	-1	-2
Remainder for □÷3	0	2	1	0	2	1	0	2	1	0	-1	-2	0	-1	-2	0

Less

□	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
Answer for □÷3	3	2	2	2	1	1	1	0	0	0	?	?	?	?	?	?
Remainder for □÷3	0	2	1	0	2	1	0	2	1	0	?	?	?	?	?	?

Q2. Do you teach Less?

Remainder

Permanence of Form

A=B x Q +r (r≧0)

□	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6
Answer for □÷3	3	2	2	2	1	1	1	0	0	0	-1	-1	-1	-2	-2	-2
Remainder for □÷3	0	2	1	0	2	1	0	2	1	0	2	1	0	2	1	0

Given an integer n > 1, called a modulus, two integers are said to be congruent modulo n, if n is a divisor of their difference (i.e., if there is an integer k such that a − b = kn).
Congruence modulo n is denoted: a ≡ b (mod n)
For example: 1 ≡ -5 (mod 3), i. e. 1 – (-5) = 2 x 3

Before Division with Reminder: Why we teach it?

Partitive vs Quotative situations
Integrate as one division operation

Permanence of Form

Proceduralization by
Compare division expressions in order

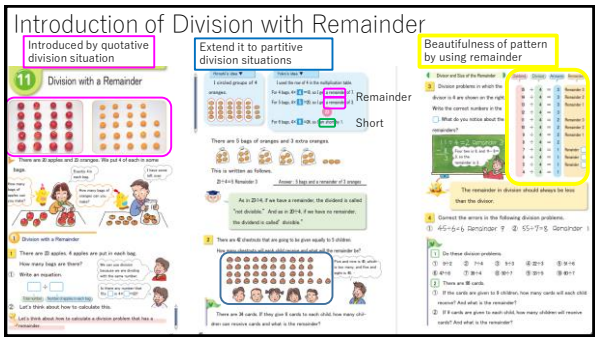
Relationship between division and multiplication

See division as multiplication

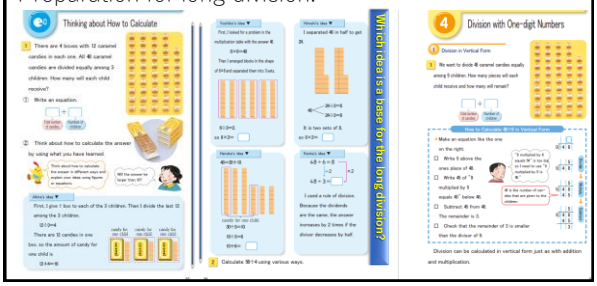
These are necessary to find the quotient.

Equivalent class of division

Introduction of Division with Remainder



Think about how to calculate: Preparation for long division.



References

References

Masami Isoda, Raimundo Olfos edited (2020). *Teaching Multiplication with Lesson Study: Japanese and Ibero-American Theories for International Mathematics Education.* Cham, Switzerland: Springer. (Open Access)

Masami Isoda, Aki Murata (2020). *Study with your friends: Mathematics for Elementary School (12 vols.).* Tokyo, Japan: Gakko Tosho.

Masami Isoda, Aki Murata, Aida Yap (2015). *Study with your friends: Mathematics for Elementary School (12 vols.).* Tokyo, Japan: Gakko Tosho.

Masami Isoda, David Tall (2019). *Mathematics for Junior High School (3 vols.).* Tokyo, Japan: Gakko Tosho.

Dominador Dizon Mangao, Nur Jahan Ahmad, Masami Isoda edited (2017).*SEAMEO basic education standards (SEA-BES): Common core regional learning standards (CCRLS) in mathematics and science.* Penang, Malaysia: SEAMEO-RECSAM. http://www.recsam.edu.my/sub_sea-bes/images/docs/SEAMEO-ASEAN-Curriculum-SEABES-CCRLS-Standards.pdf

Maitree Inprasitha, Masami Isoda, Patsy Wang Iverson, Ban Har Yap (2015). *Lesson Study: Challenges in Mathematics Education.* New Jersey, USA: World Scientific

Masami Isoda, Shigeo Katagiri (2012). *Mathematical Thinking: How to develop it in the classroom.* New Jersey, USA: World Scientific

TEH Kim Hong, ISODA Masami, GAN Teck Hock (in printing). *Mathematics Challenges for Classroom Practices at the Lower Primary Level.* Penang, Malaysia: SEAMEO-RECSAM

ISODA Masami, TEH Kim Hong, GAN Teck Hock (in printing). *Mathematics Challenges for Classroom Practices at the Upper Primary Level.* Penang, Malaysia: SEAMEO-RECSAM

GAN Teck Hock, ISODA Masami, TEH Kim Hong (2021). *Mathematics Challenges for Classroom Practices at the Lower Secondary Level.* Penang, Malaysia: SEAMEO-RECSAM

Hosomizu, Y, translated by Gould, P, Isoda, M., Foo, C. (2010). *Red dragonfly mathematics challenge.* Department of Education, New South Wealth. https://schoolsequella.det.nsw.edu.au/file/20a29ac1-c6f3-4ca3-84b1-2d8488a4cbcd/1/reddragonfly.zip/index.html

Hosomizu, Y, translated by Gould, P, Isoda, M., Foo, C. (2011). *Companion: Mathematics Challenges.* Department of Education, New South Wealth. https://www.mansw.nsw.edu.au/shop/pre-k-8-books/dragonfly-companion