筑波大学
University of Tsukuba

Free Program for SEAMEO School Network
from the University of Tsukuba, Affiliate Member of SEAMEO (lesson 20)

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking: How do you teach? Why?

Isoda, Masami, Faculty of Human Sciences
Director of CRICED, The University of Tsukuba, Japan
With collaborations of
Maitree Inprasitha, Marcela Santillan, Nguyen Chi Thanh,
Aida Yap, Erlina Ronda, TEH Kim Hong, Wahid Yunianto
Raimundo Olfos and Soledad Estrella

Adopting a 21st Century Curriculum — Revitalising Teacher Education

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Review — Do you remember my questions? What Terminology did we used for?

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values:
Generality and Expandability
Reasonableness and Harmony
Usefulness and Efficient
Simpler and Easier
Beautifulness

Mathematical Attitude attempting to:
See and think mathematically
Pose question and develop explanation such as why and when
Generalize and extend
Appreciate others' idea and change representation to conceptualize

Habits of mind for Citizen to live:
Reasonably and critically with respecting and appreciating others
Autonomously Creatively and innovatively in harmony
Judiciously using tools such as ICT
Empowerly in imagining the future through lifelong learning

Appreciations

Mathematical Thinking and Processes

Mathematical Ideas for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization and Proceduralization
Representation and Sharing

Reflection

Content

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations
- Extension of Number and Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs
- Number & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability

Acquisition

Curriculum Standards: SEABES-CCRLS(by SEAMEO-RECSAM (Mangao, Ahmad, Isoda;

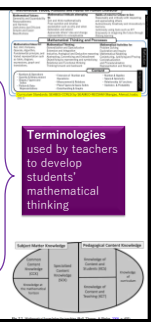
Mathematical Thinking

Review

- General Objective
  - Principles for Curriculum and Textbook Writers
- Specified in the Textbook
  - Embedded as task sequence by Authors
  - Revised by teachers and students through using it
- Planed process by Teacher
  - Teachers' thinking embedded into process by given tasks and sequence
  - Considered how it will be possible to appear from students
- Actual Thinking by students in classroom
  - Students thinking in the classroom which explained by them
  - Teachers observe and assess them
- Discussed by teachers after observation of Class
- Mathematical Thinking as Data based on Researchers' Framework for Journals on Social Scientific Studies

Terminologies for Curriculum Designers

Terminologies used by teachers to develop students' mathematical thinking



How learned terminology is useful for?

We discussed the development of mathematical thinking/values/attitudes by using various terminology instead of using higher order thinking, metacognition and so on.
Why?

- To explain objective of teaching which includes mathematical thinking, values and attitudes.
- To explain conceptual differences
- To explain task sequence
- To explain repetitions for learning from the process which includes ways of thinking and learning
- To explain what students already learned
- To explain the preparation of future learning such as the usefulness of today's learning content/representations/models for future learning
- To explain when we re-conceptualize and proceduralize: what and when.
- To shift from concrete objects to appropriate representations/models which children are able to draw for themselves.

Japanese textbooks have been developed through lesson study with terminology.
It's trying to

- Developing students who learn mathematics for and by themselves.
- Focusing on sense making through developing students who make sense for themselves by using what they learned which including various representations and translations, instead of making sense by teachers.
- Internalization of the process of learning by the repetition of the similar learning processes for enabling students to reflect on and appreciate what they already learned.

Questions: For curriculum/designing/assessments theory of Mathematics Education

- How terminology on this program is useful?
- How Japanese textbooks and ideas are useful on your countries?

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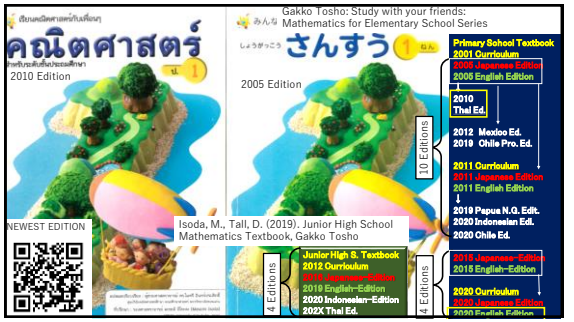

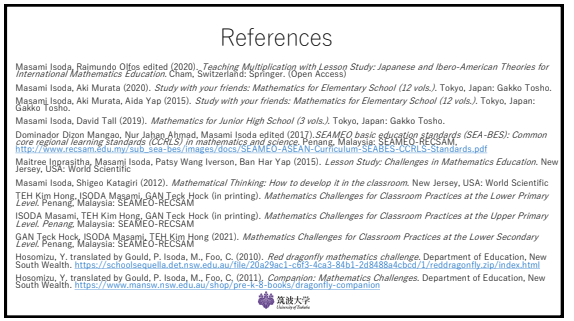

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