


**筑波大学**  
 University of Tsukuba

Free Program for SEAMEO School Network  
 from the University of Tsukuba, Affiliate Member of SEAMEO

**Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:  
 How do you teach? Why?**

**Lesson 3: What is Number?**

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Adopting a 21<sup>st</sup> Century Curriculum

Revitalizing Teacher Education

**Numerals In English as a language**  
**How about in your language?**

**Cardinal Number (set number):**  
 One, two, three, ..., nine, ten, *eleven, twelve, ...*,  
 nineteen, **twenty, twenty-one (20+1), ...**, thirty,  
 thirty-one, ... **not** exact base ten system?

**Ordinal Number:**  
 First, Second, Third, Forth, ..., ninth, tenth, eleventh,  
 ..., nineteenth, twentieth, twenty first, ...

**Multiple:**  
 Once (one time), twice (two times),  
 thrice (three times), ...  
 Half, double (twofold), triple (threefold), ...

In Japanese

Cardinal Number (set number):  
 Ichi, ni, san, ..., jyu (ten),  
 jyu (ten)-ichi (one) (eleven), jyu-ni  
 (ten+two) ... **exact base ten  
 system**

Historically Hihotsu, Futatsu, Mittsu, ..., To (ten)


Ordinal Number : C.N. + 番 (ban)  
 Ichi-ban, ni-ban, san-ban, ...

Multiple: C.N. + 倍 (bai)  
 Ichi-bai, ni-bai, san-bai, ...

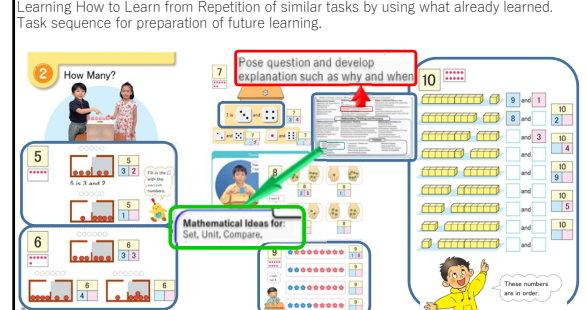
Number shows *magnitude* (size) but  
 not show the quantity.

*Quantity* (amount/measure) is  
 represented by number *with  
 denomination*.

\*3 apples\* are **denominated number**:  
**an apple is the unit for counting  
 (measurement)**, 3 is a number.



Learning How to Learn from Repetition of similar tasks by using what already learned.  
 Task sequence for preparation of future learning.



**Mathematical Values, Attitudes and Habits for Human Character**

**Mathematical Values:**  
 Generality and Expandability  
 Reasonableness  
 and Harmony  
 Usefulness and Efficient  
 Simpler and Easier  
 Beautifulness

**Mathematical Attitude attempting to:**  
 See and think mathematically  
 Pose question and develop  
 explanation such as why and when  
 Generalize and extend  
 Appreciate others' idea and change  
 representation to conceptualize

**Habits of mind for Citizen to live:**  
 Reasonably and critically with respecting  
 and appreciating others  
 Autonomously Creatively and innovatively in  
 harmony  
 Judiciously using tools such as ICT  
 Empowerly in imagining the future through  
 lifelong learning

**Mathematical Thinking and Processes**

**Mathematical Ideas for:**  
 Set, Unit, Compare,  
 Operate, Algorithm,  
 Fundamental principle, and  
 Varied representation such  
 as table, diagram,  
 expressions, graph and  
 translations.

**Mathematical Thinking:**  
 Generalization and Specialization  
 Extension and Inquiry  
 Inductive, Analogical and Deductive reasoning  
 Abstracting, Concretizing and Embodiment  
 Objectifying by representing and symbolizing  
 Relational and Functional thinking  
 Thinking forward and backward

**Mathematical Activities for:**  
 Problem Solving  
 Exploration and Inquiry  
 Mathematical Modeling  
 Conjecturing, Justifying and Proving  
 Conceptualization  
 and Proceduralization  
 Representation and Sharing

**Content**

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations

- Extension of Number and Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs

- Number & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability

Curriculum Standards: SEABES-CCRLS (by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017))

**What is the number?**  
**We usually teach:**


- Existence and necessity
- Order/Larger or Smaller/ Greater or Less
- Operations

How do you teach?  
 Make sense?  
 Acquisition of proficiency?

For what?  
 Number sense?

In Japan:  
 Make sense (understand meaning)  
**Think about how to  
 calculate/operate/find the  
 easier way to answer**  
 Acquisition of proficiency

**Try to teach how to extend  
 the number**



**Counting unit is not necessary to be a one.**

Count by one, Count on

Which one, do you teach?

- Count by two
- Count by three
- Count by four
- Count by five

Why?

Contents

**Numbers**

1 Numbers up to 10 ..... 8

2 How Many? ..... 26


3 Numerical Order ..... 32

4 Addition ..... 34


5 Subtraction ..... 46

**Shapes**

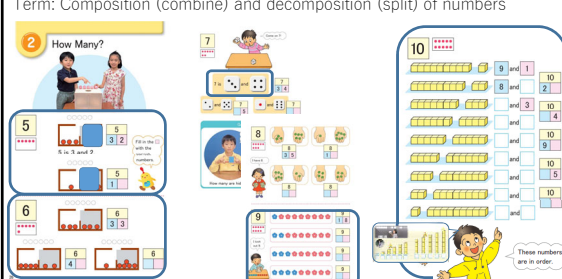
6 Shapes ..... 60




**Set (cardinal) Number VS Ordinal Number, only that?**



**How many?**  
 Term: Composition (combine) and decomposition (split) of numbers



TOPIC 1: INTRODUCTION	L1: Introductory discussion to develop mathematical thinking (17/03/21)	
TOPIC 2: NUMBERS	L2: How to introduce number (20/03/21)	L3: What is number (24/03/21)
TOPIC 3: ADDITION AND SUBTRACTION	L4: How to introduce addition (27/03/21)	L5: What is addition (31/03/21)
	L6: How to introduce subtraction (03/04/21)	L7: What is subtraction (07/04/21)
TOPIC 4: EXTEND NUMBER TO 100 WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10 (10/04/21)	L9: How to extend addition (14/04/21)
	L11: How to extend number to more than 100 (13/04/21)	L12: How to introduce column addition (24/04/21)
		L13: How to introduce column subtraction (28/04/21)
TOPIC 5: MULTIPLICATION	L14: How to introduce multiplication (01/05/21)	L15: How to develop multiplication table (05/05/21)
	L16: What is the multiplication table (08/05/21)	L17: How to introduce column multiplication (12/05/21)
TOPIC 6: DIVISION	L18: How to introduce division (15/05/21)	L19: How to extend division with remainder (19/05/21)
TOPIC 7: REFLECTIVE DISCUSSION	L20: Panel-Reflective discussion for summary (22/05/21)	



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