

 Free Program for SEAMEO School Network
 from the University of Tsukuba, Affiliate Member of SEAMEO
**Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
 How do you teach? Why?**
Lesson 5: What is addition?
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 With collaborations of
 Nguyen Chi Thanh, TEH Kim Hong and Wahid Yuniarto
 Adopting a 21st Century Curriculum  Revitalizing Teacher Education

What didn't you learned yet? Review

Number

- Existence and necessity
- Order
- Larger or Smaller
- Greater or Less
- Operations

In Japan:
 Make sense (understand meaning)
 Think about how to calculate/operate/find the easier way to answer
 Acquisition of proficiency
Try to teach how to extend the number

Contents

Let's study following your interest.

Numbers

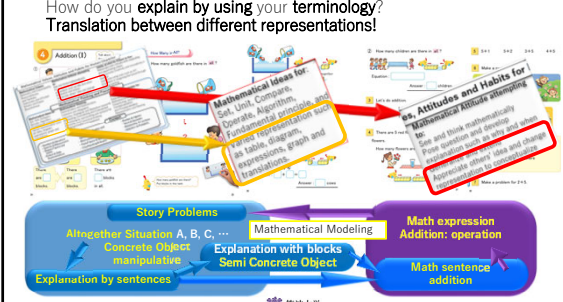
- 1 Numbers up to 10 8
- 2 How Many? 26
- 3 Numerical Order 32
- 4 Addition 34
- 5 Subtraction 46

Shapes

- 6 Shapes 60

Story Problems: Altogether Situation A, B, C, ...
 Mathematical Modeling: Explanation with blocks, Semi Concrete Object
 Math expression: Addition, operation
 Math sentence: addition

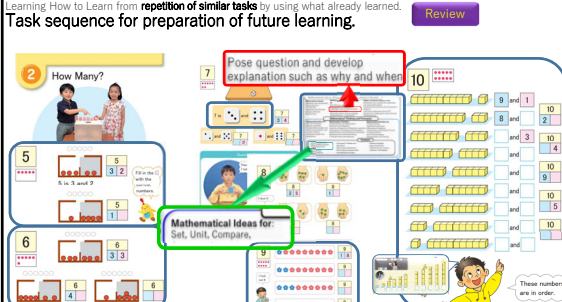
How do you explain by using your terminology?
 Translation between different representations!



Explanation of the task sequence

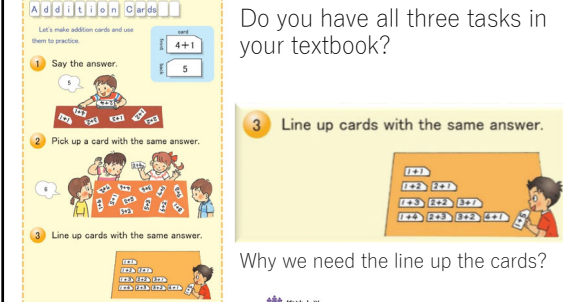
Review	L1: Introductory discussion to develop mathematical thinking (17/03/21)	
TOPIC 1: INTRODUCTION	L2: How to introduce number (20/03/21)	L3: What is number (24/03/21)
TOPIC 2: NUMBERS	L4: How to introduce addition (17/03/21)	L5: What is addition (31/03/21)
TOPIC 3: ADDITION AND SUBTRACTION	L6: How to introduce subtraction (03/04/21)	L7: What is subtraction (07/04/21)
TOPIC 4: EXTEND NUMBER TO 100 WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10 (10/04/21)	L9: How to extend addition (14/04/21)
	L10: How to extend subtraction (17/04/21)	L11: How to extend number to more than 100 (21/04/21)
TOPIC 5: MULTIPLICATION	L12: How to introduce column addition (24/04/21)	L13: How to introduce column subtraction (28/04/21)
	L14: How to introduce multiplication (01/05/21)	L15: How to develop multiplication table (05/05/21)
TOPIC 6: DIVISION	L16: What is the multiplication table (08/05/21)	L17: How to introduce column multiplication (12/05/21)
	L18: How to introduce division (15/05/21)	L19: How to extend division with remainder (19/05/21)
TOPIC 7: REFLECTIVE DISCUSSION	L20: Panel-Reflective discussion for summary (22/05/21)	

Learning How to Learn from repetition of similar tasks by using what already learned.
 Task sequence for preparation of future learning. Review



Mathematical Ideas for Set, Unit, Compare.

Do you have all three tasks in your textbook?



Why we need the line up the cards?

Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values: Generality and Expandability, Reasonableness and Harmony, Usefulness and Efficient, Simpler and Easier, Beautifulness.

Mathematical Attitude attempting to: See and think mathematically, Pose question and develop explanation such as why and when, Generalize and extend, Appreciate others' idea and change representation to conceptualize.

Habits of mind for Citizen to live: Reasonably and critically with respecting and appreciating others, Autonomously Creatively and innovatively in harmony, Judiciously using tools such as ICT, Empowerly in imagining the future through lifelong learning.

Mathematical Thinking and Processes

Mathematical Thinking: Generalization and Specialization, Extension and Integration, Inductive, Analogical and Deductive reasoning, Abstracting, Concretizing and Embodiment, Objectifying by representing and symbolizing, Relational and Functional thinking, Thinking forward and backward.

Mathematical Activities for: Problem Solving, Exploration and Inquiry, Mathematical Modeling, Conjecturing, Justifying and Proving, Conceptualization and Proceduralization, Representation and Sharing.

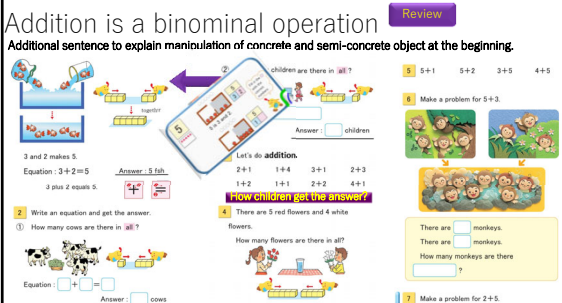
Content

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations
- Extension of Number and Operations
- Space & Geometry
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs
- Number & Algebra
- Relationship & Functions
- Statistics & Probability

Curriculum Standards: SEABES-CRRLS (by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017))

Addition is a binomial operation Review

Additional sentence to explain manipulation of concrete and semi-concrete object at the beginning.



Translation from addition expression to story problem

What is addition?

On situations: It is the object for addition.


- Altogether, Increase, Different Sets: the ways of translations and conceptualization
- It can be answered by counting.
- If counted, it is not the answer of addition but the answer of counting.

On the world of number: not denominated number/not quantity

- Number(Existence/ necessity, set/cardinal), Order, Operation
- Addition is a binomial operation;** It cannot be answered by counting but answered by memorized calculation such as composite numbers.
- Operation put the **structure** into Number Set!
 Equivalence class:
 $4 - (-2) = 2$, $4 - (-1 + 3) = 4 - (-3 + 1)$
 $(2 + 2) - (1 + 3) = (1 + 3) - (3 + 1)$?
 Commutativity, Associativity
- Addition is the **inverse operation** of subtraction.

Let's see the class of grade 1-second month by Mr. Takao Seiyama, Elementary School at U.T.

- Operation put the **structure** into Number Set!
Equivalence class:
 $4 \sim (2+2)$, $4 \sim (1+3)$, $4 \sim (3+1)$
 $(2+2) \sim (1+3)$? $(1+3) \sim (3+1)$?
Commutativity, Associativity
- Why children can answer the **reason why**?
From the video, can you find what children learned before?
What task sequence did you imagine to develop children who are able to answer the reason why?
What activity after the video you will imagine?



Why we teach number on this sequence?

Number

- Existence and necessity
- Order
- Larger or Smaller
- Greater or Less
- Operations

In Japan: Make sense (understand meaning)
Think about how to calculate/operate/find the easier way to answer
Acquisition of proficiency

Try to teach how to extend the number

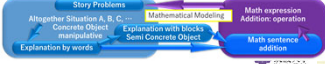
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6 Shapes	60
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References





Let's display the cards of the addition!

SEIYAMA, Takao
First grade, 7 years old

The Attached Elementary School
University of Tsukuba

Let's see the class of grade 1, second month by Mr. Takao Seiyama, Elementary School at U.T.

From the video, can you find what children learned before?

What task sequence did you imagine to develop children who are able to answer the reason why?

What activity after the video you will imagine?

References

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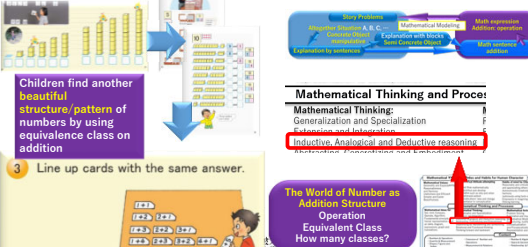
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GAN Teck Hock, ISODA Masami, TEH Kim Hong (20aug21). *Mathematics Challenges for Classroom Practices at the Lower Secondary Level*. Penang, Malaysia: SEAMEO-RECSAM



Children could explain the reason why because...
Can you imagine the ways how children are going to learn subtraction?



Children find another beautiful structure/pattern of numbers by using equivalence class on addition

Line up cards with the same answer.

The World of Number as Addition Structure
Operation
Equivalent Class
How many classes?

Gakko Toshu: Study with your friends: Mathematics for Elementary School Series




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2020 Chile Ed.

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2019 English Edition

2020 Curriculum
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2020 Indonesian Edition
2020 Thai Ed.