筑波大学
University of Tsukuba

Free Program for SEAMEO School Network
from the University of Tsukuba, Affiliate Member of SEAMEO

Teaching Mathematics to Develop Mathematical Thinking as Higher Order Thinking:
How do you teach? Why?

Lesson 6: Introduction of Subtraction

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Adopting a 21st Century Curriculum — Revitalizing Teacher Education

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Mathematical Values, Attitudes and Habits for Human Character

Mathematical Values:
Generality and Expandability
Reasonableness
and Harmony
Usefulness and Efficient
Simpler and Easier
Beautifulness

Mathematical Attitude attempting to:
See and think mathematically
Pose question and develop explanation such as why and when
Generalize and extend
Appreciate others' idea and change representation to conceptualize

Habits of mind for Citizen to live:
Reasonably and critically with respecting and appreciating others
Autonomously Creatively and innovatively in harmony
Judiciously using tools such as ICT
Empowerly in imagining the future through lifelong learning

Appreciations

Mathematical Thinking and Processes

Mathematical Ideas for:
Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Thinking:
Generalization and Specialization
Extension and Integration
Inductive, Analogical and Deductive reasoning
Abstracting, Concretizing and Embodiment
Objectifying by representing and symbolizing
Relational and Functional thinking
Thinking forward and backward

Mathematical Activities for:
Problem Solving
Exploration and Inquiry
Mathematical Modeling
Conjecturing, Justifying and Proving
Conceptualization
and Proceduralization
Representation and Sharing

Reflection

Content

- Numbers & Operations
- Quantity & Measurement
- Shapes, Figures and Solids
- Pattern & Data Representations
- Extension of Number and Operations
- Measurement & Relations
- Plane Figures & Space Solids
- Data Handling & Graphs
- Number & Algebra
- Space & Geometry
- Relationship & Functions
- Statistics & Probability

Acquisition

Review

HOTS is Math. T.

MATHEMATICAL THINKING

Curriculum Standards: SEABES-CCRLS(by SEAMEO-RECSAM (Mangao, Ahmad, Isoda; 2017)

Addition is a binominal operation.

Review

Additional sentence to explain manipulation of concrete and semi-concrete object at the beginning.

Mathematical Ideas for: Set, Unit, Compare, Operate, Algorithm, Fundamental principle, and Varied representation such as table, diagram, expressions, graph and translations.

Mathematical Attitude attempting to: See and think mathematically Pose question and develop explanation such as why and when Generalize and extend Appreciate others' idea and change representation to conceptualize

Story Problems
Altogether Situation A, B, C, …
Concrete Object manipulative
Explanation by sentences
Mathematical Modeling
Explanation with blocks
Semi Concrete Object
Math expression
Addition: operation
Math sentence addition

How many times did children experience?

Counting VS Adding

How do you teach subtraction? Why?

Left-Situation

5 Subtraction (1)

Left / Difference v.s. Difference / Left

If children write the situation by 2+2–4 instead of 4-2–2 at the introduction of subtraction, how do you assess?

If children answer the situation 7+5 =12 by counting, what preparation is missing in your teaching?

Task sequence what students already knew.

Difference-Situation

What Is the Difference?

1 How many more dogs are there than cats?

7 is 2 more than 5.

Equation : 7− 5=

Answer : more

Subtraction of Different Sets

2 How many more pieces of cake are there than dishes?

Equation : =

Answer : more

By using what children learned! Task sequence for preparation of future

Review

Number
- Existence and necessity
- Order
  Larger or
  Smaller
  Greater or Less
- Operations

In Japan:
Make sense (understand meaning)
Think about how to calculate/operate/find the easier way to answer
Acquisition of proficiency

Try to teach how to extend the number

Contents

Numbers
- Numbers up to 10 ····· 8
- How Many? ····· 26
- Numerical Order ····· 32
- Addition(1) ····· 34
- Subtraction(1) ····· 46
- Numbers Larger than 10 ····· 64

What children already prepared?
- On content
- On Thinking and Process
- On Values and Attitudes

Shapes
- Shapes ····· 60

Story Problems
Altogether Situation A, B, C, …
Concrete Object manipulative
Explanation by words
Mathematical Modeling
Explanation with blocks
Semi Concrete Object
Math expression
Addition: operation
Math sentence addition

Task Sequence of Addition (1)
Do you remember how 0 was introduced in L2?
Getting proficiency of meaning and skill

Conceptualization and Proceduralization Representation and Sharing

Permanence of Form

Review

Using what you already knew!

TOPIC 1: INTRODUCTION	L1: Introductive discussion to develop mathematical thinking		
TOPIC 2: NUMBERS	L2: How to introduce number	L3: What is number	
TOPIC 3: ADDITION AND SUBSTRACTION	L4: How to Introduce addition	L5: What is addition	
	L6: How to introduce subtraction	L7: What is subtraction	
TOPIC 4: EXTEND NUMBER TO 100 WITH ADDITION AND SUBTRACTION USING COLUMN FORM	L8: How to extend number to more than 10	L9: How to extend addition	L10: How to extend subtraction
	L11: How to extend number to more than 100	L12: How to introduce column addition	L13: How to introduce column subtraction
TOPIC 5: MULTIPLICATION	L14: How to introduce multiplication	L15: How to develop multiplication table	
	L16: What is the multiplication table	L17: How to introduce column multiplication	
TOPIC 6: DIVISION	L18: How to introduce division	L19: How to extend division with remainder	
TOPIC 7: REFLECTIVE DISCUSSION	L20: Panel-Reflective discussion for summary		

Task sequence for preparation of future learning:

Review

Questioning to others (Attitude), Easier & Beautifulness (Value)
Composite and de-composite of numbers(content), Cardinal Number (content, idea), Compare (idea)

2 How Many?

Pose question and develop explanation such as why and when

Mathematical Ideas for: Set, Unit, Compare,

Simpler and Easier Beautifulness

Addition activity for what?

Let's make addition cards and use them to practice.

1 Say the answer.

2 Pick up a card with the same answer.

3 Line up cards with the same answer.

Children find another beautiful structure/ pattern of numbers by using equivalence class on addition

What is addition?

Review

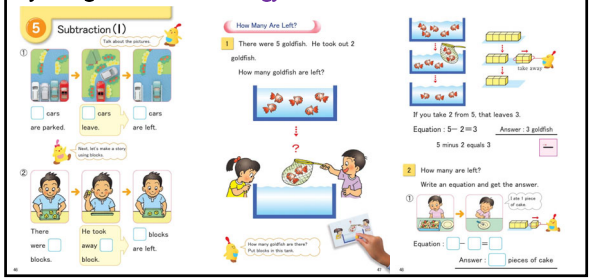
On **situations in our life**: It is the object for addition.
- Altogether, Increase, Different Sets: the ways of translations and conceptualization
- It can be answered by counting.
- **If counted, it is not the answer of addition but the same answer with counting on natural number.**
- Counting is necessary for the confirmation of sum on natural number.

On **the world of number (mathematics)**: not denominated number/ not quantity
- Number(Existence/ necessity, set/cardinal), Order, Operation
- Addition is a **binominal operation**:
  It cannot be answered by counting but answered by memorized calculation such as composite of numbers.
- Operation put the **structure** into Number Set!
  Equivalence class:
  4~(2+2), 4~(1+3), 4~(3+1)
  (2+2)~(1+3)? (1+3)~(3+1)?
  Commutativity, Associativity

Mathematical Thinking and Proce
Mathematical Thinking:
Generalization and Specialization
Inductive, Analogical and Deductive reasoning

- Addition is the **inverse operation** of subtraction.

Could you explain task sequence for subtraction (1) by using learned terminology on Lesson 1~6?



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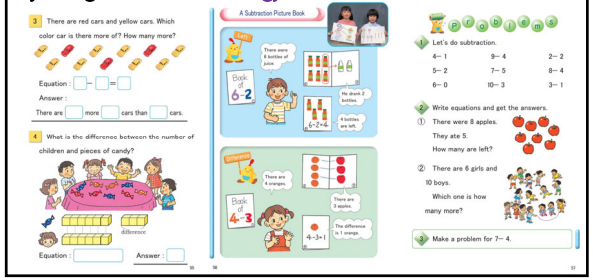

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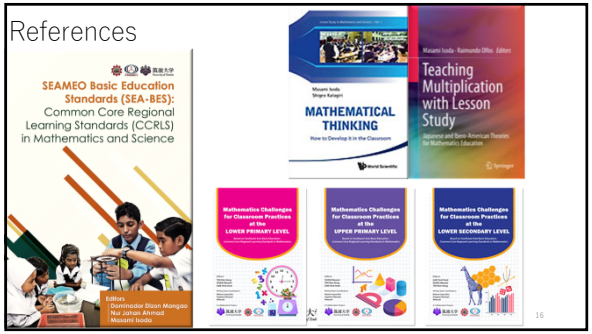

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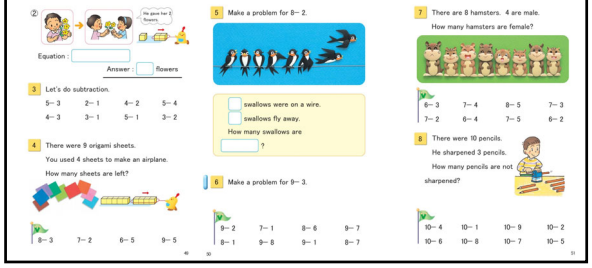

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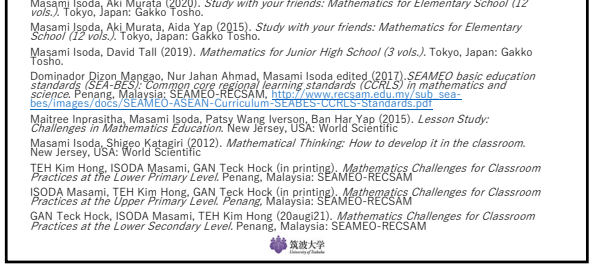

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