UNDERSTANDING NARRATIVES: A PATHWAY TOWARDS RESOLVING ISSUES AND CHALLENGES IN INTERNATIONAL COOPERATION IN MATHEMATICS EDUCATION

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This paper explores the issues and challenges in international cooperation in mathematics education by examining its history and the ecosystem of its projects. Historically, bilateral international cooperation has existed for many centuries but it was only in the last century that multilateral international cooperation has been established, with two main purposes, that of increasing international communication and serving as a platform for international aid. The ecosystem of international cooperation in mathematics education embodies a community where agents from various institutions and organisations interact at international as well as local levels. The complexity in this ecosystem has given rise to many issues which is the result of globalisation as well the possible different agendas of the agents and the institutions involved. The issues and challenges in international cooperation, briefly categorised as outer and inner issues according to its relatedness to mathematics education, are then briefly discussed. The complexity in the international cooperation ecosystem suggests that in order to initiate a pathway to resolve the issues and challenges, it is necessary that the narratives and voices of the agents at all levels be heard with the aim of creating solutions that are relevant, practical, just and sustainable to the community. The initial emergent framework towards resolving these issues and challenges comprises dispositions in the narratives towards the various aspects of knowledge and beliefs of the agents: Views of self towards authority, views of knowledge, views of knowledge production, and views of management.

Keywords: International cooperation; Mathematics education; Issues and challenges; Globalisation; Narratives; Knowledge and beliefs

INTRODUCTION

INTERNATIONAL COOPERATION AS COMMUNICATION

The formation of the *Internationale L'Enseignement Mathématique* in 1908, regorganised later as the *International Commission on Mathematical Instruction* (ICMI) in 1952, has been acknowledged as a watershed moment that formalised the beginning of international cooperation in mathematics education (Karp, 2013). Cross-border activities on mathematics education had long existed before 1908 where mathematicians and educators have engaged in the sharing of mathematics educational materials and ideas but these activities were manily bilateral and carried ou on a personal and informal basis (Karp, 2013). The formation of ICMI thus provided the thrust for formal multilateral communication among countries, beginning first with countries in Europe (Furinghetti, 2014). In examining the effects of ICMI on international cooperation, Karp (2013) and Furinghetti (2014) noted the emergence of two key purposes in international cooperation: Internationalisation and communication. As a result, international cooperation was enhanced through increased formal

communication across borders. This initial motivation for increased formal international networks was soon utilized as a gateway towards internationalising the mathematics curriculum. This would later lead to the debate as to whether the mathematics curriculum should be internationalised or localised. After World War II, more international organisations and meetings were established as platforms for international cooperation. The Royaumont Conference in 1959 was the most notable of these meetings. Though only represented by 18 countries, it led to the popularisation of the new mathematics movement across the globe (Singh & Ellerton, 2013). The formation of other international organsations followed. The International Association for the Evaluation of Educational Achievement (IEA) was formed in 1967 arising from the first meeting of psychometricians in Hamburg in 1958, and the Organisation for Economic Co-operation and Development (OECD) was formed in 1961. Both the IEA and OECD have evolved and have led the world mathematics education community to participate in two international cooperation assessments, the Trends in International Mathematics and Science Studies (TIMSS) and the Programme for International Student Assessment (PISA). Interest in TIMSS later led to more cross-border international cooperation initiatives notably the TIMSS video study (Stigler et al., 1999) which generated world-wide interest in cross-cultural studies and lesson study as areas of research on new innovative approaches to improve the teaching and learning of mathematics. What have been discussed in this section is but a sampling of the expansive cross-border initiatives in international cooperation that have taken place since the formation of ICMI. In essense, these projects were facilitated by the affordances made availabe through increased cross-border communications leading to more formal platforms for international cooperation.

INTERNATIONAL COOPERATION AS ASSISTANCE AND AID

In the period after World War II, there were several developments across the globe that led to the emergence of a different kind of international cooperation.

1. New aspirations to raise living standards emerged, leading to reconstruction, rapid development and more affluence in many advanced countries. This period is often refferred to as the *golden age of capitalism* (United Nations, 2017).
2. Many new countries gained independence. However, many of these countries which were former colonies lacked administrative structure, infrastructure and know-how for development.
3. The establishment of multilateral international organisations, such as the United Nations (UN) and its associated organisations, for exampe, the United Nations Educational, Scientific and Cultural Organisation (UNESCO) and the United Nations Development Programme (UNDP) became multilateral advocacy platforms for assistance to be provided to less developed economies.

These events were followed by augmented efforts and movements to improve education and to provide assistance to less developed countries across the globe. After the setting up of UNESCO and UNDP, many affluent nations soon established international aid agencies of their own. For example, the Norwegian Agency for Development Cooperation (Norad) was set up in 1968, and the Japan Japanese Overseas Development Assistance (ODA) which began in 1954 was later reconsituted as the Japan International Cooperation Agency (JICA) in 1974. These international agencies operated independently of each other. It was the UN and its associated agencies that provided the leadership towards providing a unifying vision for the work of international cooperation. Advocacy towards using

education as a tool to improve living standards continued to grow and eventually led to the World Conference on *Education for All* (EFA) held in Jomtien, Thailand in 1990. The Jomtien EFA declaration was founded on the premise that education is a fundamental human right for everyone (UNESCO, 2000). Ten years later, this vison was reaffirmed, formalised and adopted in the *Dakar Framework for Action* at the World Education Forum in 2000 (UNESCO, 2000). The Dakar Framework outlined several goals for the attainment of the EFA vision. Among them were the improvement of early childhood education, improving access to education for marginalized groups, improving adult literacy, achieving gender equality, and improving all aspects of the quality of education. Further, the EFA declaration envisioned that quality and excellence in education can be achieved especially in literacy, numeracy and essential life skills (UNESCO, 2000). One implication of the *Dakar Framework for Action* was the emergence of mathematics education as a key theme in international cooperation. It can be seen that the *Dakar Framework for Action* which led to the launch of the *United Nations Millennium Development Goals*, and later superseded by the *Sustainable Development Goals* (SDG) (United Nations, 2015) was to provide an internationally recognised platform for international cooperation. The backdrop for the establishment of this cooperation was a globalised world which had created, perhaps unintended, increasing inequality among the world's economies, and the awareness among institutions that international cooperation could be used as a platform to acquire economic benefits for all (Atweh et al., 2008). It is against this backdrop that more international cooperation in mathematics education were established.

MATHEMATICS EDUCATION IN INTERNATIONAL COOPERATION

The universal importance of mathematics is reflected by its inclusion in the school curriculum worldwide. Children learn mathematics as early as in pre-school and grade 1 in elementary school. Moreover, mathematical competencies are currently seen as necessary for the advancement of computational sciences and big data analysis which are essential for the economic development of future societies. Often referred to as the queen of the sciences, the utility of mathematics in various fields especially those related to science and technology is indisputable. The goals of education, however, go beyond just preparing citizens for economic development. Ultimately education must aim for the creation of a well-being and sustainable society (United Nations, 2015; OECD, 2018). In reality, while mathematics education has been successful in empowering and liberating society, it has sometimes brought about undesired consequences that are detrimental to the environment and humanity (Skovsmose, 2007, 2016). While mathematics has often been viewed as the least value-laden of school subjects and thus the least susceptible of being misused as a means for propaganda, Bishop (1990) challenged this myth by purporting that the mathematics brought by the authorities for use in the colonies was embedded with hidden elements of western culture, and was in fact a tool for the imposition of western culture. Moreover, as recent history has showed mathematics has been misused, for example, in wars and to maximize profits in planned computational experiments (Skovmose, 2016). Similar arguments have been raised in the field of international cooperation in mathematics education which includes various concerns about its effectiveness and outcomes (Atweh & Keitel, 2007). The most commonly highlighted concerns are those of equity and social fairness. To address these concerns of the diverse community who participate in international cooperation projects in a holistic manner, there is a need for equitable and authentic solutions to be instituted.

COMPLEXITY OF THE INTERNATIONAL COOPERATION ECOSYSTEM

The planning and implementation of international cooperation projects often require the need to mobilise a diversity of organisations and their agents at various levels. These organisations comprise international institutions that fund and coordinate cross-boundary projects which includes those linked to the United Nations or other international agencies established by more developed and affluent countries. At the national level, the services of various organisations such as the education ministries, teacher colleges, curriculum development centres, state and regional educations offices, and local authorities are often sought to provide expertise and to manage the implementation of the projects. Frequently, local and foreign universities also participate by offering their knowledge and expertise. At every level, the institutions and their agents contribute a myriad of purposes and agenda into the design, planning, implementation and assessment of the projects. Moreover, the agents' personal ideas, knowledge and agenda become included and embedded as part of the narratives in the ecosystem.

A complex ecosystem such as this, thus encompasses a variety of knowledge in its implementation, such as scholarly knowledge, knowledge of educational systems as well as knowledge of practice (Artigue, 2018), which do not always work in synchronicity with one another. This milieu that constitutes the knowledge developed in this ecosystem is dynamic. Its dynamic nature is not unexpected as the development of this knowledge involves agents coming from various institutions with diverse specialisations and training. Further, as this knowledge passes through the different levels of implementation, a variety of its meanings and interpretations may emerge and not always leading to a convergence of ideas. From one level to the next, there is a transposition and modification of the knowledge and ideas (Chevellard, 2019). Even mathematical knowledge, which is supposedly objective, undergoes this transposition of knowledge across levels, resulting in various forms and formats (Chevellard, 2019). As Tall (2013) observed, mathematics knowledge has transposed into three forms by the time it has crossed various levels from university to school, namely the axiomatic, theoretical and practical forms of mathematics. This milieu of institutions, agents and knowledge contributes to the complexity often seen in international cooperation. The pictorial representation in Figure 1 describes the project phases, the organisations and agents involved and the types of knowledge that can be found in international cooperation.

Metaphorically, the work in international cooperation in mathematics education is akin to an ecosystem where the agents serve as organisms that learn to accommodate and adapt to each other in order for the complex ecosystem to thrive towards achieving its goals and objectives. The development of ideas and plans in this ecosystem occurs not through individual effort but rather through socially constructed discourse in both formal and informal forms (Ernest, 1991). To better understand how negotiated solutions emerge in this complex ecosystem it would be therefore be necessary to examine and understand the narratives of the agents at various levels of the projects.

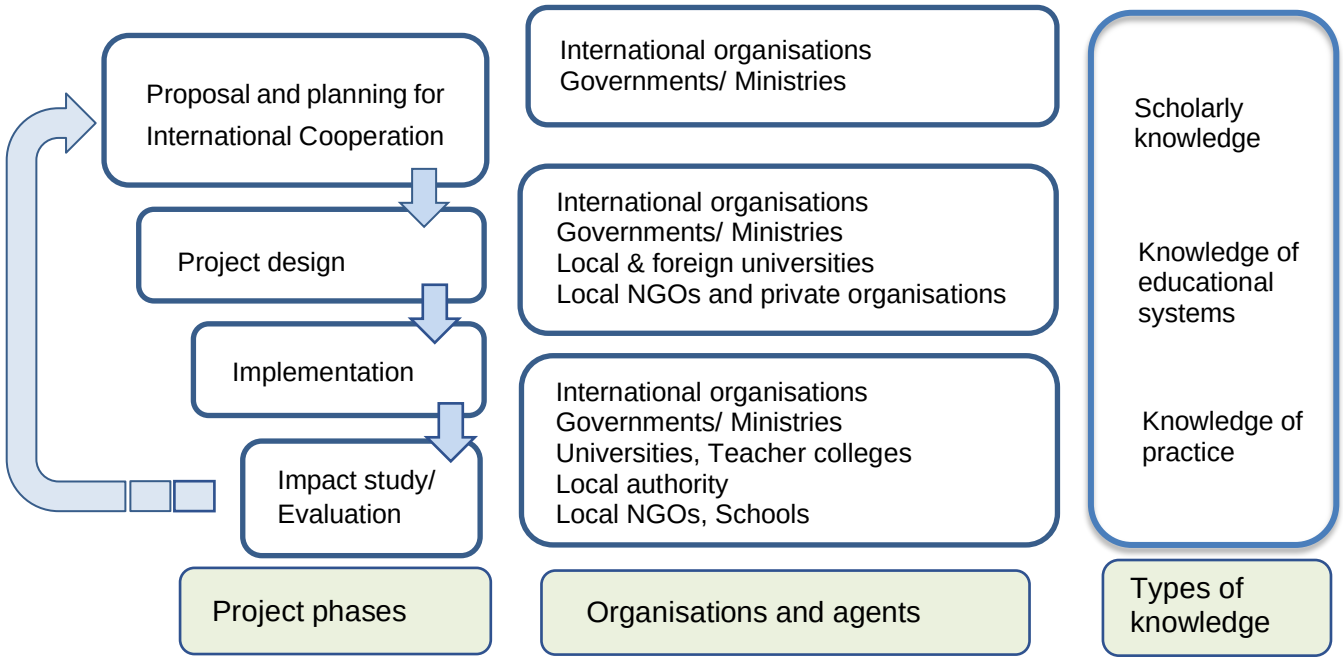

Figure 1. Project phases, organisations, agents and types of knowledge in international cooperation discourse.

ISSUES AND CHALLENGES IN INTERNATIONAL COOPERATION IN MATHEMATICS EDUCATION

Issues and challenges relating to international cooperation in mathematics education were first highlighted and discussed at the 10th International Congress on Mathematical Education (ICME 10) (Atweh et al., 2008). Most of the challenges that were discussed drew attention on emergent general issues in international cooperation and were not specifically related to mathematics education. Issues relating to mathematics education, however, could be gleaned from other literature that dwelt on cross-national and cross-cultural issues in mathematical instruction (Atweh et al., 2007; Montecillo et al., 2018; Shimizu & Vithal, 2018a; Doig et al., 2019). Together, these literatures provide a wider perspective and bigger picture that covers both international cooperation and mathematics education. The concerns and challenges affecting international cooperation in mathematics education can be categorised into two main realms. The first covers issues related to international cooperation in general which are labelled in this paper as outer issues. Outer issues although indirectly related to mathematics, are sometimes key to the solutions in international cooperation in mathematics education. The second area is labelled as inner issues which, as the name suggests, indicates its direct relation to mathematics education. Specifically, all the inner and outer issues and challenges in this paper will be discussed in relation to the professional discourse among the agents. It is by means of professional discourse that ideas and plans are argued, reconciled, reconstructed and implemented. Arguments, ambiguities, and variations in ideas and opinions emerge through professional discourse. It is also through professional discourse that reconciliations, solutions and resolutions are found. The analysis and understanding of the narratives of the agents in this discourse provide the essential elements towards providing solutions to the issues and challenges in international cooperation.

Outer Issues

Among some of the outer issues mentioned by Atweh et al. (2008) were financial aid reaching and achieving its targeted agenda, dominance of English over local languages, conflicting cultural norms and values, and the locus of authority which is often dependent on whether the agents represent the donor or recipient countries. Based on these issues and challenges raised and through our personal experiences in relation to international cooperation, the following aspects of general professional discourse were identified.

Locus of authority. The plans and implementation of projects are often influenced by the extent the voice of authority dominate professional discourse in shaping ideas, plans and implementation. The power to command authority is dependent on several abilities of the authority that are specifically related to the ability to provide or impose reward, coercion, legitimacy, expertise, reference, and information (Raven, 1992). Besides the ability to command authority, decision making in discourse is influenced by several other factors which relates to the personality of the other participants in the discourse. Generally, decision making is influenced by the dynamics of the group whether its members resist or conform to authority, or are empowered in the decision-making process. Personality traits such as adherence to traditional values, conscientiousness and agreeableness tend to support higher conformity to authority while it can be expected that moral reasoning and social intelligence tend to predict resistance to the authority figure (Bègue et al., 2015).

***Institutional agenda vs. personal agenda*.** Agents in international cooperation often play multiple roles. Some of the participants act as agents of organisations they represent while others participate in their own personal capacity. Their views may express the views of the institutions they represent yet at the same time represent their own personal beliefs or expertise. Even when the agents represent their organisations their views may not necessarily be inconformity with that of the organisations they represent. In contrast, academic scholars contribute their expertise in particular areas which are generally based their personal ideas and agenda. Moreover, the agenda of the different institutions at various levels may not always be synchronous which may lead to challenges to be resolved.

Practical ideas vs. Planned/Political ideas. The planning of projects in international cooperation is often initiated at the international level that necessitates adherence to specific performance indicators and standard operating procedures as laid out by the organisations. At times political agenda takes precedence in the planning process. International cooperation projects are very much affected by changes in the local politics. Changes in government, for example, often leads to changes in governance policies which may result in projects being discontinued. Moreover, the selection of agents and also the project site is very much dependent on political expediency. But as the international projects move into the implementation stage, the participation of more agents from other local institutions are normally sought. In projects involving mathematics, local agents who are recruited are often local teacher specialists in the subjects whose practical ideas may be at variance with initial plans that were drawn up in the initial planning stage. It is also noted that values and norms relating to cultural practices often vary both across boundaries and across different levels of the projects.

Inner Issues

In the era of globalisation and post-modernism, views about mathematics education and its aims and purposes keep evolving (Ernest, 2016; Skovsmose, 2016). Widely acknowledged as the queen of the

sciences, mathematical ability is generally accepted as a must have for the progress of the modern world (D'Ambrosio, 2016). Yet in the globalised world mathematics has now come to accommodate new roles in order to achieve a wider spectrum of aims and purposes. Mathematics education itself is continuously influenced by the commodification of knowledge where schools and universities are driven by performance and managements standards (Ernest, 2016). Given this scenario, it is apparent that the design, planning and implementation of mathematics education projects in international cooperation would be influenced by these new standards and perspectives. It is foreseeable that the agents' views about mathematics education would be just as diverse. The concern is to understand how these differing views about mathematics education are manifested and resolved in the implementation of international cooperation projects. We suggest five inner issues of mathematics education which may currently exist in international cooperation.

Behaviourist - Constructivist approaches. The debate between behaviourist and constructivist approaches was at its peak at the turn of the century. Constructivist approaches highlights designing and providing learning experiences for learners to construct mathematical ideas whereas behaviorist approaches focus on sequencing mathematical competencies for learners to master mathematical skills in the learning process. While the sequencing and attaining of mathematical competencies provide logical steps towards acquiring and applying algorithms, this instrumental approach has sometimes led to unresolved misconceptions. Moreover, an important emphasis in behavioral approaches was the extensive use of behavioural objectives to assess learning. The misuse of behavioural objectives eventually resulted in the shift towards constructivist approaches which instead, emphasise the constructions of mathematical ideas by the learners themselves. The ensuing debate had also led to changes in the perception and characterisation of teachers' functions, from authoritative to facilitative roles. This transformation from behaviourist to constructivist approaches, however, is not happening at the same pace across the global community of mathematics educators. Some are more progressive than others who may tend to adhere to the more traditional behaviourial approaches. It would be not contentious to presume that serious argument and disagreements would emerge when agents with differing views on the behaviourist-constructivist discuss, plan and decide on the suitability and appropriateness of the didactics of mathematics to be used.

General education theories - mathematics education theories. One peculiarity in the implementation of international cooperation projects is that it often brings together agents from various specialisations, some with general education background and others who specialise in mathematics education. The difference between the emphases of general education theories and mathematics education theories may be subtle but it often leads to discords in practice. The well-known mathematician Han Fruedenthal espoused disagreements in using educational research theories in designing the mathematics curriculum (Gravemeijer & Terwel, 2000). General education theories focus on general learning psychology whereas mathematics educators focus on mathematical thinking and problem solving. General educational theories such as mastery learning, experiential learning, multiple intelligence and 21st Century learning skills aims to enrich the learning environment for learners but does not emphasise the learning trajectory as envisaged by mathematics education theories where mathematising and problem solving are the main focus. The teaching strategies resulting from these different theories are at times divergent in its approaches. In planning for 21st Century skills, for example, the general education approach would emphasise managing the class to create a learning

environment that focuses on creativity, collaboration, critical thinking, problem solving, and communication. To fit into this structure, the mathematics educators would need to adapt and fit the mathematics content and processes into this paradigm. In the process, often the mathematical process like conjecturing, justifying, proofs, modelling, symbolizing and schematizing are de-emphasized.

***Pure mathematics - applied mathematics*.** In pure mathematics, mathematics is viewed as a system with its own set of axioms, rules and theories immersed in logical and deductive thinking. It has its roots on Platonism with little emphasis on applications. This emphasis on pure mathematics was the main focus of the new mathematics movement in the 1960s. While this approach keeps the purity of the mathematics discipline, it was not able to make mathematics relevant to learners in the school context (Kline, 1973). Emphasis on applications on the other hand takes the approach of mathematical ideas being developed through real life applications which would make it more interesting for school children. The balance of emphasis between the pure and applied approaches would very much depend on the ability of the learners to grasp the level of abstraction. This concern continues to be a hotly debated issue in the development of the mathematics curriculum in the global mathematics education community (Shimizu & Vithal, 2018b).

***Universal mathematics – ethnomathematics continuum*.** Ethnomathematics assumes that it is possible for practices of mathematics that to emerge from the local culture. While mathematics is often accepted as a universal value-free discipline, yet its practice among non-literate children and adults provide evidence that people are able to conceptualize mathematical ideas through their own culture outside of the school setting (D'ambrosio, 1985; Terezinha et al., 1993; Gerdes, 1994; Barton, 1996). The formal universal mathematics curriculum is therefore not necessarily the only path which makes learning mathematics more meaningful. Projects in international cooperation should be cognizant of local practices and recognize that that these practices can be incorporated into the design of the mathematics education projects. One of the challenges in international cooperation is to decide how ethnomathematical practices can be incorporated into the development of the formal mathematical instruction.

Mathematics classroom culture. One of the vital aims of mathematics education is to help teachers create a learning environment that would provide children with the opportunities to develop mathematical ideas. As a norm, there are three aspects that contribute towards this end, by designing instruction that help children engage in mathematical activities, enabling teachers engage learners in mathematical discourse, and helping learners acquire the ability to reason with mathematical tools and symbols (Cobb et al., 2011). Towards this end there have been numerous efforts to develop guidelines to help teachers develop a classroom culture that would be conducive for children to learn mathematics (National Council of Teachers of Mathematics, 2000; Lewis et al., 2009). Yet this process of building a classroom culture is complex. The aspects of culture in a mathematics classroom would include the types and sequence of classroom activities, the nature of mathematics discourse, the classroom environment and teacher-student and student-student interactions which are accepted as the norm in particular cultures. Inevitably, in constructing a vision for a mathematics classroom culture there is a need to consider the activities and practices of the learners and teachers outside classroom, perhaps even to examine the greater culture of the community. The beliefs and practices of the greater culture of the learners and teachers matter. Views of accepted classroom culture inevitably vary across countries. For example, Doig and Groves (2011) found that there were cultural impediments in

adopting the Japanese lesson study in Australia. While the sharing of teaching practices in community is widely accepted in Japan, Australian teachers view classroom practice as a private responsibility, thus creating a constraint to its implementation. One of the challenges in international cooperation is for the agents to resolve the conflicting cultural values that may impede the adoption of new teaching practices.

UNDERSTANDING THE REALITIES IN INTERNATIONAL COOPERATION

Role of Narratives in Discourse

Situations in international cooperation at times present positions of disequilibria in authority especially where donor-recipient relationships exist. This state of disequilibrium in authority and empowerment could well create circumstances that lead to bottlenecks of opinions in the implementation of international cooperation projects. Mindful of these possibilities, a pertinent first step towards resolving the issues in international cooperation would be to better understand the discourse in the complex ecosystem of international cooperation. In situations where there is an imbalance of power, a broad spectrum of views of the self towards the authority is likely to exist. This can range from taking a position of silence as a receiver of knowledge to one who is able to constructively contribute ideas and knowledge (Belenky et al., 1986). Given the existence of different dispositions towards authority, it is therefore essential that the interactions and discourse amongst the various agents be examined in order to uncover the voices that are hidden in the narratives. Understanding the narratives of the various agents would form a vital step towards understanding these interactions. Narratives are accounts or interpretations of events that are expressed to reflect particular theories or one's point of view (Bruner, 1991). Narratives abound in both formal as well as informal settings. Bruner (1991) proposed several characteristics that can be commonly found in narratives. Narratives are diachronic in nature. As opposed to synchronicity, the diachronic nature of narratives implies that understanding narratives requires the need to take into account its history and that its meaning evolves through time. Its contextual character also means that its meanings need to be interpreted within the context in which it is told. Moreover, narratives carry the narrator's intentional state, yet possess hermeneutic composability (Bruner, 1991). Narratives can be based either on true events or they could be fiction, in which case it would have the characteristic of truth likeness. Narratives are told to convey the intentions of the narrator, sometimes explicitly or at other times in subtle ways so as to protect the interest of the narrator. Viewpoints and thoughts can be subtly conveyed through narratives in situations when explicit statements are inappropriate especially when these statements could result in negative consequences. One useful way to detect a narrative in a discourse is through events where the narrator conveys a narrative which indicates a breach in canonicity or which is against accepted norms and opinions. In the area of international cooperation, in situations where agents may not wish to explicitly express their views for fear of reprisal, examining narratives could provide a pathway towards uncovering the hidden views of the agents.

Dispositions of Narratives Related to Knowledge and Beliefs

Everyday narratives cover a wide range of topics. In order to examine narratives as a pathway towards resolving the issues in international cooperation, it would be pertinent to focus on specific dispositions towards knowledge and beliefs that are found in these narratives. Knowledge and beliefs of mathematics have been acknowledged to influence mathematics instruction, not only in ways that

mathematics is learnt and applied to solve problems but also how it is taught (Ernest, 1991; Ball et al., 2008; Schoen & LaVenia, 2019). Knowledge and beliefs of mathematics provide the structural foundations of thought that allows one to learn and acquire new mathematical knowledge, and to do mathematics. In the current era of globalisation, Ernest (2016) reasoned that several aspects of mathematical knowledge and beliefs influence the way the teaching and learning of mathematics is practiced. It is our view that the mathematical knowledge and beliefs of the agents would similarly impact on the way projects in international cooperation in mathematics education are designed, planned and implemented. In view of its importance, this paper will focus on the dispositions in the narratives that relate to the various aspects of knowledge and beliefs of mathematics (See Table 1.).

Views of self towards authority. Perceptions of self and views of teaching and learning are influenced by biographical, contextual and interactional factors (Cheah, 2001; Cocklin, 1991). The context where the workplace and the home influence the way these perceptions are constructed. Situations where there is an unequal balance of power and authority cultivate perceptions that view the self as inept in constructing knowledge. Belenky et al. (1986) outlined various epistemological perspectives developed by people living and working in varying conditions where there is an imbalance of authority and power. On one end, in situations where the authority gap between the authority and the subordinates is wide, the encultured perspective is that of silence and being receivers of knowledge. The subordinate would cultivate a self-perspective as being less able to contribute towards the construction of knowledge. As the authority gap narrows and people become more empowered, the perception moves towards that of constructed knowing, being one who is able to contribute and construct knowledge. A disposition of silence or receiver of knowledge could imply taking a position of a follower during discourse. In the case of international cooperation, some possible causes for adopting this disposition are: 1) cultural view of respect for superiors, 2) ineptness in communicating in the language of discourse, 3) lack of knowledge in mathematics content or mathematical didactics, and 4) personal benefit. The reason of personal benefit succinctly differs from the other three reasons as it hints more of one's personal volition rather than an imposition by environmental factors.

Views of knowledge. The modernist and postmodernist views of knowledge are two views commonly used to describe personal philosophies of thought that emerged during the transition of society following the industrial revolution (Ernest, 1991; 2016). The modernist views knowledge as objective, depersonalised, value-neutral and easily transferable between persons; and prioritise the products of learning. This belief translates into a behaviourist didactical approach. Teaching and learning mathematics from the modernist view is therefore skewed towards fixed and predetermined procedures where the infallible knowledge of mathematics is transmitted from the teacher to the learner. This is in contrast to postmodernist view of knowledge as one that prioritises processes and the human dimensions of learning. The postmodernist view of knowledge posits that it is the learners who construct mathematical ideas in the learning process.

Table 1

Disposition of Narratives Towards Aspects of Knowledge and Beliefs

Aspects of Knowledge and Beliefs	Disposition
Views of self towards authority	Receiver of knowledge Constructed knowing - Able to create knowledge based on the context using subjective and objective strategies
Views of knowledge	Modernist view of knowledge - Knowledge is objective, depersonalized, value-neutral and easily transferable between persons; prioritize the products of learning Postmodernist view of knowledge - Prioritize the process and human dimensions of learning
Views of knowledge production (Gibbons et al. 1994)	Mode 1 – Source of knowledge is in academic disciplines Mode 2 – Source of knowledge is in the community of practitioners
Views of management	Aligning beliefs and knowledge to accomodate elements of Performance-based management Aligning elements of Perfomane-based management to accomodate beliefs and knowledge

Views of source and production of knowledge. Gibbons et al. (1994) proposed that there are two types of knowledge in regards to its source and production. *Mode 1* knowledge refers to the production of knowledge that is anchored in knowledge that is created by the community of researchers and academics within the confines of academia in a specific discipline. The production of knowledge in *Mode 2*, in contrast, theorises that the production of knowledge occurs in the real world where practitioners become active contributors in the production of knowledge. A belief in *Mode 1* production of knowledge would emphasise a belief that knowledge about mathematics is created by researchers in academia whereas a belief in *Mode 2* production of knowledge leads to a belief that practitioners play an important role in the creation of knowledge in mathematics education. Practitioners would be able to meaningfully add on and to expand knowledge especially in the practical area of the applied knowledge. Currently, there is a trend towards involving teachers actively in education projects. This involvement of teachers as practitioners in education is purposively intended to provide practical ideas to complement educational theories from academia. In this regard, having a Mode 2 disposition towards the production of knowledge would serve as an enabler for the agents as opposed to Mode 1 knowledge which would tend to inhibit the practitioner's willingness to contribute towards discourse.

Views about management. Management practice too has evolved and is currently trending towards a performance-based approach (Ernest, 2016). In performance-based management (PBM), the success and effectiveness of the project is evaluated and managed by setting key performance indicators and evaluating whether these targets have been attained. Performance-based management may or may not be aligned to management practices that prioritise educational goals. Agents in international

cooperation will thus be persuaded to adopt and adapt practices and plans so that they become aligned to performance-based management indicators, forcing them to re-examine their values and beliefs about the teaching and learning of mathematics. In response to the implementations of performance-based management, the dispositions of the agents towards management could include: 1) rejecting performance-based management which would lead to positions of conflicts, 2) accepting performance-based management by either prioritizing PBM and adapting their beliefs and knowledge of education to accommodate PBM, or prioritising their beliefs and knowledge of education and adapting the elements of PBM to accommodate their prior beliefs and knowledge.

Accessing and Analysing Narratives

There are some characteristics of narratives make accessing and analysing them challenging. It is possible to access the narratives either as first-person narratives or as third-person narratives through participatory observers. First-person narratives would require deep introspection to be carried out by the agents and the ability to articulate them either verbally or in writing. This would be possible with some of the agents at some levels in the international cooperation ecosystem. However, many agents may not have the ability or volition to do this task. There is also the issue of reliability and trustworthiness to consider. Accessing the narratives through participatory observers would be a better choice. Ethnographic approaches could perhaps offer a more naturalistic setting to access these narratives. The diachronic and hermeneutic properties of narratives could entail a long period of data collection, notwithstanding the possibility of a multitude of interpretation that needs verification. These difficulties suggest that adopting a qualitative and interpretivist method to analyse narratives could offer a more suitable approach than purely quantitative approaches. The interpretivist approach adopts the assumptions that research is bounded by value, time and context, and that there are multiple perspectives towards reality (Denzin & Lincoln, 1998). In the interpretivist approach, the researcher and the participants mutually interact and are not expected to be detached as in positivist approaches. Recommendations to ensure trustworthiness of the data analysis include prolonged engagement, persistent observation and triangulation (Lincoln & Guba, 1985). Data collection methods could include participant observations, ethnographic interviewing, search for artefacts and researcher introspective reflections (Cheah, 2000). The standards and assumptions in the interpretivist approach fit well with the context of international cooperation projects which are very often multicultural in nature.

CONCLUSION

Trends in education in the twenty-first century are changing in response to changes in society that is undergoing transformation due to economical, technological and cultural shifts. Current trends point to approaches that encourage and favor skills such as communication, collaboration, critical thinking and creativity. Thus some dispositions may seem to be in the vogue while others may be seen as traditional. The aim of examining the disposition in the narratives of the agents is not to endorse one disposition over another as being superior. Rather, the aim of examining these narratives is to clarify differences of opinions in discourse, taking into account the narratives of all the agents involved in the international cooperation projects. Examining the narratives allow us to determine the dispositions that could be cause of these differences.

The discussion in this paper offers a hopeful expectation that this discussion would lead to deeper emprirical investigations into the narratives that exist in international cooperation in mathematics education. Narratives provide a window into the thoughts of the agents in international cooperation. This will provide a deeper understanding of the issues and challenges in international cooperation leading to a resolution of these issues. Some immediate emergent questions about narratives relate to its cultural source: What narratives are specific to particular cultures and what narratives are common across culttures? How does culture influence the ways the narratives are voiced? How can we create environments that allow open dialogue so that agents can voice their narratives without fear of reprisal? What are strategies that can be used to succesfully resolve differences in the narratives? Ultimately, the aim for understanding these narratives is to strive for sustainability and improvements in the quality of mathematics education globally.

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